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# Teachers' noticing to promote students' mathematical dialogue in group work

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**Abstract**
How can teachers refine their strategies for purposefully engaging students in mathematical discussions when students are working in groups and the teacher enters an ongoing group conversation? In three educational design research cycles, four teachers collaborated with a researcher for one year to analyse, design and evaluate strategies for engaging students in small-group mathematical discussions. The idea of noticing (Mason in Researching your own practice: the discipline of noticing, RoutledgeFalmer, London, 2002; Sherin et al. in Mathematics teacher noticing: seeing through teachers' eyes, Taylor & Francis, New York, 2011) was used to organize the findings—by paying attention to aspects in the mathematical discussions and interpreting the interactions, teachers could together refine their own actions/responses to better support students' work. The Inquiry Co-operation Model of Alrø and Skovsmose (Dialogue and learning in mathematics education: intention, reflection, critique, Kluwer Academic Publishers, Dordrecht, 2004) was used as a theoretical base for understanding qualities in mathematical discussions. Ehrenfeld and Horn's (Educ Stud Math 103(7):251–272, 2020) model of initiation-entry-focus-exit and participation was for interpreting and organizing the findings on teachers' actions. The results show that teachers became more aware of the importance of explicit instructions and their own role as facilitators of mathematical questions to students, by directing specific mathematical questions to all students within the groups. In this article, by going back and forth between what happened in the teachers' professional development group and in the classrooms, it was possible to simultaneously follow the teachers' development processes and what changed in students' mathematical discussions.

**Keywords** Group work · Inquiry co-operation model · Noticing · Promoting mathematical dialogue · Student interaction · Questions in mathematics

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## Group work for mathematics learning

In recent decades, participating in mathematical discussions while working together with other students has been a preferred alternative for students to learn mathematics (Brandt & Schütte, 2010; Cobb et al., 2001; Sfard, 2015; Sfard & Kieran, 2001). The adoption of socio-cultural theories of learning (Radford, 2013; Sfard, 2015; Vygotsky, 1978) as well as the related emphasis in different curricula on the development of competences such as mathematical reasoning or communication have challenged silent individual work. Organized interactions between students are seen as central for learning. Arguments that teachers can use small-group work to provide the context for social and cognitive engagement with mathematics (Walshaw & Anthony, 2008a) have also contributed to the use of group work as an important pedagogical strategy.

However, research has shown that the extent to which group work indeed leads to better mathematics learning is questionable. Deen and Zuidema (2008) found that although students have more opportunities to talk compared to whole-class discussions, "group work proves to be not a sufficient condition for learning" (p. 171). One problem is that teachers only get a glimpse of students' learning during small-group work when they enter an ongoing conversation. For instance, Esmonde and Langer-Osuna (2013), discussed the difference between group work when a teacher is present compared to when it is student-led. Teachers get little insight into what happens when they are not present, and it is hard to gain an oversight of all groups in a classroom simultaneously. Furthermore, from the students' point of view, group work is not always seen positively. Fuentes (2013) found that student interaction is affected by lack of communication between all students in a group, poor communication patterns, and other norms that impact students' learning negatively. Horn (2017) concluded that the learning students can gain from group work should be weighed against, for instance, the social risks that students might be exposed to. One problem could be that certain students are left out of conversations, or do not participate actively, and hence have fewer opportunities to learn (Barnes, 2005; Cohen, 1994).

The role of the teacher is important for mathematical group work to promote meaningful conversations and support students' learning through monitoring student group work (Ehrenfeld & Horn, 2020; Stein et al., 2008). So, what can teachers do to support group work? Effective group work has many aspects to take into consideration, for instance:

> teachers need to select appropriate tasks that allow all students access to the mathematics; use instructional strategies that prompt participation by all students; and support high quality mathematics conversations within groups. Not accomplishing any one of these can exclude some students from participating in interactions necessary to support their learning. (Staples, 2008, p. 351)

Hintz and Tyson (2015) also found that teachers can model behaviour and encourage students by being curious about their mathematical thinking and asking questions to amplify their ideas; and Esmonde and Langer-Osuna (2013) stressed the importance of teachers carefully listening to student groups to learn more about their interactional styles as well as their mathematical thinking. When students explain their thinking, they get the opportunity to "hear and respond to one another's ideas about the mathematics" (Hintz & Tyson, 2015, p. 305).

In spite of their documented importance, teachers' routines for monitoring student small-group work are understudied (Ehrenfeld & Horn, 2020). More generally, research on small-group mathematical conversations in upper secondary school is limited (Walshaw and Antony 2008b). Staples (2008) concluded that more research is needed on effective use of group work, especially at the high school level.

Although group work is both considered important and is practiced in many classrooms, the question of what makes learning together in small groups powerful and durable still remains open (Sfard, 2015), and it is not yet researched enough if and how students can benefit from dialogic teaching (Resnick et al., 2019). In particular, it is an open question how teachers can promote discussions in mathematical small-group work, and what can be done to support teachers in doing so.

## Aim and research question

This article aims to contribute to the research on how teachers can promote discussions in mathematical small-group work. This is done by focussing on the process of change in teachers' awareness as the teachers move back and forth between their classrooms and students' group discussions, and a professional development group in which their teaching was discussed. Frequently research concentrates its gaze in one main site of practice: the classroom to study students, teachers or their interactions; or spaces of teachers' professional development and their relations to colleagues and researchers (Lefstein et al., 2020). In this article, the moving between these settings allows us to capture how teachers' changed awareness takes place through the process of noticing (Mason, 2002; Sherin et al., 2011). Noticing is often defined by three parts – attending to what is happening, interpreting it, and deciding how to act/respond (Jacobs et al., 2011; Kazemi et al., 2011). By noticing what happens in the classroom, teachers can create a "movement or shift of attention" (Mason, 2011, p. 45). However, as much research using noticing is done for pre-service teachers, focussing cognitive processes and not how in-service teachers can respond to situations or make decisions within the classroom (Santagata et al., 2021), there is a need to better understand how this movement of teachers' attention can be implemented.

In this article, the process of noticing is used in two settings that complement each other. In the classroom setting, teachers pay attention to certain pre-decided aspects of mathematical discussions, interpret them and decide how to act/respond. However, noticing is also used outside the classroom, in a professional development group setting. Here attention is directed towards aspects of the mathematical discussions. The latter are reviewed and analysed through video recordings, and the spontaneous interpretations from the classroom are complemented by interpretations connected to theories and reflections within the teacher group. Decisions on how to promote student interactions are discussed in a cyclic process in cooperation between teachers and researcher.

By going back and forth between these two settings, it is possible to follow how teachers' changes in awareness affect how students' interactions are promoted and vice versa. The teachers' analysis of students' interactions can help teachers understand more about their roles as teachers. Following teachers' encounters with students' mathematical conversations in small groups can contribute to changes in teachers' roles and deliberate actions in students' group work (Goldsmith & Seago, 2011).

The research question guiding the study is: how can teachers refine their strategies for purposefully engaging students in mathematical discussions when students are working in groups and the teachers enter an ongoing group conversation?

## Supporting and understanding mathematical interactions in group work

In order to understand how teachers can promote mathematical discussions, two analytical frameworks were connected to the noticing processes and used in this study: the Inquiry Co-operation model (IC-model) of Alrø and Skovsmose (2004), and the initiation-entry-focus-exit and participation framework, in this article called the Teacher Moves model (TM-model) of Ehrenfeld and Horn (2020). The notion of teachers' noticing is central in articulating the way teachers were moving between settings. Even though noticing rarely is used in educational design research or connected to other theoretical frameworks from mathematics education (Santagata et al., 2021), here it will be used to analyse the changes in teachers' awareness with support of the two analytical frameworks. Before going into details about the study, we briefly present the frameworks.

### The IC-model: dialogic acts to understand interactions

One way of understanding how conversations in the classroom produce opportunities to learn mathematics is proposed by Alrø and Skovsmose (2004) with their Inquiry Co-operation Model, which captures the characteristics of mathematical dialogue. The model contains eight dialogic acts (written in italics in this article) which support mathematics learning when used actively as part of a conversation: *getting-in-contact* (preparing for interaction), *locating* (understanding the problem), *identifying* (finding the mathematics in the problem), *advocating* (examining ideas), *thinking aloud* (making perspectives and thoughts visible), *reformulating* (clarifying and rephrasing), *challenging* (questioning ideas) and *evaluating* (looking back at the problem).

In previous studies (e.g., Alrø & Skovsmose, 2004; Mueller et al., 2020; Sjöblom, 2015), the IC-model has been used to analyse mathematical dialogue by looking at which IC-acts are present and how they affect the conversation. The IC-acts could be connected to students developing mathematical abilities and through the conversations learning mathematics. Malasari et al. (2020) used the model in relation to mathematical literacy proficiency, claiming that medium- or high-achieving students who receive teaching based on the IC-model learn more compared to conventional teaching. This was explained by the fact that these students maximised the *advocating* stage in exchanging ideas, asking questions and supporting each other. However, Malasari et al. (2020) found that this was not the case for low-achieving students, who were unable to maximize the *advocating* stage. Also, Weng and Jankvist (2017) found the IC-model especially useful for teachers when talking about mathematics to mathematically gifted students.

Likwambe (2018) used the IC-model to explore the nature of dialogue in a calculus lecture room from the lecturer's viewpoint, where the different dialogic acts resulted in questions from the lecturer that made students think mathematically. Likwambe did not use the model from a student viewpoint.

Sjöblom (2018) used the IC-model to study how student-to-student interaction shifted focus from finding correct answers in mathematics to also trying to understand each other's mathematical reasoning and pose questions to each other. However, when students talk to each other, it is not likely that all IC-acts are occurring (Alrø & Skovsmose, 2004; Sjöblom, 2015). Hence, teachers are left with the challenge of facilitating the emergence of a variety of dialogical acts to promote richer mathematical conversations for all students, particularly in situations where students work in groups and their conversations are expected to result in learning. Depending on many complex factors, such as the classroom context, students' cooperation and previous knowledge, what acts to promote differ from situation to situation, classroom to classroom.

## The TM-model: teachers' moves to promote student interaction

Ehrenfeld and Horn (2020) developed a framework on initiation-entry-focus-exit and participation (in this article called the TM-model) for illuminating common moves teachers work with when they engage with groups of students solving mathematical problems. In the model, "moves" are related to actions that teachers take when interacting with students working in small groups. In the framework, there are five key categories of moves (written in bold in this article), namely:

1. *Initiation* of conversation with students
2. *Entry* into student conversations
3. *Focus* of the interactions
4. *Exit* from student conversation
5. *Participation* patterns

Ehrenfeld and Horn (2020) claimed that these moves can be used to understand the complexity of group work and inform teachers' and researchers' understandings of how to support students' collaborative mathematical sense-making. However, as all classrooms are different, it is not possible to use the framework to find a model for what is *the* best practice for teachers. One part of the complexity also relates to the fact that when student interaction is the focus and is made an important activity in the mathematics classroom, teachers also give away a bit of their control of the lesson, as the students get to decide, for instance, what questions to ask, when to involve the teacher and how to work together in their groups.

The different moves are useful for studying what teachers do when engaging student groups. The *initiation* move is about how teachers approach groups and initiate conversations. The *entry* move is about what teachers first say when they enter the student conversation. The *focus* move is about what teachers focus on in the interaction with the group—is it about participation or mathematics or something else? The *exit* move is about how teachers exit the conversation and if they leave students with open-ended or close-ended questions. Finally, the *participation* move is about students' opportunities to participate actively in conversations.

## Using the frameworks together

The two frameworks have several connections. For instance, the way teachers *initiate* and *enter* conversations can be connected to the IC-act *getting-in-contact*. When analysing the *focus* of the conversations, Ehrenfeld and Horn (2020) concluded that this was not always about mathematical content, but also related to participation, instructions on tasks, or answering technical questions. When it was more about mathematics it could, for instance, be about listening or asking for a summary of the results (opening up for *reformulating, challenging* or *evaluating*), or mathematically asking students questions/answering student questions to *locate*, *identify*, *advocate* or *think-aloud*. As regards *participation*, this could again be connected to *getting-in-contact* and involving all students in mathematical conversations.

In this article, when working with teacher noticing (Mason, 2002; Sherin et al., 2011), the two frameworks contribute in different ways to understand the interaction. The IC-model can help both to decide what to attend to in the interaction and to interpret what is going on in the mathematical discussions. The TM-model can help both interpreting and designing teachers' actions/responses. According to Miller "attention is always limited" (2011, p. 53), and teachers need to make a selection of what parts of students' interactions to attend to, and in this article, attention is limited to the IC-acts and the TM-moves.

In the analysis, the IC-model helped characterizing and interpreting what happened in the mathematical conversations by identifying dialogic acts used by students and teachers. It was also used to help teachers identify what kind of dialogic acts they needed to attend to and promote, for instance *getting-in-contact* is important for *participation*. By working with IC-acts so that all students were encouraged to ask questions and include everyone in the discussions, the IC-model was considered a way of avoiding students being seen as outsiders (Barnes, 2005).

In the analysis, the TM-model was a tool to organize data and interpret the actions/responses of teachers when they engaged with student groups, for instance how they *initiated* conversations or how they worked with *participation* patterns. As it is a framework that illuminates teachers' common moves, it was used to understand and organize the story of what happened in the classroom. Thereafter, by reflecting in the teacher professional development meetings about teachers' and students' interactions and connecting this to the IC-acts, conclusions could be drawn about how teachers' actions/responses affected the conversations and what needed to be changed.

## Methodology

### Research design

In this study, the methodology was chosen to help teachers notice and become more aware of how their own actions in the context of their classrooms affected students' opportunities for mathematical discussions. Building on educational design research, EDR, (Cobb et al., 2003; McKenney & Reeves, 2012; van Den Akker et al., 2006), the study was conducted together with a group of four teachers. The teachers, who worked together in a public upper secondary school in a city in Sweden, volunteered to participate in the project during one school year since they wanted to learn more about interaction in mathematics. The study was led by the first author of this article, who had the double role of leading the teachers' professional development processes and being researcher in the EDR-project.

In EDR there are three phases in each cycle: the analysis/exploration phase, the design/construction phase, and the evaluation/reflection phase (McKenney & Reeves, 2012). The results of EDR-projects are both theoretical as well as practical, in that they are to lead to both theoretical understandings and a maturing intervention (McKenney & Reeves, 2012). The group conducted three EDR-cycles during the school year, and all three cycles were closely connected to the context of teachers' classrooms as well as their professional development group. The work was organized with clear goals and followed the structure of EDR-methodology. The overarching goal was to promote interaction with a focus on encouraging students to actively listen to each other, ask questions and present their mathematical thinking while working together in small groups. The 16–19-year-old students were attending university preparatory programmes. During the year, teachers met regularly with the first author almost every week at scheduled times during working hours and conducted tasks in their classrooms in-between.

In the teacher group, the focus was both on students' and teachers' actions, since being able to promote student interaction requires teachers to be aware of their own role in and impact on students' mathematical discussions. The teachers were involved in all three phases of the EDR-cycles—actively exploring the needs of their students in group work, deciding and designing what to try out in their classrooms and taking part in analysing the results and reflecting on what to try in the next cycle. Hence, EDR was used as a way of organizing and supporting the teachers' noticing processes. In the analysis/exploration phase, teachers attended to certain aspects of students' interactions in the classroom, and interpreted the results both in the classroom but also together in the professional development meetings. Thereafter, in the design/construction phase they planned and implemented lesson activities as an action/response to the students' needs. The design phase included plans for mathematical tasks, how teachers structured their lessons and gave students instructions, and how teachers talked with students in small-group conversations. Finally, in the evaluation/reflection phase, the teachers again interpreted and decided how to act/respond in the coming cycle.

## Data collection

The data collection included data from both the classroom setting and the teacher professional development setting. It consisted of audio recordings of 30 teacher meetings, on average 45 min each, in which the designs were planned and analysed within the group of teachers. Two of the teachers were also video recorded when the designed lessons were conducted.[1] The teachers taught mathematics in either the Natural Science (NS) programme, year 10, or the Social Science (SS) programme, year 12. Six groups of students, usually three groups in each classroom and with every group having three or four students, were included in the recordings, with separate cameras for each group. The groups were randomly chosen amongst the students that had consented to be part in the research project. The video recordings were used in the analyses within the teacher group as a way of promoting rich discussions about productive learning environments (Schoenfeld, 2017). Table 1 provides a summary of the collected data.

[1] The teacher group did not have time to watch videos from all four classrooms and hence the data collection was limited to two classrooms. The two teachers most positive to being video recorded volunteered to have cameras in their classrooms.

**Table 1** Data collection in two different settings

| Cycle | Cycle 1 (C1) Fall 2018 | Cycle 2 (C2) Spring 2019 | Cycle 3 (C3) Spring 2019 |
|---|---|---|---|
| *Classroom setting* Interactions in classrooms[a] | Four video-recorded lessons with 5 NS groups and 6 SS groups (824 min) | Three video-recorded lessons with 6 NS groups and 2 SS groups (472 min) | Six video-recorded lessons with 9 NS groups and 8 SS groups (1099 min) |
| *Professional development setting* Teacher group meetings[b] | 16 meetings (706 min) | 6 meetings (280 min) | 8 meetings (385 min) |

Recordings from the classroom are referenced, for instance, (C1, 181,024, NS2, 20:27)—Cycle 1, 24th of October 2018, student group NS2, at 20 min, 27 s into the recording

Recordings from the teacher meetings are referenced, for instance, (Teacher meeting, C1, 181,128, 41:30)—teacher meeting from Cycle 1, 28th of November 2018, at 41 min 30 s into the recording

### Methods of analysis: a two-step procedure

The material was analysed in a two-step process. In the first step, parts of the classroom data were discussed during the three cycles within the teacher group. Sherin and Dyer (2017) claim that watching and discussing videos provides opportunities for teacher learning and it is also a common method in teacher noticing studies (Santagata et al., 2021). Hence, it was a conscious choice to watch videos together, as this supported the work of noticing student interaction. The analysis together with the teachers was done differently in each cycle, depending on the teachers' choices about what to focus and notice. In the first cycle, the first author of this article selected and showed clips of student interactions, focussing on how they did or did not listen to each other and how the IC-act of *getting-in-contact* played out, since this was what the teachers wanted to attend to initially. In the second cycle, the teachers selected and watched one film each of a group work situation (approximately 1 h per film) and summarized to each other in the coming meeting how the interaction worked out. Here, the area of attention was questions that could be related to several different IC-acts. In the third cycle, short clips of what happened before, during and after the teacher entered the groups were discussed. By analysing the teachers' objectives as they designed the activities, what took place in the classroom, and then reflecting upon what happened in the classroom within the teacher group, it was possible to understand more about what happened when a teacher entered a conversation. Again, the first author of this article selected and showed video clips of teachers with focus/attention on how teachers engaged students in group work in their classrooms as this was what teachers focused in the third cycle.

The second step of the analysis was conducted by the three authors of this article once the three cycles were completed. By identifying and selecting instances of teachers' work with engagements in students' conversations in the teacher meeting transcripts, a general picture of what happened when teachers visited groups emerged. All instances in the teacher meeting transcripts that contained discussions about going in and out of groups were highlighted. These instances were then connected to transcripts of video recordings in the classroom showing how teachers' actions while engaging with groups developed during the EDR-project. Not all video recordings from the classrooms were used, instead a selection was made with a starting point of situations discussed in detail in the teacher meetings. The TM-model was used to organize the data material, and the IC-model to analyse the mathematical discussions. By using the TM-moves, teachers' actions were organized and described. Some of the teacher meetings contained questions from the first author of this article to the teachers about their views on mathematical dialogue. These questions made it possible over time to find out what teachers claimed had changed in their awareness and teaching in relation to promoting interaction in students' small-group work.

## Analysis and findings in the EDR-cycles: teachers visiting student groups

Across the three cycles, teachers tried, with various foci, to promote student interaction. The cyclic EDR-process gave the teachers a chance to rethink their actions, and to progress in how they acted when encouraging students to engage in mathematical conversations. In the following sections, we summarize teachers' noticing processes by going back and forth between the classroom setting and the professional development setting to follow the process of how teachers reflected together and became more aware and refined their strategies for purposefully engaging students in mathematical discussions. The transcripts used in the cycles have been selected from both these two settings to illustrate the processes.

## Cycle 1: Focus on listening

The first cycle did not initially focus on teachers' actions in small-group work, but rather on listening. The teachers wanted to attend to and interpret not only how students talked, but also how students' listening affected the group work. Since the teachers were not aware of what challenges to be solved in the beginning of the project, the objective in the first cycle was an open attempt to find out what affected student-to-student interactions. The teachers did not deliberately plan how to act in the student groups, since it was first towards the end of the cycle that they became aware that interesting things happened within the groups as they passed by or briefly talked to them.

There were two recurring scenarios when teachers interacted with student groups, that were selected for the video analysis at the teacher meetings. The first scenario was teachers passing by without saying anything, often being ignored by the students (not *getting-in-contact*). In the first cycle, no predetermined actions about how teachers *initiated* or *entered* student conversations were planned for in the design process. Even if a group was stuck, they often did not ask the teacher for help. On one occasion, a group had discussed for more than five minutes whether a square root is always positive or if it can be both positive and negative (*identifying, advocating, challenging*). When the teacher asked them how they were doing, one of the students who had not been particularly active in the group work, answered, "good" and the teacher went on to another group, not understanding that there were unresolved problems in the group (C1, 181,024, NS2, 20:27). However, it was not only the students who were avoiding interaction. In the analysis, teachers explained that they did not want to disturb students while they were active and that when there were discussions going on, they often choose just to listen and not to interfere to give the students a chance to finish their process (Teacher meeting, C1, 181,128, 10:35). They also said that there were other groups needing help more (Teacher meeting, C1, 181,114, 25:40), thus prioritising which groups to interact with.

The second common scenario, connected to the TM-moves about *focus* and *participation*, was a teacher entering a group with one of the students, usually the one who had talked most during group work, interacting with the teacher while the others were quiet. The group focus changed from discussion to "presentation mode", where the same student explained to the teacher either what the group had found out, or his/her own thoughts. One example was when four students were discussing how to solve, assess and grade the solution given by an anonymous student to a problem in a previous mathematics test in the class. Before the teacher arrived, all four students were active, although two of them, Student 1 and Student 2, were talking more. The following conversation took place (C1, 181,109, SS3, 27:16) when discussing how to understand and grade the student solution:

*Transcript 1*

| | |
|---|---|
| Teacher: | Have you managed to grade the task? |
| Student 1: | Yes, no, we have more… this [student] has not reached a complete solution. The student has only simplified. I do not know what to say. |
| Teacher: | Yes. |
| Student 1: | Ehhh, I could not follow what this person had done. |

| | |
|---|---|
| Teacher: | No. Can you find something that is mathematically wrong even though you cannot follow [the solution]? |
| Student 1: | Yes… I mean… it is… no… mathematically wrong? I didn't focus on that. Have you seen, said something about that? [Student 1 asks the group] |
| Student 2: | But we are supposed to write E-points and C-points,[2] right? |
| Teacher: | Yes, exactly. |
| Student 1: | Yeeees. |
| Teacher: | On every task, if you had been the teacher… |
| Student 1: | [interrupting]… okay, I would not have given this [solution] any points at all. |

In this transcript, the teacher *initiated* and *entered* the discussion with an open question to better understand what the students had been doing. Initially, Student 1 was the only one talking, trying to explain and present what they had done. Student 1 often used the pronoun "I", indicating that he was presenting only his own thoughts. The teacher consequently directed questions to the entire group, using the Swedish plural form of "you". In the middle of the conversation, Student 1, realizing that he did not know the answer, reached out to the group for help, but when he understood what to do, he tried to take over the conversation again. The teacher then *exited* the discussion and left the students to continue their work. Students 3 and 4 did not *participate* actively in the conversation with the teacher. During the teacher's engagement, little mathematics was discussed and few IC-acts coded; there was one attempt at *getting-in contact* when Student 1 reached out to the group and another when the teacher posed questions to get students to work with *locating* what to do with the scoring. It was not until the teacher left the group that the other students became active again.

In this example, it was hard for the teacher to know what had happened before he entered and after he left the conversation. When analysing the video clip, the teacher remembered that he had thought that it was mainly Student 1 who had talked during the group work (Teacher meeting, C1, 181,128, 41:30).

Analysing the *focus* of the interactions revealed that several of the student discussions, both in this group and in others, were about finding the correct answer, and depending on whether the students succeeded with this or not, they expressed either feeling good or stupid. This affected teachers' options for *initiating* or *entering* conversations. Sometimes students tried to push teachers away to disguise that they did not understand.

Analysing the *participation* patterns in the videos, teachers reflected that one important technique for getting quiet students to participate actively in the conversations, could be to pose a question that included the quiet student. This was clear both from instances when students asked each other questions and from when teachers entered the conversations and asked questions. On several occasions, the teachers found that questions made discussions continue and deepened the mathematical reasoning. Questions are also important parts of all IC-acts (Alrø & Skovsmose, 2004). One conclusion, in relation to the research question and teachers' actions/responses in the classroom, was that the teachers needed to think more about how questioning could more purposefully engage students in mathematical discussions, leading them to focus on the use of questions in the second cycle.

[2] In Sweden grades are on a scale from A to E, and in mathematics tests it is usual to give points on A, C and E-levels.

## Cycle 2: Focus on the use of questions

In the second cycle, teachers' objective was to attend to and make students more aware of the importance of mathematical questions, and in that way increase the use of IC-acts in the conversations. The intention was to get both students and teachers to use questioning as a way of changing *focus* and *participation* patterns within the groups. One reason for focusing on this was to support active participation of all students. Bishop (2012) wrote that students "do not always know how to, or choose to, interact equitably, productively, and positively" (p. 70), and that it therefore is important to have discussions about how small group work is done.

The teachers designed, as an action/response to the results in the first cycle, a meta-discussion in their classrooms—students first thought by themselves, then discussed in their small groups, and then everyone in the class discussed together. Focus was to ask students meta-questions (Pimm & Keynes, 1994) to make them more aware of participation and questions in mathematical dialogue. The discussions were about when students choose to actively participate in group discussions, whose responsibility it is that everyone is active, and what the difference is between *how*-questions and *why*-questions. When designing the meta-discussion, the IC-model served as inspiration, as different kinds of questions can be connected to different IC-acts depending both on the mathematical content and on the purpose of the questioning. Asking, "how can we find the information we need?" could be about *identifying* or *locating*, while asking, "why are you using that strategy?" could be about *advocating*, *challenging* or *evaluating*. A conclusion the teachers came to from the meta-discussion was that students often choose to be quiet when they do not understand. The teachers tried to argue that it is precisely when one does not understand that one should ask questions and not be quiet. Many students still claimed that it was better to just listen to those who actually had an idea about a solution. One of the teachers reflected afterwards that teachers raising thoughts about asking more why-questions was useful for the students in the long run (Teacher meeting, C2, 190121b, 3:50), because if it could help normalize the asking of questions and make it okay to admit that one does not know, then such a strategy could help prevent students from feeling stupid when they cannot find the answer to a question. Such a discussion could help when *initiating* conversations with the students. If students were not afraid of making mistakes, they might choose to involve the teachers more frequently when there was a need for help.

The teachers also designed an activity in which the mathematical tasks were more open than usual, so that students had more ways of solving the problems. One task was about building mathematical models in different ways to predict the future; for example, to determine how many cars there might be at different points in time. The openness was supposed to change the focus from finding the correct answer to seeing several possible ways of thinking. The teachers decided in the design phase that when they engaged with the groups, they would be aware of and attend to their own use of *why*-questions and try to use them to *initiate* the discussions and hinder students from going into "presentation mode". They wanted to challenge students to think and justify their answers and so decided to interfere a little more with the students compared to the first cycle. A strategy that one of the teachers used spontaneously (it was not planned in the teacher group), was to ask quiet students in the groups, "did you understand why X said that?" or, "I see that X is not convinced about your reasoning, can you talk more to her and explain why you reason the way you do?" This encouraged the quiet students to take part in the discussions and the teachers could promote *participation* patterns that supported more equal participation.

When designing the lesson about mathematical models, the group of teachers came up with asking "why did you chose that model?" which they considered to be a good *why*-question for the students. One of the teachers reflected:

> Maybe if they get the why question from us, then they will not have to come up with any of their own. Maybe there is a danger in it, but maybe we should pose some why-questions. Just to steer them a little in that direction. (Teacher meeting, C2, 190128, 32:11)

In the following lessons, the teachers related to questioning in different ways. One of the teachers was still mostly not actively participating in the small-group mathematical discussions and students were not involving this teacher when they became stuck on a problem. The TM-moves about *initiating* and *entering* were not really visible. The teacher tried to ask questions, but mostly without mathematical content, such as, "how are you doing?" or "are you finding something out?" Several of the groups did not understand the concept 'mathematical model' or how to work with models on their calculators/computers, but despite their uncertainties, they still did not ask the teacher, and so it was hard for the teacher to find out what they had become stuck on and thus to determine what mathematical questions to ask.

Transcript 2 is from a group with three students, discussing different models for working out how many cars there might be in year 2010 and 2018, respectively. Student 3 is trying a linear model on her calculator, while the others, although following her work, are mostly inactive. Then they get stuck on what other models they could use but do not tell the teacher about their problem, so the teacher's attempts at initiation are not successful (C2, 190,212 NS1, 19:05):

*Transcript 2*

Student 4: Okay, do we have another line or mathematical model we can work with?
Student 5: We can use those [points towards models written on the white board].
Student 4 Yes, because we got one.
Student 5: y equals c e to the power of x [$ce^x$].
Student 4: Can we work with that one?

[Teacher is passing by; all students look busy and Student 6 starts reading from the handout with the task]

Student 6: What limitations does the model you chose have?
Teacher: Have you tried different [models]?
Student 4: We have worked with one.
Student 5: We are working with that now. [Teacher walks on to another group]

In this transcript, Student 5's last claim can be interpreted in two different ways—either that the students are about to start working with more models, or that they are doing it. For the next fifteen minutes the students struggled to come up with another mathematical model and they did not ask the teacher any questions even though she walked by several times. The teacher, seeing that they were working, did not interfere. Fifteen minutes later (C2, 190212 NS1, 35:03) the following conversation occurred:

*Transcript 3*

Teacher: Have you found something out?
Student 4: We have only figured out how to work with one model.
Teacher: Yes, have you tried another model?
Student 4: No, we do not know how to work with it.
Teacher: You use your calculator [shows them].
Student 4: Okay [laughing].
Student 5: Makes sense, makes sense.

[Teacher walks on to another group]

Student 6: Wait, what did she say?
Student 5: You are doing it on your calculator. No. We are so slow.

Then the group started working. Since the group did not ask the teacher any questions on the first visit, it was hard for her to understand what the group had become stuck on. Also, in other groups in the classroom students' problem was to work out how to use their calculators correctly, something the teacher assumed they already knew how to do, since they had worked with similar tasks in Chemistry and Physics.

At the end of Transcript 3, Student 5 remarked that they were "so slow". One question here is about whether asking the teacher questions is admitting that you do not know how to do the mathematics and whether there is a social risk in asking questions. The same students said in the meta-discussion that they did not consider asking the teacher to be an option. The students in this class focussed often on finding the right answers, not on engaging in conversations as a learning opportunity. During the activity, there were episodes in which students worked individually, trying to find an answer, without interacting at all.

In the analysis afterwards, the teachers interpreted what happened in the interaction. They came to realize that their use of questions while *initiating/entering* group conversations could directly affect what happened, since the questioning helped teachers understand what students were struggling with and hence made it easier to support students' mathematical discussions. Questioning could also have an indirect effect, since the way teachers asked questions and discussed the use of questions with students could make students more aware of their own questions. Hence, teachers wondered whether they could be role models for how to ask mathematical questions. Reflecting on the results in the second cycle during the video analysis phase, teachers concluded that there were two important factors about their actions that affected what happened in the group work after their visits. Firstly, how long they stayed in the groups had an effect on how much they discovered and understood of the students' discussions. Secondly, the types of questions they asked the students affected what direction the mathematical discussion took both during and after their visit. Asking more mathematical questions revealed more information to the teachers and made it easier for them to support the students to deeper mathematical discussions. Questions could therefore affect all IC-acts and TM-moves positively or negatively, depending on how they were used. The conclusion related to the research questions in the second cycle was, that the way teachers used questions within the groups to purposefully engage students in mathematical discussions was crucial. This needed to be further investigated in relation to what happened when they visited student groups in the third cycle and acted/responded to students' interactions.

## Cycle 3: Focus on teachers' moves

In the third cycle, the objective and focus of teachers' designs were mainly about noticing what teachers should do when talking to students in small groups. In cycle two, the teachers had tried to make students more aware of the benefits of asking questions, but it was harder to change students' use of questions than their own. Therefore, in the third cycle, the teachers chose to focus on their own role and how to engage student groups in mathematical conversations in a more structured way. The teachers discussed how to *enter* and *initiate* conversations with groups, how to get students to *focus* on mathematical issues and how to get everyone to *participate*. There was no focus on the TM-move about how to *exit* conversations in this study.

The design for this cycle included teachers predicting what mathematical questions it would be possible to ask in relation to the difficulties they anticipated students would have within particular mathematical content areas. These questions would then be used when teachers *entered* the groups. For instance, in a task on inscribed angles, the teachers knew that students often find it hard to understand what arc the inscribed angle originates from and so they prepared a mathematical control question on this to *initiate* a mathematical discussion. They thought this would help the *locating* and *identifying* processes, as well as *challenge* students' thinking if they were on the wrong track. The control question could then be used as a first question within each group that would help the teachers to see whether the students had understood the problem and to find an appropriate way into the students' mathematical conversations.

Before the activities, groups were rearranged to consist of only three students instead of four. This was to influence *participation* patterns, as it was assumed that it would force students to sit closer together and avoid talking in pairs (as a result of the first two cycles). Within the groups, teachers were to attend to and actively engage all three students in discussions while they were present within the group. A quiet student would get a direct question. The teachers' focus was on *getting-in-contact* and getting all three students engaged in *advocating* and *challenging*, by asking them questions that promoted these dialogical acts. By *reformulating* what students had said and posing a question, this was to help them to *think aloud.* They decided, as a general rule, to talk to the students as a group and not individually (Teacher meeting, C3, 190,401, 12:30).

In the third cycle, teachers followed the same type of strategy when planning three mathematics lessons in succession. The result was that students experienced the same kind of arrangement concerning tasks and ways of working together three times in a row. The assumption behind this was that it is not enough to try an arrangement only once, and that students need to grow accustomed to a particular way of working. Teachers constructed group activities that consisted of an individual task to prepare everyone for interaction, followed by a group task (involving mathematical *why*-questions) in which students discussed together, and a summary in which the group had to think about what they had learnt and what questions they had used. Teachers also tried to limit the time students were given for the group work, so that periods of inactivity would be minimized. In the first round of filming in the third cycle, there were several examples of what happened when the teachers did not achieve the intention of talking to all three students. In one instance, when a teacher was talking to two of the students in a group, one student appeared to be listening to the conversation and nodded and acted as if she understood, but as soon as the teacher left the group this proved not to be the case. In Transcript 4, this group of students was discussing how to find coordinates that fulfil three mathematical inequalities simultaneously (C3, 190,503, SS2, 17:24):

*Transscript 4*

Student 7: Now we must find three different coordinates that are part of the system of inequalities.
Student 8: What does that mean?
Student 7: Is it just three random coordinates? … [inaudible]

[Teacher arrives]

Student 8: In which area? Is it in this area? [to the teacher]
Teacher: Yes. If you fill in the handout exactly as you did in your individual papers.
Student 8: How is it now? You filled in down here [to Student 7; filling in one area]. I filled in here [filling in another area]. And you [looking at the paper from Student 9] filled in here [filling in the last area].
Student 7: Then it is that triangle, right?
Teacher: Yes.
Student 8: Yes, exactly. It is that one? [points to triangle]
Teacher: And now we see why we fill in the area that we are not interested in. We see easily what area is left.
Student 8: God, this is good.

[Teacher walks on to another group]

Student 8: Okay. Three coordinates here.
Student 7: They are points. Just mark three points somewhere so we can see where they are.
Student 9: But wait. Now I do not understand what we were doing.

In this transcript, the conversation was not *initiated* by the teacher. Instead, students posed a question to the teacher as he passed by. The students wanted help to *locate* what they should be doing. Student 8 led the discussion and the teacher offered information and supported the process. The teacher talked to the entire group, but only Student 7 and Student 8 were taking part in the discussion, while Student 9 followed what Student 8 was doing on the paper in front of her. Student 9 was quiet during the conversation and the teacher left without listening to her or asking her questions, not knowing if she had understood or not.

When the group of teachers realized this during the video analysis, it encouraged them to be even more aware of the *participation* patterns, and especially how quiet students acted. If students were quiet, teachers interpreted the situation and considered that this might be because they did not understand or because they might have a question. They decided to act/respond to this by making sure to involve everyone in the conversations while visiting the groups in the following lessons. The analysis showed how teachers were more aware of the importance of talking to all students and subsequently did this purposefully when visiting the groups. This gave them the opportunity to spend a little longer time in the groups, and make sure all students were on track.

When reflecting on the results in the third cycle in relation to the research question, it was not only how long teachers spent in the groups, but also what they attended, interpreted and *focussed* on when talking to the students that affected their opportunities to engage students in mathematical discussions. They did not only ask "how are you doing?", but instead planned purposeful specific mathematical questions that would get students to actively discuss mathematics. Teachers often used students' names as a response to group work that did not include all students. For instance, when one student was in 'presentation mode', the teacher calmly asked another student in the group, "X, what do you think about this?", with the intention of getting everyone engaged in the discussion.

## Summarizing what was noticed

In the three cycles, results from going back and forth between the classroom setting and the professional development setting were used to answer how teachers can refine their strategies for purposefully engaging students in mathematical discussions. These results emanate from the analysis of the EDR-cycles and are both theoretical and practical as they should be following EDR-methodology (McKenney & Reeves, 2012). In Table 2, both the professional development setting and the classroom setting inform the conclusions as part of teachers' noticing processes across all three EDR-cycles. The column "attending" summarizes what the teacher group focused on in the three cycles, "interpreting" what was discussed in teacher professional meetings, and "acting/responding" what they decided to do or think more about in the future. The results in the latter column would not have been achieved if the previous two had not been scrutinised.

In the next sections, we discuss what theoretical and practical results came out from the teachers' noticing processes across the three EDR-cycles.

### Theoretical results

In the study, the more theoretical results relate to the use and combination of the two theoretical frameworks (Alrø & Skovsmose, 2004; Ehrenfeld & Horn, 2020) to clarify what was happening both in the classroom setting as well as in the teacher professional development setting. The changes in teachers' awareness were a result of the noticing processes in both settings. In some ways, the analysis became more complex, since it included larger amounts of data, but since interaction in mathematics classrooms is multifaceted, the combination of being in different settings with the two theoretical models could be one way to deepen the understanding of this complexity.

In the study, the IC-model helped to create a common understanding of what is quality in mathematical discussions and identify what parts of students' interactions teachers wanted to encourage. Across the three cycles, paying attention to the teachers' as well as to students' engagement in IC-acts was a way of refining teachers' awareness of different options for promoting small-group discussions. Across the cycles, much attention was paid to the IC-act of *getting-in-contact*. When everyone was involved in the conversations, teachers considered students' activity to be increasing their opportunities for learning mathematics. It also improved teachers' possibilities to attending and interpreting students' mathematical thinking and understanding what was mathematically challenging for them.

**Table 2** Summary of teacher noticing across the three EDR-cycles

| Cycle | Attending | Interpreting | Acting/responding |
|---|---|---|---|
| 1 | Open start—what happens in the interaction? | Students sometimes avoid talking to teachers to not admit that they are stuck<br>When questions are asked, either from students or teachers, this can include more persons in the discussions and change interaction patterns | Think more about how students' and teachers' use of questions can influence the mathematical conversations<br>Design meta-discussion about questions in which both students and teachers can use more mathematical questions |
| 2 | Students' and teachers' questioning (or lack thereof) | It is hard to make students use questions, it might be easier to change teachers' actions<br>When specific mathematical questions are asked, students' reasoning is deepened | Ask more specific mathematical questions, connected to areas that teachers know are problematic to students<br>Include all students in the conversations<br>Make students work in groups of three to avoid them dividing work between pairs |
| 3 | Teachers' moves while visiting student groups | The importance and complexity of including all students in conversations to ensure everyone is following the conversation | Always talk to all three students within each group and include students that are quiet<br>Think more about the importance of clear instructions, how teachers can facilitate students' questioning, and how to act when entering a mathematical conversation (connected to for instance TM-moves) |

By purposefully asking how- and why-questions, students were engaged in *locating, identifying, advocating, challenging* or *evaluating* depending on what mathematical content and purpose the questions had. When teachers asked students questions, they often *reformulated* what students had said, and asked them to *think aloud.* Hence, the IC-model provided insights for the different kinds of questions teachers ask students, which could help them to refine their strategies for promoting mathematical discussions.

The TM-model was helpful for interpreting and understanding the findings about what teachers did to promote student interaction. Both students and teachers *initiated* conversations. The video analysis revealed that students were sometimes reluctant to talk to the teachers, as they did not want to admit that they did not know what to do. Another result, however not visible in the codes of the TM-model, was that a good way to prepare for an *entry* was to start a conversation with a specific, pre-planned mathematical question, connected to some part of the problem-solving process that the teachers anticipated might be difficult for the students. There were many similarities in this study to Ehrenfeld and Horn's (2020) study with respect to *focus*, with findings about how teachers asked both mathematics and non-mathematics-related questions. For instance, the mathematics probing in the model could be related to how teachers in this study worked with how- and why-questions. As in Ehrenfeld and Horn's (2020) study, much of the students' focus was on results and finding the right answers. This was often because teachers frequently asked for results when they engaged with the groups, but students also did this in their conversations without the teacher, revealing that they put a lot of effort into finding the right answers. When the *participation* patterns did not include all students, teachers concluded that this could lead to some students not understanding what was going on. While Ehrenfeld and Horn (2020) did not always include everyone in the conversation through the TM-moves, the teacher group in this study felt that in their context it was important before *exiting* the group to talk to all three students to make sure that the understanding was there.

Both the IC-model and the TM-model are conceptual frameworks that can make visible the complexity of teachers' engagement in students' mathematical group work. By analysing what IC-acts appeared in the mathematical dialogue and what TM-moves teachers made when entering an ongoing student discussion, it was possible to follow and draw conclusions about how teachers' actions affected and promoted student interaction, and hence what teachers needed to do to refine their teaching. Both models informed the noticing-process, and hence, another result of this study is the combination of theoretical frameworks from mathematics education with teacher noticing as a way of understanding mathematical dialogue.

## Practical results

The more practical results in this study relate to the actions/responses that the teachers worked with as a result of the structured noticing process. These are results connected to the specific EDR-cycles and the specific focus areas chosen by the participating teachers, and hence, the results must be seen in relation to the context of their classrooms and their professional development group. As the summary in Table 2 shows, the practical results of how teachers worked with refining their teaching, consisted in teachers becoming more aware of the importance of clear instructions, trying to get an overview of all groups and their own role as facilitators of mathematical questions, and to listen not only to the students talking but also to the students being quiet and involve them in conversations. Also, one conclusion was that a group size of three students in a group promoted cooperation between all students within the group and also made it possible for teachers to talk to all students. The teachers realized the importance of purposefully asking mathematical questions to all students, as well as planning their instructions and actions when entering a group discussion.

The noticing process in itself can also be seen as a practical way of changing teachers' awareness when it comes to promoting students' mathematical group work. Comparing the cycles shows that in the first cycle the focus of attention was unclear, changing to become more specific for each cycle. Considering that there always are limitations in what to attend to (Miller, 2011), it seemed important to choose the focus areas by interpreting what happened and step-by-step trying to improve the actions/responses and specificity in coming cycles.

In the interpretation stages of noticing, teachers became aware of a variety of factors affecting the small group work, both when they were present in the groups and when students worked alone. The video analysis process revealed certain aspects affecting students' interactions that were hard for teachers to recognize when visiting groups for a few minutes, which is in line with Schoenfeld's (2017) claim that video enables teachers to capture and discuss classroom phenomena that they otherwise might miss. Hence, for teachers to be part of the analysis process, it was important for them to understand the challenges related to mathematical conversations that occurred in the classroom. As in Fuentes' (2013) study, there were problems with lack of communication between all students and sometimes poor communication patterns, for instance, with students not participating actively or going into 'presentation mode' when teachers were visiting the groups. Another problem was the norms that impeded students' interactions, such as students only focussing on finding correct answers or feeling stupid when they could not solve a task. These problems were clarified and interpreted at the teacher meetings when analysing video recordings. Thereafter, this resulted in the practical action/responses.

## Concluding remarks

Previous research presents both benefits and problems with small-group work in mathematics. Although group work can provide an engaging context for mathematics learning (Alrø & Skovsmose, 2004; Walshaw & Anthony, 2008a), there is uncertainty about what happens when the teacher is not present and how teachers can best promote student interaction (Ehrenfeld & Horn, 2020; Sfard, 2015; Staples 2008; Walshaw and Antony 2008b). This article contributes to the field by analysing how a group of teachers working with noticing student interaction (Mason, 2002; Sherin et al., 2011) and using EDR-methodology, can refine the way they purposefully engage students in mathematical discussions. This is connected to attending certain aspects of the interaction, interpreting what is happening in the interaction, and deciding how to act/respond differently as a teacher.

In addition to many other studies that either focus the classroom setting or the professional development setting, this study moves back and forth between both settings. In contrast to for instance Ehrenfeld and Horn's (2020) study, this study focuses not only what is happening in the classroom, but also, by following discussions in teacher meetings, why it is happening. This gives insights to how teachers' noticing processes can be connected to changing teachers' awareness and promoting students' interactions in mathematics. It also contributes to the identified gap in research (Santagata et al., 2021) regarding the third step in noticing (responding/acting), as the setup with the two settings creates a possible way of making informed decisions and try new ways of teaching across the EDR-cycles.

However, changing the way teachers act in classrooms is a long-term process that is not easily achieved. By working with professional development in a structured way, the teachers were able to reflect upon their own role in relation to the students, which previous research claims is important to succeed with school improvement work (Harris, 2014; Timperley, 2011). Thus, this project should not be seen as a finished quest, but rather as the beginning of a process.

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