Degree Project with Specialisation in Mathematics

A comparative study of the probability content in upper secondary school textbooks in Sweden and China

En komparativ analys av sannolikhetslärande av svenska och kinesiska matematikläromedel för gymnasiet

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Abstract

The purpose of this study is to examine the textbook contents pertaining to probability content at the upper secondary school in Sweden and China. Four mathematics textbooks are analysed with respect to three research questions: 1) what are the similarities and differences of the probability content, 2) how are the worked examples of the probability content presented with respect to Polya’s problem-solving model, and 3) how do the practice exercises in the selected textbooks differ in relation to mathematical, contextual and performance requirements in the selected textbooks in Sweden and China. Also, three existing frameworks are applied for implementing the comparative content analysis. The results show that the Swedish and Chinese textbooks are similar in structures but differ in visual appearance and mathematical language usage. It is also found that there is a larger number of probability exercises in the Swedish textbooks, a total number of 189 practice exercises comparing to a total number of 86 exercises in the Chinese textbooks. The Swedish textbooks are characterized with student-oriented material, covering more problem-solving modelled examples and procedure-focused exercises, while the Chinese textbooks are more subject-related covering exercises that require a more problem-solving process, mathematical reasoning and analytical thinking skills. Future research can focus on areas, such as problem types in the mathematics textbooks, problem-solving process, programming knowledge in upper-secondary school, and exercise problems that require different cognitive levels in the Swedish textbooks.

Keywords: China, comparative content analysis, Li’s dimensions of problem requirements, mathematics textbooks, Polya’s problem-solving model, probability, qualitative study, Rezat’s microstructural levels, Sweden
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1. Introduction

There have been some changes in mathematics education in the Swedish curriculum for upper secondary education during the last decade. These changes are, for example, the introduction of different mathematics momenta (Spår A, B and C) in 2011, the emphasis from heavy tedious memorization and drilling of mathematics formulas to problem-solving and integrating the mathematics problems together with concepts into daily life. The problem-solving technique is not only a core requirement in the curriculum but also one of the most significant abilities in mathematics learning and teaching.

The subject probability is a fundamental tool in applied mathematics and mathematical modelling and it is being taught in the mathematics curricula in Sweden. The students are supposed to be familiar with using the language of probability, understanding the concept of possible outcomes, random events and the calculation of likelihood. Unlike many other mathematics topics, which require deriving from formulas or theorems to apply calculation, the subject of probability involves sufficient knowledge or information of an event in order to compute the likelihood, the expectation and correlations. It is common for textbooks to combine probability and statistics together because probability refers to predicting the likelihood of future events while statistics refers to analysing the frequency of earlier events. In studying probability at the university, the term Stochastics model is used to refer to the combination of probability and statistics. Probability and statistics are one of the core contents apart from number system and arithmetic, algebra, geometry and problem-solving in senior secondary mathematics in Sweden (Skolverket, 2011).

The Third International Mathematics and Science Study (TIMSS) analysis consisted of a huge content analysis of curriculum guides and textbooks. This study demonstrated that textbooks act like the potentially implemented curriculum between the tripartite model of curriculum. The research on textbooks has indicated that mathematics textbooks serve as an important instrument for both teachers and students and it can be viewed as a bridge between the national goals and the practice (Valverde, Bianchi, Wolfe, Schmidt, and Houang, 2002). Valverde et al. (2002) expressed that textbooks
were considered “a key role in mediating between systemic intentions and classroom instruction” (Valverde et al., 2002, p. 4). Figure 1 below shows the relationship between the intended curriculum, which compose of the national aims and goals, and the implemented curriculum, which are the classroom practices, activities and strategies.

![Diagram of the tripartite model](image)

**Figure 1: Textbooks and the tripartite model. (Source: Valverde et al. 2002, p. 13)**

After studying in Sweden for several years, I soon will finish my teaching education in mathematics at Malmö University. As a Chinese, I find similarities in both the Swedish and Chinese mathematics classes, for example, the teachers use the textbooks as a guideline for lecture planning, teaching and preparing for the national exams while the students use the textbook exercises for learning, reviewing and building up the understanding of knowledge in mathematics class in both countries. One of my professors once told me that if one wanted to know a subject well, he or she should have written a thesis about it. As a mathematics teacher-to-be, I want to know more about mathematics textbooks, such as the characteristics of the contents and how do the worked problems and exercises assist teaching and learning in the class. With this intention, I analyse and compare the mathematics textbooks between Sweden and China in this thesis project to gain new pedagogical perspectives.
2. Purpose and research questions

Most of the textbooks usually provide worked examples for explanation together with the practice exercises and review questions for training the acquired knowledge. These features make textbooks an important role in the teachers’ pedagogical instruction and the students’ achievement in mathematics. The focus and importance of the textbooks make it valuable to study and compare the content. In most countries, the textbooks are the sole source of material for studying mathematics. According to Valverde et al. (2002), Nordic countries rely more on textbooks than other countries. Different studies have been made concerning the study of textbooks for example curriculum studies, classroom studies and comparative studies internationally. Despite the fact that the textbooks are imperative, little research has been done in Norway and other Nordic countries in terms of mathematics textbooks studies (Grevholm, 2011).

In a thesis project, Wallén (2018) studied the change and development of probability and statistics content of the upper secondary school in Sweden through comparing three different versions of mathematics textbooks between the year 2000 and 2018. The findings showed that the content of probability and statistics hasn’t changed much over the last two decades. The topic probability has gained much more recognition in the mathematics areas. There is a limited amount of study on the subject probability, not to mention a comparative study between Sweden and China. Therefore, the study of probability is the focus of this study. Some interesting questions have occurred during the pedagogical internship in my teaching education. For example, how is the probability topic treated in Swedish mathematics textbooks? How do the worked examples and practice exercises help develop the student’s competences like problem-solving and mathematics reasoning? However, the primary focus in this study is to find out how are the subject probability presented at the first year of upper secondary school in Sweden with the comparison to China, instead of seeking why the textbooks are presented the way they are.

The primary purpose of this study is to analyse and compare four selected textbooks for the teaching and learning of probability knowledge at the upper secondary school in
Sweden and China. Both Sweden and China depend heavily on textbooks in mathematics class. Also, the sequence of both countries’ textbook shares similar structures, for example first presenting the theories and then the worked examples next followed by the practice exercises. It is hoped that this study can present some new perspectives through exploring the treatment of probability topic, worked examples as well as practice exercises of the representative mathematics textbooks for both countries. More specifically, the main research questions of this study are as follow:

1. What are the characteristics of the probability content in the selected mathematics textbooks in Sweden and China?

2. How are the worked problems of the probability content presented with respect to Polya’s problem-solving model?

3. How do the practice exercises in the selected textbooks differ in relation to mathematical, contextual and performance requirements?
3. Theoretical perspective

This section elaborates the theoretical parts used in this study and will be presented in the following order: a review of probability; some fundamental probability concepts; teaching and learning probabilistic thinking; the selected education systems and their curriculum studies.

3.1 A review of probability theory

There are various explanations of probability theory, but Konold (1991), Shaughnessy (1992), and Borovcnik, Bentz and Kapadia (1991) identify it with the help of mathematical history into three conceptions: classical, frequentist and subjectivist probability. Classical probability also called theoretical probability, is when an event assumed to be “the ratio of the number of alternatives favourable to that event to the total number of equally-likely alternatives” (Konold, 1991, p. 142). This theory is limited to the experiments such as coins, dice and spinners (Konold, 1991) that the chances of each event occurring are equal or equiprobable. Frequentist probability, also known as empirical or experimental probability, claims that probability can only be revealed through observations. In mathematics, it involves the theory of limit and convergence (Shaughnessy, 1992). Subjectivist probability is that everyone has a degree of belief, and the probability is a measure of a person’s belief in an event (Borovcnik, Bentz & Kapadia, 1991).

3.2 Some fundamental mathematical probability concepts

In this section, we will present some basic mathematical probability concepts that are relevant for the senior secondary school. Probability often refers to the chance that an event will occur and it can be expressed as a ratio, fraction, decimals or percentage. Let \( \Omega \) denotes the sample space, which contains the set of all possible outcomes from a random experiment. \( A \) is an event that occurs in the sample space, such probability of any event \( A \subset \Omega \) is given by \( P(A) \). \#A indicates the number of elements in the event A. When \#\Omega<\infty, we define the probability of event A as:
The probability measure obeys that $0 \leq P(A) \leq 1$ and $P(\Omega) = 1$. As for the calculation of probabilities, there are four basic operations ($+, -, \times, \div$):

**Addition:** If events $A$ and $B$ are mutually exclusive, which is $A \cap B = \emptyset$, then the probability of $A$ or $B$ is defined

$$P(A \cup B) = P(A) + P(B)$$

**Subtraction:** The complement of $A$ can be written as $A^c = \Omega \setminus A$ and the probability of the complement of $A$ is,

$$P(A^c) = 1 - P(A)$$

**Multiplication:** The events $A, B$ are said to be independent if

$$P(A \cap B) = P(A)P(B)$$

**Division:** If $P(B) > 0$, the conditional probability of $A$ given $B$ is defined by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

### 3.3 The selected education systems

Although Sweden and China have some similarities in mathematics education and school system, they differ in many aspects, for example cultural, political, economic and educational system and students’ performance in mathematics. Sweden has a decentralized education system while China a centralized one. The decentralized education system means that the municipalities have the responsibility of the personals in the schools.

Upper secondary education is non-compulsory and comprises three years of schooling from grade 10 to 12 in Sweden, following nine years of compulsory primary and junior high school. The same school system applies to China. In Sweden, there are 18 national programmes in the senior secondary school, consisting of 12 vocational programmes and 6 programmes preparatory for higher education including natural science, humanities programme, etc. (OECD, 2015). The students will study different mathematics momenta depending on their programmes in Sweden. The Chinese
mathematics curriculum structure is divided into two major categories: senior high school and senior vocational school. In the first year of senior secondary education, all the students need to study four compulsory mathematics courses irrespective of which programme the students choose (Yang, 2012).

The mathematics textbooks in Sweden are chosen by the school or the individual teachers while required to follow the intended curriculum. On the other hand, the selection of textbooks in China is chosen by the provinces or the local schools from a series of textbooks, which are required to be approved by the Ministry of Education of China, published by the appointed publishers. The mathematics textbooks in Sweden are thicker, providing abundant material for teaching and learning. With the idea that the textbooks are the essence of knowledge the students are supposed to learn and understand all the essential material. The textbooks in China are thus much more compact. In China, the teaching is carried out in a large class size of 40 to 60 students depending on the provinces while in Sweden it is much smaller. The teachers have more opportunities to assist individual students during mathematics classes in Sweden comparing to China.

3.4 The curriculum goals

Since the 1960s, there has been an increasing recognition that probability and statistics should be included in the mathematics curricula for both primary and secondary school. Bruner (1960) stated the following in his research *spiral curriculum*:

If the understanding of number, measure, and probability is judged crucial in the pursuit of science, then instruction in these subjects should begin as intellectually honestly and as early as possible in a manner consistent with the child's forms of thought. Let the topics be developed and redeveloped in later grades.

(Bruner, 1960, p. 53-54)

The topic probability has gained much more recognition in the mathematics areas in recent years. In Sweden, apart from understanding and applying mathematics
knowledge, the students will need to prepare and pass a national exam (Nationella prov) in the central contents. Thus, it is important to follow the requirements written in the intended curriculum. The requirements for probability content in the first year for the natural science programme are that the students should understand:

The concepts of dependent and independent events and methods for calculating probabilities in multiple-step in a random experiment with examples from games, risk and safety assessments.

(Skolverket, 2011, p.105, my translation)

The requirements for probability content in the Chinese curriculum are:

I). Events and probabilities: 1) understand the uncertainty of random events and the stability of frequency, definition of probability and the difference of frequency and probability; 2) understand the mutually exclusive event and be able to use the addition operation in probability.

II). Classical probability: 1) understand the classical probability and the calculation formulas; 2) can calculate the number of events and the probability of occurrence of some random events.

III). Random number and geometric probability models: 1) understand the definition of a random number and can use simulation to estimate the probability; 2) understand the definition of geometric probability models.

(Ministry of education of the People’s Republic of China, 2003, p. 10, my translation)

It appears that China has more extensive and specific requirements for the probability content while it is more concise and less content are required for the Swedish curriculum. Probability can become very complex, especially when it is applied in a wide variety of areas. Thus, the curriculum for probability should cover a wide range of learning areas. It is shown above that the requirements for probability content are very comprehensive and strict for the Chinese intended curriculum. It is constructive and profitable for the teachers and students to have clear and explicit goals written in the curriculum, even though it minimizes the teachers’ creativity and space for preparing the lessons. Differently, the requirements for the probability content in Sweden are
concisely written and have fewer requirements. It gives the teachers much more flexibility and space to interpret the goals.

Both the Swedish and the Chinese mathematics’ intended curricula have seven goals aiming to develop an appreciation of mathematics in its scientific, cultural and practical aspects (see Appendix 1). There are more similarities than differences in the goals. For example, both countries focus on the ability of problem-solving, the implication and reasoning of mathematics. In Sweden, it is stressed the skill of communicating mathematics while it places more emphasis on individual knowledge process and acquisition in China. Data processing is included in the Chinese curriculum goals, where the students need to collect, organize and process big data to extract useful information regarding research questions in order to make a reasonable judgment.

3.5 Research reviews

In this section, a study of previous research will be presented. The search of literature is implemented through content analysis and comparative textbook study using databases such as ERIC (Education resources information centre), ResearchGate and library resources.

Olaniyan, Omosewo and Nwankwo (2015) carried out an investigation of 120 senior school students and they were interested to find out the students’ achievement when exposed to Polya’s (1957) problem-solving model comparing to those who exposed to lecture method. The result showed that the students who were exposed to problem-solving model had better performance. It was encouraged by Olaniyan et al. (2015) that Polya model should be promoted in teaching and learning of Physics in senior secondary school as well as textbooks should use the problem-solving model when editing.

Rezat (2006) used the analytical and comparative study to investigate the structure of eight German mathematics textbooks. He was interested in finding out the content structure and characteristics in the textbooks. Rezat analysed these textbooks in terms of his five microstructural elements: content, linguistic and visual characteristics,
pedagogical functions and situative conditions (Rezat, 2006). His study showed that the structure of the examined textbooks shares more similarities than differences, for example, the structures are almost in the same sequence.

In order to find out the characteristics of the integer addition and subtraction problems, Li (2000) compared five American mathematics textbooks and four Chinese textbooks using three dimensions of problem requirements: a) mathematical features, b) contextual features and c) performance requirements. The findings showed that both countries had a similar amount of practice problems that characterized with purely mathematical context, but the problems in Chinese textbooks were of the higher level of mathematical content. Anyhow, the American textbooks covered more varied problem requirements and conceptual problems than Chinese textbooks according to Li (2000).

Also, Özer and Sezer (2014) made a cross-cultural comparative study of the practice questions in the 8th-grade mathematics textbooks between three countries: Turkey, Singapore and America based on Li’s framework. The idea was to discover how to improve the 8th-grade mathematics textbooks. A total of 6,772 questions were analysed from the three countries. They proposed “to increase both the number and variety of questions in the Turkish mathematics books, especially those with a high level of cognitive demand” (Özer and Sezer, 2014, p. 419).
4. Method

In order to attain the objectives of this study a comparative content analysis in which a qualitative research approach is used. In this section, the methodology, the selected materials, the analysis framework and the implementation, limitation and reliability in the study are described.

4.1 Methodology

The empirical data of this research is the probability content from the four representative mathematics textbooks. This study uses a qualitative approach, implementing a comparative content analysis of the textbook contents to fulfil the purpose of the research.

Content analysis is widely used in social science and educational analysis with the aim to study the content of research materials in order to gain new research perspectives. Accordingly, the application of content analysis is valuable for education with respect to the understanding and evaluation of important content as well as improving the educational purpose. According to Bryman (2012), comparative content analysis is a research technique for comparing and studying two different cases with similar methods and the purpose is to “seek explanations for similarities and differences or to gain a greater awareness and a deeper understanding of social reality in different national contexts” (Bryman, 2012, p. 72).

According to Hsieh and Shannon (2005), a qualitative analysis technique goes beyond quantifying words in the text like the quantitative content analysis does. The qualitative content analysis aims to “examining language intensely for the purpose of classifying large amounts of text into an efficient number of categories that represent similar meanings” (Hsieh & Shannon, 2005, p. 1278). With the help of inductive reasoning, Hsieh and Shannon (2005) classified three methods for qualitative content analysis. The conventional qualitative content analysis refers to coding categories derived directly and inductively from the original data. Directed content analysis method refers to when the
researchers code the text using an existing theory and relevant research framework as
guidance. The last method, summative content analysis, begins with counting some
words and then aiming for latent content analysis in the text.

The qualitative research approach is an inductive, interpretive and constructive method
according to Bryman (2012). The empirical analysis in this study implements a directed
content analysis and is carried out in the following way: first, only the probability
content in the four representative textbooks are reviewed carefully at least three times;
next, the relevant content such as the worked examples, the practice exercises and
solutions are evaluated and classified using the existing theory and frameworks as
guidance. All classified and identified material is then written down in a table for
further validation and comparison.

4.2 Selection of material

The empirical data for this study is the four representative mathematics textbooks used
for the students studying the natural science programme in the first year of the post-
secondary school in Sweden and China. Among these four textbooks, two textbooks are
from Sweden and two from China. The most common used mathematics textbooks in
the upper secondary school in Sweden are, for example, Matematik 5000, Origo,
Exponent and M-series. Two of the Swedish textbooks Exponent 1c (Gennow,
Gustafsson and Silborn, 2011) and Matematik M 1c (Holmström, Smedhamre, and
Sjunnesson, 2012) are chosen in this study because they were used in the school where I
had my pedagogical internship. Among the Chinese textbooks, the best known and most
used textbooks in China are selected: the People’s Education Press B version (Gao,
2007) and Beijing Normal University Publishing Group (Xin & Jiao, 2008). For
convenience, the abbreviations for the textbooks: E1c, MM1c, PEPB and BJNUPG will
be used in the following analysis.

As mentioned earlier, all the students in China have to study four compulsory
mathematics courses regardless of which programme the students choose. Instead of
compiling a single volume with all the topics, the Chinese textbooks are produced in a
systematically and subject-related design. Accordingly, they are smaller in size and
divided into four booklets. In this way, the textbooks can be used separately or in combination with each other. In compulsory course 3, the students learn about statistics, fundamental programming skills and probability contents. On the other hand, the Swedish textbooks are designed as a single volume with a great variety of topics which covered in the intended curriculum, such as number system and arithmetic, algebra, geometry etc. Accordingly, both the Swedish textbooks have more than 300 pages while the Chinese ones are around 150 pages on average for the selected textbooks. The topic of Probability is often paired together with another topic Statistics in the same chapter in the Swedish textbooks. However, in the Chinese textbooks, each subject has its own separate chapter. The probability content of the empirical data will be presented in Table 1 below.

Table 1: Probability topics of the selected mathematics textbooks.

<table>
<thead>
<tr>
<th>Books</th>
<th>Year</th>
<th>Publisher</th>
<th>Probability content pages</th>
<th>Probability topics</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1c</td>
<td>2011</td>
<td>Gleerups Utbildning AB</td>
<td>31</td>
<td>1) Random trials; 2) Relative frequencies; 3) Attempts in multiple steps; 4) Dependent events and probability.</td>
</tr>
<tr>
<td>MM1c</td>
<td>2012</td>
<td>Liber</td>
<td>23</td>
<td>1) Chances; 2) Independent events; 3) Events in multiple steps; 4) How often does one event happen?</td>
</tr>
<tr>
<td>BJNUPG</td>
<td>2008</td>
<td>Beijing Normal University Publishing Group</td>
<td>42</td>
<td>1) The probability of an event; 2) Classical probability model; 3) The application of probability.</td>
</tr>
</tbody>
</table>

Despite the textbooks’ different compiling designs, three of the representative textbooks (E1c, PEPB and BJNUPG) cover more than 30 pages on the probability content. All the textbooks cover a similar number of topics. It can be seen that the Chinese textbooks are structured more systematically. For example, first introducing the concepts of
probability and event before exploring a classical probability model and lastly the application of a geometric probability model. Moreover, the Chinese textbooks integrate the development of probability history into the content study. For example, it mentioned the term classical probability. On another hand, the Swedish textbooks emphasize more on concrete and fundamental topics, for example, dependent and independent event, complement event and relative frequency and tree diagram. Some of them do not need previous knowledge to build up the next, for example, tree diagram and relative frequency can be treated independently.

4.3 Limitations of the study

This study is limited to a selection of sample size four mathematics textbooks pertaining to probability content intended for the first year in the natural science programme (Naturvetenskapsprogrammet) of upper secondary school in Sweden and China. Despite the fact that the most used textbooks in China are chosen, the small sample size of the selected textbooks is not representable for the mathematics textbooks in both countries. Thus, there might be some bias in the study when analysing and interpreting the contents. Another limitation of the study is the versions of the textbooks. The selected textbooks for this analysis for Sweden is between 2011 and 2012 while for China is between 2007 and 2008. These textbooks are older versions and there is a need for analysing newer textbooks produced by both countries. Last but not least, this study’s main interest is to find out how the probability content is presented in the first year of post-secondary school in Sweden with the comparison to China instead of why the textbooks are presented the way they are.

4.4 Implementation

This section started with an analytical microstructural model by Rezat (2006) to characterize the patterns of the probability content in the selected textbooks. Next, Polya’s (1957) problem-solving strategy is used to evaluate and analyse the worked samples. Lastly, Li’s (2000) framework of dimensions of problem requirements is conducted to study the practice exercises in the textbooks. This study is interested in only the probability content in the four selected mathematics textbooks in the first year.
of post-secondary education in Sweden and China. Also, these textbooks are considered for the same age group.

**Research Question 1:**
To discover the characteristics of the probability content in the selected textbooks, the Rezat (2006) model is chosen to investigate the microstructural level in the respective chapter. By looking at the five extensive aspects of the microstructure in the textbook content, including characteristics in terms of content; linguistic characteristics; visual characteristics; pedagogical functions and situative conditions, Rezat model aims to find out the similarities and differences presented in the textbook contents. Table 2 below shows the microstructural level and definition.

<table>
<thead>
<tr>
<th>Level</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Characteristics in terms of content</td>
<td>This level states the structure of each chapter, such as introductory activities, definitions and theorems, worked examples and exercises.</td>
</tr>
<tr>
<td>Linguistic characteristics</td>
<td>This function refers to the suitability and conciseness of the use of language in the content.</td>
</tr>
<tr>
<td>Visual characteristics</td>
<td>This level refers to how the contents are displayed visually in the section, for example, boxed definitions or distinct marked different sections for identification.</td>
</tr>
<tr>
<td>Pedagogical functions</td>
<td>This function refers to how the textbook contents assist the teachers in engaging the students actively in the class.</td>
</tr>
<tr>
<td>Situative conditions</td>
<td>This function refers to activities, examples and puzzles that help the students to discover mathematics on their own.</td>
</tr>
</tbody>
</table>

**Research Question 2:**
After finding out the characteristics of the probability content, an analysis of the worked examples will be implemented based on Polya’s problem-solving model. There are four stages accounting for Polya’s (1957) problem-solving process. For determining if the worked problems meet the criteria of all four stages, a stage named *All phases* is added to this round of analysis. The feature, *All phases*, aims to find out the frequency of all the worked problems with respect to Polya’s four problem-solving stages.
The worked problems are explicitly marked in the representative textbooks which make it easy for selecting when implementing the study. Almost all the probability topics in the representative textbooks provide two or more worked examples for demonstrating the process on how to arrive at an answer. A total of 26 probability worked examples in the Swedish textbooks and 32 probability worked examples in the Chinese textbooks are examined in this round of analysis. First of all, each worked example in the representative textbooks will be read thoroughly at least three times to fully understand the question design, the strategy and process solving the problem and arrive at a solution. Next, the worked problems and solutions will be analysed to determine if they model each of the four phases of Polya’s (1957) problem-solving phases of understanding, devising, carrying out and looking back. In the end, all the worked examples will be checked over again to see if they meet Polya’s four problem-solving phases. Table 3 below presents Polya’s problem-solving model together with definition applied for the analysis of the probability worked problems in this study.

<table>
<thead>
<tr>
<th>Polya’s phase</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understanding phase</td>
<td>Reading the problem: the problem or question is clearly stated such as the data and the unknown.</td>
</tr>
<tr>
<td>Devising phase</td>
<td>Analysis of the problem: a solution plan is suggested for exploring the connection between the data and the unknown.</td>
</tr>
<tr>
<td>Carrying out phase</td>
<td>Implementation of the solution: a solution is given clearly.</td>
</tr>
<tr>
<td>Looking back phase</td>
<td>Verification of the solution: the review of the solution, showing different approaches to help students recognise and discover mathematical patterns and strengthen students’ understanding.</td>
</tr>
<tr>
<td>All phases</td>
<td>The worked examples display all the problem-solving stages.</td>
</tr>
</tbody>
</table>

**Research Question 3:**

To answer the research question *how do the practice exercises in the selected textbooks differ*, Li’s (2000) framework *dimensions of problem requirements* is used for analysis. Mathematical features, contextual features and performance requirements are the three main categories in Li’s (2000) framework. The Swedish textbooks provide more than double the number of exercises than in the Chinese textbooks. According to the
textbooks, the exercises are divided into different difficulty levels. The problem types of the practice exercises in textbook *Elc* mentioned explicitly in the textbooks according to the Swedish intended curriculum goals (see Appendix 3), which helps the identification based on Li’s dimensions of problem requirement. A diagram below is provided to show the distribution of the difficulty level of the exercises in the selected textbooks. Level 1 questions refer to the fundamental problems, level 2 questions refer to a bit more difficult problems while level 3 questions require higher analytical skills. In *PEPB* has only two difficulty levels but it is found that some of the problems are required higher analytical abilities. It can be seen in Figure 2 that the Swedish textbooks provide a much higher number of easy problems for strengthening the students’ understanding before moving on to new knowledge.

![Distribution of the exercises at different difficulty levels in the textbooks.](image)

Figure 2: Distribution of the exercises at different difficulty levels in the textbooks.

To implement this round of analysis, two steps will be implemented. First, the mathematical and contextual features will be evaluated. Each exercise and solution are read and analysed thoroughly for determining whether the exercise requires a single or multiple computation procedures to arrive at an answer. Next, the problems are examined again to find out whether they are described using a purely mathematical context or any illustrative context with pictures. The mathematical and contextual features and definition of the practice exercises are displayed in Table 4 below.
Table 4: Mathematical and contextual features and definition in Li’s framework.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Problem requirements</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical</td>
<td>Single computation procedure required</td>
<td>The exercise requires a single computation to be solved.</td>
</tr>
<tr>
<td>feature</td>
<td>Multiple computation procedures required</td>
<td>The exercise requires multiple computations to be solved.</td>
</tr>
<tr>
<td>Contextual</td>
<td>Purely mathematical context in numerical or word form</td>
<td>The exercise presents purely in a numerical table or in plain word form.</td>
</tr>
<tr>
<td>feature</td>
<td>Illustrative context with pictorial representation or story</td>
<td>The exercise presents through a picture, graph, or diagram or story form.</td>
</tr>
</tbody>
</table>

Next, the practice exercises and the solutions are reviewed carefully again to find out which of the performance requirements that the problems are best described. Li (2000) categorized the performance requirements into two sub-categories, the response type and cognitive requirements. For the response type, the exercise solutions will be analysed in order to determine the answer types. Table 5 below illustrates the three answer types and definition in Li’s framework.

Table 5: Li’s sub-category response type and definition in performance requirements.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Problem requirements</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performance</td>
<td>Numerical answer only</td>
<td>The answer requires only a number.</td>
</tr>
<tr>
<td>requirements</td>
<td>Numerical expression only</td>
<td>The answer contains only algebraic expressions such as equations which combine letters, numbers and symbols.</td>
</tr>
<tr>
<td>Response</td>
<td>Explanation or solution required</td>
<td>The answer requires a solution with an explanation.</td>
</tr>
<tr>
<td>type</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For the cognitive requirements, both the practice problems and the solutions are evaluated for finding out the cognitive levels the students are required for solving the problems. For problems that can not be identified with the predetermined features, they are labelled into a new feature which captures the characteristics of the problem type. One new feature, *simulation*, is added to the sub-category cognitive requirement. In this
way, all the exercise problems are examined in the selected textbooks to increase reliability. Table 6 below illustrates the cognitive requirements and definition in Li’s (2000) framework.

Table 6: Li’s sub-category cognitive requirements and definition in the performance requirements.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Problem requirements</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performance</td>
<td>Cognitive requirements</td>
<td></td>
</tr>
<tr>
<td>requirements</td>
<td>Procedural practice</td>
<td>The exercise focuses on handling procedures and solving practical problems of standard characteristics.</td>
</tr>
<tr>
<td></td>
<td>Conceptual understanding</td>
<td>The exercise focuses on understanding mathematical concepts and relationships in order to solve it.</td>
</tr>
<tr>
<td></td>
<td>Problem solving</td>
<td>The exercise requires the ability to work through mathematical problems using selected strategies and methods.</td>
</tr>
<tr>
<td></td>
<td>Special requirement</td>
<td>The exercise requires mathematical reasoning and critical thinking.</td>
</tr>
<tr>
<td></td>
<td>Simulation</td>
<td>The exercise requires programming tools and skills to perform.</td>
</tr>
</tbody>
</table>

The analysis result will be presented in the next chapter. To sum up, the probability content in the four selected textbooks is studied and analysed in order to answer the three research questions raised in the second chapter. Three existing theory and frameworks, Rezat (2006) model, Polya’s (1957) problem-solving model and Li’s (2000) dimension of problem requirements, are applied to examine the textbook contents. It is hoped that this study will be useful for understanding what probability concepts should we teach in post-secondary school and how should it be taught to the students.

4.5 Reliability in this study

According to Bryman (2012), instead of reliability and validity, the qualitative researchers have tended to “develop evaluative criteria for assessing research” (Bryman,
According to Patton (2001), the researcher in a study is the most important factor in a qualitative study. He states that the reliability of the qualitative research often depends on the effort, competence and experience of the researchers.

When selecting the existing theory and frameworks, I have to read different papers and material for choosing comprehensive and relevant models for the analysis. This study contains my interpretations of the probability content, the worked examples and the practice exercises in the selected textbooks for determining and validating the predetermined framework parameters. To make the study more reliable, I have also discussed with my peers the investigated probability material as well as revised my analysis over and over again with the purpose to make the interpretations and results both as fewer errors and as more trustworthy as possible. Last but not least, I have consulted my supervisor and others to check if the constructed analysis for making sure they are in relation and reliable to the researched questions I have been working with.
5. Results and analysis

In this section, only the result of the analysis of the study is presented in the order of the research questions proposed in the second chapter.

5.1 Research question 1

In order to study and answer the research question, *the characteristics of the probability content in the selected textbooks*, the probability content were examined for classifying the five elements of Rezat’s model (2006): characteristics in terms of content; linguistic characteristics; visual characteristics; pedagogical functions; and situative conditions.

*Characteristics in terms of content*

The probability topics in the selected textbooks are analysed and derived the following characteristics. The characteristics in terms of content pertaining to the probability for the representative textbooks are a) chapter opener, b) introductory samples, c) theorems, d) worked problems, e) practice exercises, f) activities/reflection and discussion, g) chapter summary, h) revision test.

It can be seen that both countries share a similar structure in designing the topics, for example starting with a chapter opener then introductory examples before the worked examples demonstrated. All the textbooks offer different levels of practice problems to meet the students’ different levels and needs. It can be seen that the Chinese textbooks offer more problems modelling daily situation, such as the probability of winning the lottery, the likelihood of rainfall, the risk of influenza transmission. In the textbook *BJUNPG* there is a structured teaching and learning guidance for the teachers. It is designed in the following order: a) posing an artificial question, b) the students discuss, explore and experiment the topic with the teacher’s assistance in the class, c) the teacher teach out the relevant definition and theorems and demonstrate worked examples, and finally d) the students do some reading comprehension and exercises. This learning process is designed in each topic and it appears to be subject-related, organized and students motivated.
Linguistic characteristics

The language used in the Swedish textbooks are concise and accurate, for example, it explains the definition clearly and connects the texts and pictures in a story form. On the other hand, a slightly higher frequency of mathematical symbols is seen in the Chinese textbooks comparing to the Swedish textbooks. Here are examples of the definition of probability from both textbooks to show the different usage of mathematical symbols (see Appendix 2 for original texts):

All outcomes are of equal probability when throwing a symmetric dice, and we call it a uniform probability distribution. This means that you theoretically can calculate the probability of an event as in the example:

\[
P(A) = \frac{\text{number of favourable events}}{\text{total number of possible outcomes}}
\]

(Exponent 1c, 2011, p. 278, my translation)

In the classical probability, a random event A in a trial is usually composed of some basic events. If all possible outcomes (basic events) in the trial is n, and the number of basic events in random event A is m, then the probability of event A is:

\[
P(A) = \frac{\text{number of possible events in event A}}{\text{total number of all events in a trial}} = \frac{m}{n}
\]

(Beijing Normal University Publishing Group, 2008, p. 136, my translation)

The language of the mathematical terms in the Swedish textbook above is concise and accurate, for example, we can apply a uniform probability distribution when throwing a symmetric dice, and we calculate it by a division between the number of total events and the number of total outcomes. On the other hand, language usage in Chinese textbooks applies more mathematical terms and symbols, such as classical probability, random event and symbols \( m \) and \( n \). This application can make the students become more familiar with mathematical structure and symbols.
**Visual characteristics**

In the Swedish textbooks, there are pictures connecting to real life, tables and diagrams to help the students to understand the content. Different colours are used to illustrate different kinds of topics, for example, the colour blue for worked examples, orange for definitions and theorems. The definitions and theorems are distinctly boxed and highlighted. To show the differences, worked examples using tree diagram are chosen from both countries.

The Swedish example on the left in Figure 3 below:

*In a basketball match, Jim got two penalty shots. What is the probability that he can score both shots? If it is known that Jim usually scored 80% at penalty shots*

(Matematik M 1c, 2012, p. 253, my translation).

The Chinese example on the right in Figure 3 below:

*There are four identical balls in a bag, two in white and two in black. Four students draw a ball from the bag in order. What is the probability that the second student get a white ball?*

(Beijing Normal University Publishing Group, 2008, p. 140, my translation).

Figure 3: Visual features in the Swedish and Chinese textbooks. (Source: Matematik M 1c, 2012, p. 253 and Beijing Normal University Publishing Group, 2008, p. 140)
It can be seen that the content is clearly structured with basketball pictures showing the probability of scoring a shot 80% and missing a shot 20% in the tree diagram. The students can see where the solution is given because it is marked in blue colour and started in a new paragraph to bring to the students’ attention. After the solution, a definition of multiplication principle is boxed and in orange colour, which can catch the students’ eye. The tree diagram is printed in black and white without giving the probability of drawing the balls from the bag. Unlike the Swedish textbooks, there is no new paragraph or different colour before a solution is given but it is marked in bold characters.

In Figure 4, it can be seen that there is a lot of text in plain colour in the Chinese textbook. For students who have trouble reading a lot of texts, they have to go through all the long text, filtering and extracting the important information themselves. In other words, the Chinese textbooks are less student-friendly and more subject-centred. Even though there is also quite a lot of text in the Swedish textbooks, but it provides a picture of the dice to bring some visual effect to the students. One thing worth mentioning is that for the younger age of the students in China use also this printing style for the textbooks. If the students have developed difficulty at an early age, this could lead to a more difficult situation for the students who have difficulty learning mathematics and filtering important information. Because mathematics textbooks tend to contain more information and text in the upper-secondary school.
Pedagogical functions

These pedagogical functions refer to how the textbook contents assist the teachers in motivating and engaging the students actively in the mathematics class. There is the teachers’ guide in mathematics aside from the textbooks to assist the teachers in teaching in both countries. In China, the school often provide the teachers’ guide for teachers. In Sweden, while the school were supposed to provide the teachers’ guide for the responsible teachers. Due to possibly economic budget and other reasons, not all the teachers have access to the teachers’ guide. However, the focus in this study will mainly be on the mathematics textbook contents.

One of the Swedish textbooks presents a problem called *throw a pig (kasta gris)* (Exponent 1c, 2011, p. 283). The goal of this activity is to motivate the students to learn how to compute the frequency for one event through finding out how often the plastic pig land on different positions likewise the back, on the side, on the snout or on standing position. This problem is illustrated with pictures and diagram which can help the teacher to explain and engage the students’ understanding and learning when teaching. It has a higher frequency for the Swedish textbooks presenting probability examples on throwing dice, throwing a coin or drawing a coloured ball. The teacher can bring dice, coins and some coloured balls to the class to play with the students when learning in this way to engage them actively. The Chinese textbooks provide wider range examples, such as the probability of choosing a certain amount of weights from 4 different materials in a gym, throwing a drawing pin, finding out the probability of having a child with different eye colour. The teachers can bring some gym weights and drawing pins to the class for the students to try out and in this way to actively engage the students for learning during the class.

Situative conditions

Situative conditions refer to activities, examples and puzzles that help the students realize mathematics on their own. All the selected textbooks give some suggested contexts or examples to assist students’ discovery of mathematics. Here are two activity examples from both countries see Figure 5 below.
- The students are divided into a pair for carrying out this activity. They need to throw a drawing pin and record the frequency of the pointy brass up.

- The students need to throw the drawing pin five times in a row and take note of the frequency of the pointy brass up until they have recorded 30 times, i.e. they have to throw in total 150 times. And then calculate the cumulated frequency for the pointy brass up with the help of the calculating table. Add up the frequency in the calculating table to the cumulated frequency in column C and calculating the cumulated relative frequency in column D.

The students use random numbers to model the experiment of tossing a coin. Any five of the numbers between zero to nine to represent the flips of ‘landing heads up’ and the rest of the five numbers to represent the flips of ‘landing tails up’. For example, the students can choose numbers of 0, 1, 2, 3, 4 to represent the flips of ‘landing heads up’, and the numbers of 5, 6, 7, 8, 9 to represent the flips of ‘landing tails up’. The students need to implement 100 trials to complete Table 3-4, and then calculate the frequency of the flips of ‘landing heads up’ occurring. Finally, the result will be discussed in the whole class.
The activities provided in both countries are clearly instructed with the attempt to motivate the students to discover the beauty of mathematics through carrying the experiment. It can also be seen from the examples in Figure 5 that it is coherent with the previous finding that the Swedish textbooks are more student-oriented and the Chinese textbooks are subject-oriented. The Swedish textbooks provide a picture of drawing pin for the students to connect to the activity and the table is marked in colour to make it visually attractive while the Chinese textbooks are printed in black and white text and table.

5.2 Research question 2

In order to resolve the research question *how are worked examples presented in the selected textbooks help the students’ problem-solving skill*, all the worked examples pertaining to probability are chosen for analysis based on Polya’s (1957) problem-solving model. The data analysis of how frequent the probability worked examples in the selected textbooks model Polya’s problem-solving process is illustrated in Figure 6.

The data analysis shows the distribution of each selected textbook in relation to Polya’s problem-solving model. However, to fulfil the purpose of this study, the frequency percentage of each stage over the total is used for comparing the findings from the
representative textbooks between China and Sweden. From the data analysis, it is found that the worked problems are modelled slightly different between the Swedish textbooks and the Chinese textbooks. In other words, the Swedish students have more possibility to expose themselves in worked examples which are modelled using the problem-solving process. A table of numerical data for the number of worked examples that modelled the Polya’s problem-solving process in the selected textbooks is presented in Appendix 4. All the 26 probability worked problems in the Swedish textbooks model the understanding, devising and carrying out stages from Polya’s problem-solving process, but only 10 of 26 worked examples (38.46%) that model Polya’s verification stage. In the Chinese textbooks, 28 of 32 worked examples (87.50%) model the understanding phase, 26 worked examples (81.25%) model the carrying out stage and only 7 worked examples (21.88%) model the looking back phase. It can be seen that the Chinese textbooks tend to focus on the understanding and devising stage of Polya’s problem-solving model. When examining the worked problems one last round to determine if they meet all the four stages, it is found that 10 of 26 worked examples (38.46%) in the Swedish textbooks model all the phases while 6 of 32 worked problems (18.75%) in the Chinese textbooks.

As can be seen from the example in Figure 7 below that the Swedish textbooks place an emphasis on the problem-solving process, describing and analysing the problem, implementing and verifying the solution.

![Figure 7: Worked examples illustrating Polya’s problem-solving process. (Source: Matematik M 1c, 2012, p. 252)](image)
If we toss a coin two times, there are four possible outcomes according to the tree diagram (see Figure 7 for tree diagram).

Solution:

All possible outcomes = 4.

Every possible outcome has probability ‘one fourth’ chance, i.e \(1/4 = 25\%\).

Starting from the top of the tree diagram and follow the branches. The red arrow shows that we get the flips of heads landing up in for both tosses i.e. (head, head). The grey arrow shows that we first get a flip of tails landing up and then the flip of heads landing up, i.e. (tails, head).

With the help the tree diagram, we will calculate the following probability:

a) \(P(\text{head, head}) = 25\%\)

b) \(P(\text{the coin shows different sides}) = P(\text{head, tails}) + P(\text{tails, head}) = 25\% + 25\% = 50\%\).

An alternative solution for \(P(\text{the coin shows different sides})\):

All possible outcomes = 4.

The number of favourable events = 2.

\(P(\text{the coin shows different sides}) = \frac{2}{4} = 0.5 = 50\\%\). (Which means we can get either (head, tails) or (tails, head)).

(Matematik M 1c, 2012, p.252, my translation)

The worked example above has clearly stated the data and the question with a tree diagram and detailed description. A solution and the implementation is explained and given clearly. In the end, it reviews the solution by showing different or alternative approaches for helping the students discover mathematical patterns and strengthen the students’ understanding. These findings characterize research question 1 further on that the Swedish textbooks place emphasis on presenting the material in a clear and pedagogical pattern. It can be also seen from the result that the Swedish textbooks incorporate better the problem-solving models into the worked problems compared to the Chinese textbooks.

5.3 Research question 3
The practice exercises are selected and analysed using Li’s (2000) framework in order to answer the third research question: *how do the practice exercises in the selected textbooks differ in relation to mathematical, contextual and performance requirements.*

The first part of this research question examines the mathematical and contextual features of all the practice problems in the topic of probability section in the textbooks. A total of 189 exercise problems in the Swedish textbooks and 86 from the Chinese textbooks are examined in this research question. The results of analysing the mathematical and contextual features of the practice exercises in the representative textbooks are illustrated in Figure 8 below.

![Figure 8: Results of analysing the mathematical and contextual features of the exercises.](image)

The result of the data analysis shows the distribution of each selected textbook in relation to the mathematical and contextual features of the practice exercises. However, to fulfil the purpose of this study, the frequency percentage of each feature over the total is used for comparing the findings from the selected Swedish and Chinese textbooks. The finding shows that the mathematical and contextual features in the probability content for the Swedish and Chinese textbooks are surprisingly similar. A table of all the numerical data of the mathematical and contextual features is presented in Appendix 5. Among all the practice exercises, 134 of the 189 practice problems (70.9%) in the Swedish textbooks while 66 of the 86 practice problems (76.75%) in the Chinese textbooks demand multiple calculation procedures. As can be seen from the example below, multiple computation problems involve two or more calculation computations to
arrive at an answer. Meanwhile, both countries are surprisingly similar in presenting the problem exercises contextually. It’s found that 169 (89.42%) of the practice problems in the Swedish textbooks and 77 (89.54%) of the exercises in the Chinese textbooks are characterized with illustrative content or story form with pictures, graph or diagram. Unlike many other topics, probability topic can be difficult and sophisticated to grasp. Therefore, problems with pictures, diagrams or in story form make it more interesting for the students to solve probability problems, especially when the problem relates to the application of probability in everyday life.

For example, “In backgammon, a game that uses two standard dice, the black side needs to roll at least one 3 on a die and at least one 5 on the other die to be able to win the game. How is the probability of winning the game” (Exponent 1c, 2011, p. 291, my translation)? One way to approaching this problem is to understand the problem first: it is wished to determine the probability of rolling one 3 on one die and at least one 5 on another die. In another word, the students need to first calculate the total number of possible outcomes for two dice, in this case, 36 possible outcomes and then find out the number of favourable events in order to compute the probability. Another way is through drawing see Figure 9 below. All the dots represent all possible outcomes and the favourable events are marked in the figure.

![Figure 9: A drawing for all possible outcomes and the marked favourable events.](image)

The favourable events are: (3, 5), (3, 6), (4, 5), (4, 6), (5, 3), (5, 4), (5, 5), (5, 6), (6, 3), (6, 4), (6, 5), (6,6). Thus there are 12 ways to roll at least one 3 and at least one 5 with two dice and the probability of winning the game is:

$$P(W) = \frac{\text{number of favourable events}}{\text{total number of possible outcomes}} = \frac{12}{36} = \frac{1}{3}$$
The second part of this research question investigates the response type or the type of answer that the practice problems require. Figure 10 below shows the performance requirements’ sub-category response type of the practice exercises.

![Figure 10: Data analysis of the response type of the practice exercises.](image)

A table of the numerical data of the response type of the practice exercises can be found in Appendix 6. It is found that of the 184 practice exercises in the Swedish textbooks, 142 (75.13%) of them require a numerical answer and only 33 (17.46%) require an explanation or solution. In the Chinese textbooks, of the 79 practice problems, 18 (20.93%) expect numerical answer only and numerical expression while 50 (59.14%). It can be seen from the data analysis that the Swedish and Chinese textbooks place different emphasis on the response type of the practice exercises. The response feature, explanation or solution required, reveals that the exercise answers require an explanation of how a solution is found. China has a higher percentage of problems that require an explanation or a solution. For example, “Please exemplify three examples which illustrate random phenomenon around you”, “Explain what do you know about random phenomenon” (People’s Education Press B version, 2007, p. 92 and p. 101, my translation), “A symmetric coin is thrown twice in a row, explain whether the chance of ‘two times head’, ‘two times tail’, ‘one time head’ and ‘one time tail’ occurring are the same” (Beijing Normal University Publishing Group, 2008, p.138, my translation).
The third part of this research question analyses the cognitive requirements which required to solve the practice problems. Figure 11 presents the performance requirements’ sub-category cognitive requirements of the practice exercises.

![Figure 11: Data analysis of the cognitive requirements of the practice exercises.](image)

A table of the numerical data of the cognitive requirements of the practice exercises is shown in Appendix 7. The analysis shows that 96 of the 189 practice exercises (50.79%) require procedural practice and 72 (38.10%) require problem solving in the Swedish textbooks, and only 4 (2.12%) practice exercise focus on understanding the mathematical concepts in order to solve the problems. In the Chinese textbooks, 34 of 86 practice problems (39.53%) require problem-solving and 28 practice problems (32.56%) require procedural practice. The Chinese textbooks have slightly higher percentages in the cognitive requirements for conceptual understanding, special requirement and simulation. The result indicates that a large percentage of the practice exercises in the Swedish textbooks focus on the procedural operation and problem solving. This finding is coherent with the previous finding that a great amount of the probability worked examples in the Swedish textbooks are also modelled in a problem-solving process. However, there is only 2% of the exercises that require conceptual understanding and 7% of the exercises requires special requirements. This result is not compatible with the intended Swedish curriculum goals, to develop students’ concept and reasoning skills.
6. Conclusion and discussion

The purpose of this study is to compare and investigate the topic of probability content in Sweden and China upper secondary school mathematics textbooks. In this section, the results and methods of this study will be discussed as well as suggestions for future studies.

6.1 Discussion of the results

This study is based on an analysis of probability content in four mathematics textbooks of upper secondary school in Sweden and China. Specifically, the following three research questions have been asked:

1. What are the characteristics of the probability content in the selected mathematics textbooks in Sweden and China?
2. How are the worked problems of the probability content presented with respect to Polya’s problem-solving model?
3. How do the practice exercises in the selected textbooks differ in relation to mathematical, contextual and performance requirements?

To address research question 1 with respect to the characteristics of the probability content in the four mathematics textbooks, Rezat’s (2006) microstructural model was used. The model implemented on the topic of probability content showed that the two countries’ textbooks were similar in content microstructural design and situative conditions and were different with respect to linguistic, visual and pedagogical functions.

It can be concluded from the analysis that the probability content is structured similarly and have adequate examples for the students to discover and appreciate mathematics for the four selected textbooks. The Swedish textbooks attempt to use simple language and language structure to introduce a new probability topic so that the students can
completely understand each process before moving to the next one. The Swedish textbooks also place emphasis on the visual attractiveness of the material together with clearly structured and boxed theorems and definitions compared to the Chinese textbooks. On the other hand, the Chinese textbooks are printed in black and white, filled with much more mathematical symbols and provided with more teaching activities for enhancing the students’ appreciation of mathematics and engaging the students actively in the class. It can be concluded that the Swedish textbooks are more student-oriented, trying to grasp the students’ interest with fewer mathematics symbols, colourful pictures, clearly boxed theorems and definitions together with relevant diagrams and tables. The Chinese textbooks, on another hand, are more subject-related with a large proportion of black and white text and more mathematics symbols and expressions together. This could make it hard for students who have difficulty learning mathematics. The Chinese students need to have an inner drive to learn mathematics using the Chinese textbooks while the Swedish students have an attractive and external source to motivate them for learning.

For research question 2, “How are the worked examples of the probability content presented with respect to Polya’s problem-solving model?”, the textbooks in Sweden model the problem-solving process slightly better than the Chinese textbooks. The data analysis showed that the Swedish textbooks perform better in modelling the worked problems with respect to Polya’s problem-solving process. Almost 40% of the worked problems in the Swedish textbooks modelled Polya’s problem-solving process while there is only around 20% in the Chinese textbooks. The finding is consistent with the intended Swedish curriculum goal (see Appendix 1) that the mathematics class should improve the student’s problem-solving competence in mathematics.

The differentiation of the curriculum requirements pertaining to probability could lead to a different way of modelling the problems. The Swedish students are supposed to learn “the methods for calculating probabilities in multiple-step” (Skolverket, 2011, p. 105), while the Chinese curriculum appears to place emphasis on understanding the definition and formulas, such as “understand the uncertainty of random events, definition of probability and the definition of random number, etc.” (Ministry of
education of the People’s Republic of China, 2003, p. 10). This could limit the design of the worked examples in the textbooks. However, the worked examples are supposed to design in a clear structure such that the students learn the strategies to arrive at an answer either with the help of new knowledge or strengthening old knowledge.

For research question 3, “How do the practice exercises in the selected textbooks differ in relation to mathematical, contextual and performance requirements”, the finding shows that a) both countries have similar percentages of the practice exercises in mathematical and contextual features, and b) the two countries’ exercises differ greatly in the response type and share some similarities in cognitive requirements.

Interestingly, all the selected textbooks perform very closely in the mathematical and contextual features. Both countries’ textbooks place emphasis on exercise problems which require multiple computations and are stated in an illustrative context or in story form. Around 75% of the Swedish exercises require only numerical answers and 17.46% problems require explanation or description of the solution. It can be seen from the exercise problems and solutions in the Swedish textbooks that more emphasis is placed on building up the knowledge by solving exercises, which the answer only needs a numerical value. More than half of the exercises in the Chinese textbooks require an explanation or solution. The Chinese exercises are found that more focus is put on requiring theoretical explanation and asking the students to give different probability examples. This result is not consistent with the Swedish intended curriculum goal, developing the students’ mathematical communication in written form. With a higher percentage of problems that require only a numerical answer, it shifts the emphasis on the drilling of mathematics calculation. The students can improve their mathematical communication skill if they are exposed to more training of the same kind.

Both the practice exercises in Sweden and China show a higher percentage in procedural practice and problem-solving and a bit lower in special requirement and conceptual understanding. The finding is not coherent with the Swedish curriculum that mathematics education should promote the students’ concept, mathematical reasoning and communication skills. Yet, the practice exercises model heavily on the
mathematical procedures in the Swedish textbooks. It appears that the emphasis for practice exercises is too one-sided for the Swedish textbooks, while the cognitive requirements of the practice exercises are more comprehensively distributed in the Chinese textbooks. Furthermore, most of the exercises in the Swedish textbooks are about rolling dice, drawing cards from a deck, or drawing different coloured balls while the Chinese textbooks cover more variations, such as calculating gym weights, finding out the probability of having the same birthday with another person, probability on product quality control etc. There are very few exercises in the Swedish texts applying day to day life examples or situations, which is not consistent with the intended curriculum goals of applying mathematics in the social, work and daily life (Skolverket, 2011, p. 9).

This study yields the conclusion that the Swedish textbooks provided a much larger number of exercises for learning and practising and contained more worked examples which modelled problem-solving process and exercises that require routine procedures than the Chinese textbooks. However, it is found that the Chinese textbooks offer a wider variety of examples and exercises integrating day to day life and mathematical concepts than the Swedish textbooks. It can also be seen that the Swedish curriculum is less concrete and precise than the Chinese curriculum in relation to the probability content. However, it is important for the teachers to keep in mind the goals and requirements of the intended curricula but should also certainly be aware of the students’ different levels, needs and interests when teaching and planning a lecture.

6.2 Discussion of method

The textbooks are chosen as the focus of this study because textbooks are an important instrument for both teachers and students according to Valverde et al. (2002). An analysis of the textbooks is made without taking into account the teachers’ pedagogical approaches, classroom environment or the students’ achievement in mathematics in this study. This allows the study to focus on probability content in the textbooks, including the content structures, the worked examples and the practice exercises.
A qualitative study employing the technique of comparative content analysis is used for examining the textbook contents. Three existing theories and frameworks are applied as guidance to fulfil the goals of this study. The advantage of applying the existing theories in the study is that these theories give a reliable and clear framework that other researchers have tested (Hsieh and Shannon, 2005). Using the existing theories, on the other hand, brings limitations to the study because the predetermined categories in the frameworks can restrict the analysis and place an overemphasis on theory (Hsieh and Shannon, 2005).

For this reason, I have carefully chosen the theory and frameworks that contain extensive and relevant parameters for increasing the diversity of this analysis. Three theory and framework are applied in this study, Rezat’s model (2006), Polya’s problem-solving process (1957) and Li’s framework (2000). Meanwhile, I read the content of the textbooks at least three times and have discussed with my peers when uncertainty occurs during the analysis to increase reliability. A new category, simulation, is created when the practice exercises that cannot be identified to meet the existing framework’s criteria in order to capture all the essence of the textbook characteristics. Moreover, the curricula and textbook contents from China and Sweden are translated into English in this study. Even though I have tried to implement the analysis and revise the result carefully at least three rounds with the aim to minimize human errors, my interpretation of all the worked problems together with almost 300 practice exercises could still lead to some interpretation and human errors.

The comparative content analysis is a good approach to gain a deeper understanding of mathematics textbooks. If more textbooks can be analysed and compared, this would help the reliability of the study. Last but not least, this study is limited to a selection of sample size four mathematics textbooks pertaining to probability content. Future study should use a bigger sample size than four textbooks. It will allow the study to have more variety and precise.

6.3 Suggestion for future work
This study is focused on analysing and comparing the mathematics textbooks pertaining to probability content at the upper secondary school. This study attempts to find out how the Swedish mathematics textbooks organize the content for teaching and learning. Future research concerning the comparative content analysis of mathematics textbooks could continue to find out more the following areas, a) problem types in the mathematics textbooks, b) problem-solving process, c) programming knowledge, and last but not least d) exercise problems that require different cognitive levels in the Swedish textbooks.

The study of the problem types in mathematics textbooks may give teachers a better understanding of the textbooks. For example, how does the distribution of these problem types affect the students’ learning? Do the textbooks have more open questions or closed questions? Do the textbooks provide more single computation problems and problems only require numerical expressions in algebra calculation or fundamental operations? Do the textbooks have more multiple computation problems and problems that require explanation in functions or statistics? The aim is to find out how does the distribution of different problem types assist the students learning mathematics. This study would involve classroom observations and interviewing teachers and students. The result of this analysis can assist teachers when preparing lectures. If the distribution of the problem types is not balanced, the teachers can add other exercises to improve the students’ other mathematical abilities in order to make sure the lectures cover a wide range of problem types.

To investigate and find out the students’ answer performance with respect to problem-solving ability through studying the type of answers provided by the students when solving a problem can offer teachers comprehensive aspects of students’ perceived knowledge. What problem-solving process do the students perceive when they study the worked problems? What answer types do the students give after exposing to the worked problems in the textbooks? This study would involve some well-designed tests with problems that capture the problem-solving stages, interviewing the teachers and students and probably a questionnaire. The finding of this study may provide new pedagogical perspectives for both the teachers and students. On one hand, the teacher can learn more about the students perceived knowledge in order to change his or her
teaching methods, classroom instructions or providing exercises in relation to individual levels and needs. On another hand, this study can help the students to learn about their performance through the tests. They can either ask the teacher for assistance or find similar problems by themselves for strengthening the desired mathematical skills.

Programming is recently included in the mathematics curriculum in the primary school and this can influence the upper secondary school. What should be included in the post-secondary school? At what level is the programming content already included in the current Swedish textbooks? Does the programming content improve the students’ learning in mathematics and problem-solving skills? Since the introduction of programming content is relatively, it can be beneficial to study and analyse this topic from a cross-national level. This finding can be used as a support for future authors when editing new mathematics textbooks with respect to the programming section.

One final follow-up study can be involved the exercise problems that require different cognitive levels. How do the textbooks influence the students’ cognitive levels? Are the textbooks enough for developing higher cognitive requirements or is there a need for extra material and other specific training? For students who have difficulty learning mathematics, how can the teachers help improve their cognitive levels using the textbooks as a sole source? This study would involve classroom observations, interviewing teachers and students and well-designed tests. The result of this study can open up the bridge between the textbook contents and the actual classroom performance. Also, it can help the textbook authors who are not teachers themselves to get a better understanding of how do the textbooks being used to help students’ cognitive developments in mathematics.

This study has provided me with useful knowledge of using the textbook to prepare mathematics lectures. When I first came to Sweden to study mathematics, textbooks were just a tool to pass all the exams from a student perspective. Today, on my journey to become a mathematics teacher in Sweden, I have gained meaningful insight into the usage of the textbook through this study. The three frameworks I have applied in this study also increase my understanding, the characteristics of the textbooks, the worked
problems and the exercises from a pedagogical perspective. In other words, I have accumulated different tools to prepare my teaching career. As a teacher-to-be, it is very important for the teacher to be aware of the students’ different needs and levels when teaching and planning for lectures. This can be achieved only if the teachers are familiar with the textbook contents. Finally, if the teachers have time and energy, it is useful to compare different textbooks nationally and internationally to see how does the same topic presented differently.
7. References


## Appendix 1

A table of the translated Swedish and Chinese curriculum goals.

<table>
<thead>
<tr>
<th>Sweden curriculum goals</th>
<th>China curriculum goals</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Concept</strong></td>
<td><strong>Abstraction</strong></td>
</tr>
<tr>
<td>Apply and describe the</td>
<td>Recognize and process</td>
</tr>
<tr>
<td>meanings of mathematical</td>
<td>the essential attributes</td>
</tr>
<tr>
<td>concepts as well as the</td>
<td>of mathematical problems.</td>
</tr>
<tr>
<td>relationships.</td>
<td></td>
</tr>
<tr>
<td><strong>Procedure</strong></td>
<td><strong>Data processing</strong></td>
</tr>
<tr>
<td>Handle procedures and</td>
<td>Collect and process big</td>
</tr>
<tr>
<td>solve practical problems</td>
<td>data to extract useful</td>
</tr>
<tr>
<td>of standard characteristics with or without tools.</td>
<td>information for the</td>
</tr>
<tr>
<td></td>
<td>researched questions and</td>
</tr>
<tr>
<td></td>
<td>reasonable judgement.</td>
</tr>
<tr>
<td><strong>Problem-solving</strong></td>
<td><strong>Problem-solving</strong></td>
</tr>
<tr>
<td>Formulate, solve and</td>
<td>Calculate, reformulate</td>
</tr>
<tr>
<td>analyse mathematical</td>
<td>and process data according</td>
</tr>
<tr>
<td>problems as well as</td>
<td>to the problems'</td>
</tr>
<tr>
<td>evaluate the selected</td>
<td>requirements and the</td>
</tr>
<tr>
<td>strategies, methods and</td>
<td>ability to solve</td>
</tr>
<tr>
<td>results.</td>
<td>problems with flexibility.</td>
</tr>
<tr>
<td><strong>Modeling</strong></td>
<td><strong>Application</strong></td>
</tr>
<tr>
<td>Interpret a realistic</td>
<td>Apply the mathematics</td>
</tr>
<tr>
<td>situation, design</td>
<td>knowledge, concepts and</td>
</tr>
</tbody>
</table>
| mathematical model and  | methods to solve real-
| evaluate a model's       | world problems.         |
| features and limitations.|                        |
| **Reasoning**           | **Reasoning**           |
| Understand, conduct     | Improve reasoning process|
| and assess mathematical  | and both deductive and |
| reasoning.              | inductive argumentation | mathematical thinking. |
| **Communication**       | **Spatial imagination** |
| Communicate mathematical | Create correct graphics  |
| thinking both orally and | for example to visualize|
| in written form.        | the images; reveal the   |
|                          | nature of the problems,  |
|                          | decompose and analyse    |
|                          | the elements of the     |
|                          | graphs.                 |
| **Relevance**           | **Innovation**          |
| Relate mathematics to    | Find effective ways to  |
| its meaning and apply    | analyse and solve        |
| them in other subjects   | practical problems; To   |
| in a professional, social| promote mathematical     |
| and historical context. | expression and           |
|                          | communication as well as|
|                          | knowledge acquisition.   |

Appendix 2

Original texts for the definitions of probability in the selected textbooks:

Source: (Exponent 1c, 2011, p.278)

Source: (Beijing Normal University Publishing Group, 2008, p.136)
Appendix 3

The marked practice exercise types in *E1c* based on the Swedish intended curriculum goals.

Förmågor

I det övergripande syftet i ämnesplanen i matematik beskrivs 7 olika matematiska förmågor som du ska få träna på. Varje kurs har sedan ett Centralt innehåll, t.ex. tal, algebra och geometri. För att undervisningen ska bli varierad och för att du ska få ett rikt matematiskt kunnande, har de olika uppgifterna i boken märkts med vilken förmåga de avser att träna. Forskare i Sverige och internationellt anser att här finns den enskilt största förbättringspotentialen för matematikundervisningen, att gå från en procedurbetonad matematik till en mer mångsidig.

1. Begreppsförmåga
2. Procedurförmåga
3. Problemlösningsförmåga
4. Modelleringsförmåga
5. Resonemangsformåga
6. Kommunikationsförmåga
7. Relevansförmåga

Source: (Exponent 1c, 2011, p.5)

ÖVA II

6011 I ett lotteri finns det 10 rader från 1 till 100. Av de 5 nummer som innehåller minst en tre Beräkna sannolikheten man slumpvis tar en lot

Source: (Exponent 1c, 2011, p.281)
## Appendix 4

The table of numerical data for the number of worked examples that modelled Polya’s problem-solving process in the selected mathematics textbooks.

Number of worked problems model Polya’s model in the selected textbooks

<table>
<thead>
<tr>
<th>Book</th>
<th>Number of worked problems</th>
<th>Polya’s problem-solving phases</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Understanding phase</td>
<td>Devising phase</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(15 worked problems)</td>
<td>(15 worked problems)</td>
</tr>
<tr>
<td>E1c</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>MM1c</td>
<td>11</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>Sweden</td>
<td>26 (100%)</td>
<td>26 (100%)</td>
<td>26 (100%)</td>
</tr>
<tr>
<td>Total</td>
<td>53</td>
<td>32 (100%)</td>
<td>26 (81.25%)</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>22</td>
<td>17</td>
</tr>
<tr>
<td>PEPB</td>
<td>10</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>BJNUPG</td>
<td>28 (87.50%)</td>
<td>32 (100%)</td>
<td>26 (81.25%)</td>
</tr>
</tbody>
</table>
## Appendix 5

The table of numerical data for mathematical and contextual features of the practice exercises in the selected mathematics textbooks.

### Mathematical and contextual features of the practice exercises

<table>
<thead>
<tr>
<th>Book</th>
<th>Mathematical features</th>
<th>Contextual features</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Single computation</td>
<td>Multiple computation</td>
</tr>
<tr>
<td>E1c (116 exercises)</td>
<td>30</td>
<td>86</td>
</tr>
<tr>
<td>MM1c (73 exercises)</td>
<td>25</td>
<td>48</td>
</tr>
<tr>
<td>Sweden Total</td>
<td>55 (29.1%)</td>
<td>134 (70.9%)</td>
</tr>
<tr>
<td>PEPB (40 exercises)</td>
<td>12</td>
<td>28</td>
</tr>
<tr>
<td>BJNUPG (46 exercises)</td>
<td>8</td>
<td>38</td>
</tr>
<tr>
<td>China Total</td>
<td>20 (23.25%)</td>
<td>66 (76.75%)</td>
</tr>
</tbody>
</table>
Appendix 6

The table of numerical data for the response type of practice exercises in the selected textbooks.

Response type of practice exercises in the textbooks

<table>
<thead>
<tr>
<th>Book</th>
<th>Response type (A sub-category of performance requirements)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Numerical answer only</td>
<td>86</td>
<td>7</td>
<td>23</td>
</tr>
<tr>
<td>E1c</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(116 exercises)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MM1c</td>
<td></td>
<td>56</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>(73 exercises)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Sweden Total</strong></td>
<td></td>
<td>142 (75.13%)</td>
<td>14 (7.41%)</td>
<td>33 (17.46%)</td>
</tr>
<tr>
<td>PEPB</td>
<td></td>
<td>8</td>
<td>11</td>
<td>21</td>
</tr>
<tr>
<td>(40 exercises)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BJNUPG</td>
<td></td>
<td>10</td>
<td>7</td>
<td>29</td>
</tr>
<tr>
<td>(46 exercises)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>China Total</strong></td>
<td></td>
<td>18 (20.93%)</td>
<td>18 (20.93%)</td>
<td>50 (59.14%)</td>
</tr>
</tbody>
</table>
Appendix 7

The table of numerical data for the cognitive requirements of the practice exercises in the selected textbooks.

Cognitive requirements of the practice exercises in the textbooks

<table>
<thead>
<tr>
<th>Book</th>
<th>Cognitive requirements (A sub-category of performance requirements)</th>
<th>Procedural practice</th>
<th>Conceptual understanding</th>
<th>Problem solving</th>
<th>Special requirement</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1c</td>
<td></td>
<td>65</td>
<td>0</td>
<td>38</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>MM1c</td>
<td></td>
<td>31</td>
<td>4</td>
<td>34</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Sweden</td>
<td></td>
<td>96 (50.79%)</td>
<td>4 (2.12%)</td>
<td>72 (38.10%)</td>
<td>11 (5.82%)</td>
<td>6 (3.17%)</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PEPB</td>
<td></td>
<td>11</td>
<td>7</td>
<td>15</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>BJNUPG</td>
<td></td>
<td>17</td>
<td>1</td>
<td>19</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>China</td>
<td></td>
<td>28 (32.56%)</td>
<td>8 (9.3%)</td>
<td>34 (39.53%)</td>
<td>9 (10.47%)</td>
<td>7 (8.14%)</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>