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Dyscalculia/Dyslexia – A Dichotomy?

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PREFACE.

This work is dedicated to the study of two fundamental learning disabilities – dyscalculia and dyslexia. We will prove that dyscalculia is not a concept by itself, but merely yet another sad guise of dyslexia. The approach is psychological aspects of mathematical didactics, which is contributed by S. Abdelmalek, and the conclusion is a strict mathematical proof – E. Andersson - of the fact stated above. The latter part of the thesis is also under peer view for publication in a Springer Verlag periodical and is thus a part of a concluded research project.

ABSTRACT.

In this article, we analyse similarities and differences in and between two very topical issues in today's learning disabilities, namely *dyscalculia* and *dyslexia*. More precisely, we introduce the nature of mathematics as science, which - of course - is the essence of the matter. From this, we will try to prove that *dyscalculia* is not a concept by itself, but merely yet another one of the sad guises of *dyslexia*. This will completely answer the question given by the title of the thesis.

Keywords: dyscalculia, dyslexia, mathematical science;

Contents.

1. Introduction.	6
2. Aim and research question.	8
3. Method.	9
3.1. Choice of sources and source criticism.	10
4. The nature of the language and the sad guise of dyslexia.	14
5. The nature of dyscalculia.	16
5.1. Dyscalculia statistics and world view.	18
5.2. Is dyscalculia a single independent condition?	19
5.3. What are the cognitive psychological explanations behind students' difficulties in mathematical problem solving?	20
5.4. The brain activity of dyscalculic- and dyslectic individuals	23
5.5. Is dyscalculia inherited?	24
5.6. In what way is dyscalculia a comorbidity of cognitive disabilities and different neurological processes?	25
5.7. The assessment of dyscalculia.	26
5.8. Is there a treatment for dyscalculia?	29
5.9. How effective are the interventions?	31
5.10. Who decides for the assessment and helping-aids for dyscalculic students in school?	32
5.11. Inclusion or not?	33
5.12. Summary.	34
6. The natural number system $\mathbb{N}$ - The mathematical/scientific perspective.	35
7. Analysis and conclusion.	37

1. Introduction.

In this paper, we will have a closer look at two major phenomena/diagnoses, which are major causes of student learning problems and unhealth; these are dyscalculia and dyslexia. These two phenomena can be related to the two parameters, which are essential for imparting – and profiting by – the teaching of a subject namely, on the one hand mathematics core as science (*dyscalculia*), and on the other hand imparting/communicating science by means of the characters (*dyslexia*) we humans have developed over eons with our intellect. Regarding *dyscalculia*, our overview of existing theories and previous research results below will reveal that focus has been solely on the latter – the imparting/communicating of science, whereas one have completely failed in weighing in the meaning of mathematics as science and the axiomatic presentation of the natural number system $\mathbb{N}$.

In order to substantiate the possible outcome of our work, we have effectuated a pre-study consisting of a series of interviews performed among students with the two learning disabilities, where the students have been asked the following questions.

- (i) *In which part/parts of the sequence 'reading and understanding a mathematical problem/elaborating/formulating/writing/presenting a solution' do you experience - encounter with - difficulties?;*
- (ii) *Have you ever been experiencing problems with time and/or orientating yourself in everyday life and/or remembering courses of events?;*
- (iii) *Do you feel that presenting your solutions to a mathematical problem is simplified by using a computer or similar?.*

Here, the *dyslectics* will run into problem at the beginning and at the end of the sequence in question (i) above, while the student with *dyscalculia* would encounter with difficulties in the mid section. The answers obtained have *all* been of the following kinds.

- (i) *'I can read, but sometimes the letters and the words are jumping around.';*

'I can read, but sometimes I don't understand what I read and because of that, I can't solve the problem.';

'I can read digits, but sometimes it's also jumping around a bit, and then I have to read the problem many times. When I have read the problem many times, then I understand and can solve the problem.'

(ii) *'No, I can see how things happen in the right order.'*

(iii) *'Yes, it's much easier to use a computer, because I don't write very well, so I can see what I have written much better on the computer.';*

'I can read the letters much easier on the computer.';

'Multiplication is easier on the computer, because then the numbers don't jump around.';

'I don't know why, but it's much easier to read both letters and numbers on the computer.';

'I can solve problems in mathematics both better and easier on the computer.'

Considering the research ethics of the interviews, the subjects were well aware of the purpose of the study as well as how their responses (data) will matter in future research. In addition, the subjects were informed that they can stop the interview at any time they wanted and were ensured that the study would not cause any mental or physical harm, and would be used solely for these research purposes. Finally, the investigator clarified the confidentiality aspects and assured the subjects that their names and/or or responses would never be revealed unanonymously to anyone but the investigator. The researcher was unable to record the informants responses, since the subjects did not want to be recorded.

We will use the following disposition of the work. In *Section 4-5*, we will give the definition and the theoretical background of dyscalculia and dyslexia along with some results of previous research in this area. As dyscalculia is the main issue of the thesis, it will thus be given more space than dyslexia. In *Section 6*, we introduce the mathematical background, and, finally, in *Section 7*, we analyse and discuss the results from the previous sections of the thesis.

2. Aim and research question.

The above will lead us to a question, which have divided the Parnassus of experts into two camps, and whose answer will reasonably require knowledge of the second parameter – the scientific structure of mathematics – as well:

Is dyscalculia as defined an own concept or merely yet another one of the sad guises of dyslexia?

The purpose of the thesis is thereby to study existing research literature with respect to its contents and to try to find out whether or not the mathematical approach has been availed of before (which will also give us good overview of the concept of dyscalculia/dyslexia); and if not, to introduce the second parameter – mathematics as science - in an attempt to come closer to the answer of this question as formulated above. The answer will be substantiated by the outcome of the interview constituting the pre-study accounted for in *Section 1*.

3. Method.

In our search for literature, we have started in the mid 1990's, and we have tried to follow the research development of dyscalculia up until to today. This have lead us to some excellent works (*e.g.* Sjöberg 2006 and Lundberg and Sterner 2009), which give a fine overview of the subject and which are covering essentially all of the aspects focused on in our other references. We have used the keywords stated in the *Abstract* in order to retrieve adequate references from the research data base. The selection has been made with respect to the researchers' – in our opinion - impact on the field of work. *E.g.* Brian Butterworth – Butterworth 2003 and Butterworth 2008 - is a professor em. of cognitive neuropsychology, and his work has been held in high esteem all over the world. He diagnosed former president of the USA, Ronald Reagan, with Alzheimer's disease merely from hearing one of his speeches ten years before he formally received the diagnosis. This has given us a fine overview of the existing theories and previous research in the area of dyscalculia and dyslexia (*Section 4-5!*).

The articles/literature of previous research results have be analyzed with respect to their contents in order to give a good overview of the subject as well as to establish whether or not the mathematical/scientifical approach – resulting in the strict mathematical proof (*cf. Section 6, Theorem 1 and Remark 2*) answering the question above - is genuinely new, or if the same approach has been availed of before. As one of the authors is a former *Associate Professor* in mathematics (Center for Mathematical Sciences, Mathematics, Faculty of Science, Lund University), we found no need for the use of any other adequate tools for analyzing the literature in this respect. Should it have emerged that the same approach has been availed of before, we would have to compare those results with our own, in order to search and analyze the discrepancies between them. They would undoubtedly have existed. Since, as it turned out, there were no sources which took this important concept into consideration, our purpose of choice of sources was to argue against earlier theories as well as finding relevant information and findings for indirectly supporting our argument. Moreover, we examined various types of sources for realizing if there was any consistency versus contradiction for how dyscalculia is individually experienced and viewed upon today.

Therefore, we have chosen this method, which we believe will give us the best overview of the research area, existing theories and previous research results.

3.1. Choice of sources and source criticism.

The purpose of source criticism is to understand why current sources were chosen, examined and tested. Since, there were no sources which directly confirmed our theory; our purpose of choice of sources was to argue against earlier theories as well as finding relevant information and findings for indirectly supporting our argument. Moreover, we examined various types of sources for realizing if there was any consistency versus contradiction for how dyscalculia is individually experienced and viewed upon today.

Besides considering national Swedish articles which indirectly are a reflection of the needs and political system of the Swedish society, we considered international sources for investigating how other countries view and tackle dyscalculia and dyslexia. A country's definition of dyscalculia determines how the school manage dyscalculia and how much resources are invested in the helping-aids. In addition, if dyscalculia was not defined as a single learning disability, more economic- and mental efforts would be required from the responsible school authorities. Moreover, these required efforts indicates that the mathematical problems do not come from the student, but is a result from a lack in teaching methods and/or mathematical resources.

In order to broaden our knowledge and understand how dyscalculia and dyslexia develop, mutates, changes the brain activities, and how the psychological personal motivation for solving mathematics is effected and mattering in the social and educational settings, we examined sources from different disciplines. We choose to use neurological and medical articles written by medical doctors to understand and compare the brain activities and/or changes in the neural structures between people with dyscalculia, dyslexia or without any diagnose. Further, we considered the psychological developmental and learning articles which are mainly written by psychologists to understand the cognitive explanation for the development and maintenance of dyscalculia. To understand how dyscalculia is managed in school, we investigated educational and social articles which are written by specialized teachers and other school authorities who work in school. Almost each research (article) we choose is conducted and designed according to the researchers work perspective and personal beliefs in learning disabilities. For revealing possible subjectivity, bias and/or political pressure, and as mentioned before, we consider international sources which reflect different national educational beliefs, school politics and various educational systems.

In addition, to scientific journals, we have read and examined books and handbooks in which the author whom is often a researcher as well, has subjectively collected and investigated different articles for supporting his or her argument. Such books and handbooks are usually written in a “reader friendly” way which clearly explains medical, psychological and educational terminologies. Further, such handbooks target the public and are often found in schools and other public sectors for guiding and acknowledging the people about dyscalculia. The chosen sources mentioned above, which mainly come from four different disciplines and/or perspectives (medical, psychological, educational/social and governmental organizations) and target different audiences, are either first-hand and primary based or secondary and second-hand based.

Firstly, we discuss second-hand and secondary sources, such as handbooks, books and governmental organizations handouts whose base their summarized information on statistics and information from primary and first-hand articles in scientific journals. It is important to consider such secondary findings, since they target the larger number of the population due to its “user friendly” language and explanation methods. Since the aim of this paper is to change the public opinion of dyscalculia and redefine its nature as a single disability, we pay attention to what kind of information that reaches the public who do not necessarily have any education or even general knowledge about dyscalculia. Unfortunately, as the information is written in a “language friendly” manner, meaning that it does not contain difficult medical, psychological or mathematical terminology, people believe and comply with what they read and can therefore think they suffer from dyscalculia without searching for any primary and first-hand research based information. Even worse, those who assess and work with dyscalculia in schools, many times diagnose students and apply interventions based on such “user friendly” information as it is easier to understand, interpret and apply. However, the average public may not acknowledge the fact that such organizations argument and presented facts are based on statistical findings which may be incorrect and inaccurate due to insufficient research techniques and/or the researchers’ bias effect (subjectivity). Unfortunately, many times the research is formed and conducted to reach and confirm the hypothesis.

Secondly, other types of secondary and second-hand sources are social and educational articles written by specialized teachers who many times collect previous investigated and tested data and draw their own conclusions and thereby theories out of such. Again, as mentioned above, occasionally such articles or rather “collections” are for supporting the already existing educational system. In order to critically test the secondary data, we choose as will be discussed in coming section, first-hand articles to reveal and read about the original studies.

Thirdly, we discuss and support our argument based on different learning and developmental psychological articles from peer-reviewed journals which mainly discuss the psychological cognitive perspective for explaining dyscalculia. These articles are primary and first-hand data, meaning that the researcher conducted and saw the results with his or her own eyes and thereby developed a theory. However, these first-hand and primary articles also include secondary and second-hand sources for the literature reviews and to compare their findings with. In order to understand these peer-reviewed primary psychological articles, the reader needs to have some basic or rather advanced education in psychology as well as having a great deal of knowledge about dyscalculia and dyslexia. These articles usually target and are written for psychologists and/or medical doctors, who are not the ones who assess and implement helping aids in school for the students who believe they suffer from dyscalculia. In this paper, we considered the findings from a psychological perspective and compared them to the mathematical explanations. When analysing the articles, we were well aware of the researcher's effect which can alter the findings due to the researcher's subjectivity and expectations of the research. In order to eliminate such effects, we analysed many different psychological articles and compared the results. We were also aware of which context and country the research was conducted in, since the public general opinion which is formed according to the country's ideology, strongly affect the subjects' response in the research. Again and unfortunately, teachers and other responsible authorities in school who assess and are responsible for the helping aids for dyscalculic students lack higher education in psychology and therefore fail to understand the first-hand and primary psychological articles. Such teachers and personal prefer to read the "user friendly" articles which as stated above are secondary and second-hand sources which many times are conducted according to the desired and needed results which suit and match the school resources, physical structure, inclusion ideology and abilities to meet and handle dyscalculia.

Our fourth choice of sources is first-hand and primary articles from neurological- and medically based journals. When analysing such articles, we examined and discussed the biological theories for explaining dyscalculia in relation to dyslexia. Those medically and biologically based articles are written and explained through medical terminologies and require a higher education in biology and medicine in order to understand the medical terminologies. Like psychologist, medical doctors are not the ones who assess and determine for necessary implementations for dyscalculia helping-aids and therefore such medical and neurological results loses its validity when it's perceived and evaluated by a non-medically educated audience who are the ones dealing with dyscalculia students in school. When such sources are

interpreted without basic information in medicine, the reader can perceive the results and suggestions from a subjective perspective that fits the already existing beliefs and abilities to tackle dyscalculia.

The nationality of the researcher and subjects and the publishing dates of our sources did not significantly matter to our argument, since the idea that dyscalculia is an independent disability is still believed worldwide since many years. However, as mentioned above in earlier section, each countries' strategies of diagnosing and dealing with dyscalculia differ from country to country according to how much resources are spent on learning disabilities. What is relevant about all primary and first-hand sources is that they target different audiences which almost all do not directly work with dyscalculia. Those who do assess and determine for the implementation of helping-aids use secondary and second-hand data due to its "user friendly" language which do not go in depth of explaining dyscalculia. Furthermore, the "user friendly" articles inhibit the reader's ability to realize that dyscalculia is not a single independent disability, but a dark form of dyslexia.

This paper has discussed various research and findings based on different scientific methods which also come from different perspectives, disciplines and countries. It is important to examine dyscalculia from all possible different perspectives in order to reach a broader view of understanding the nature of dyscalculia and its relation to dyslexia. In addition, examining different research methods enables the researcher to reveal for possible subjectivity as well as pointing out how each country's ideology indirectly determines what kind of research is conducted for reaching the desired results that suit the country's political, school- and educational system. Today dyscalculia is still defined as an independent learning disability, which can be due to the fact that if dyscalculia did not exist, the school would be obliged to change their teaching methods and/or invest in more mathematic resources which may be an economic- and mental burden.

4. The nature of the language and the sad guise of dyslexia.

The language, spoken and written, and the body language (referred to as a concept and thus independent of any particular idiom) is something the human race - with our eminent intellect - have developed during eons in order to be able to exchange thoughts ideas, opinions and - to some extent - feelings with one another, *i.e.*, the language - spoken and written - is the human race's existing and adequate means of communication. In this note, we choose to keep focus on the language *spoken and written* (henceforth referred to as - simply - *the language*). This is also, above all others, the academic world's principal means of communication.

However, *the language* manifests significant shortcomings in potential to express mainly *i.a.* feelings. *The language* does not offer adequate means of, clear and unequivocally, expressing our inmost meanings and our inmost feelings, but these abstract phenomena every now and then evince significant discrepancies between the sequences *experience/feel* and *dress in words/by means of writing or verbally impart to the world around us*. In every sequence, there will always some *noise* and some *losses*, and if I *e.g.* get an inner picture of my neighbor's red car, and I utter the words *my neighbor's red car* before a collection of ten people, these people will probably first of all paint a picture of a red car and then try to render their associations. This will most likely lead to ten different allusions to my neighbor's red car, which was the original toe hold for take-off for the entire discussion.

This is also reflected by the fact that a pseudo-science such as jurisprudence essentially has its main part of justification in the imperfection of *the language*. We cannot in an Act clear and unequivocally express what should be ratified. Whether it is the members of the supreme court, which have to pass a verdict in a precedent or it is some company lawyers, which have *to elaborate a business agreement, in the end it all comes down to establishing principles of* interpretating the written lines in the adequate Act or in the adequate business agreement. Should *the language* be perfect in this sense, jurisprudence would lose most of its legality...

A *dyslectic* is (*cf.* Lundberg and Sterner (2009), p. 33, and Specialpedagogiska Skolmyndigheten (2012)!) a person with reading- and writing difficulties but with an - in other parts - fine working intellect. I the process of *experience/feel* and *dress in words/by means of writing or verbally impart to the world around us*, the subsequence of *experiencing, feeling and dressing in words* will not pose an obstacle. Instead, the difficulties will arise during the sequence of *by means of writing impart to - or collect from - the world around us*. *Cf.* the

quotation from *Section 1*: *'I can read, but sometimes I don't understand what I read and because of that, I can't solve the problem.'* This strongly indicates that the crux of the matter does not lie in understanding, treating and elaborating a solution to a problem, but in writing conveying the solution to the reader. This is also underlined by the fact that most dyslectics are very benefitted by the use of a computer, where the shaping and distinction of different letters becomes a smaller issue.

5. The nature of dyscalculia.

In Lundberg and Sterner (2009), pp. 20-24, (and further references therein), dyscalculia is defined as *reduced ability of determining numbers and developing a mental number axis*, but where, in other parts and quite analogously to a dyslectic, the person suffering from it has a well-behaved intellect. Thus, dyscalculia - as commonly known - is difficulties in understanding the construction of the mathematical system of the natural numbers and developing a mental number axis, *i.e.*, the set of numbers $\mathbb{N}=\{0,1,2,\dots\}$ and its mathematical structure with the ordinary binary operations of addition $+$ and multiplication $\cdot$. In the process of elaborating a mental number axis it is natural to (*cf.* Lundberg and Sterner (2009), p. 20!) logarithmically placing the numbers on the axis - the higher the numbers, the bigger the tendency of lumping them together - so that neighboring numbers will be lying closer together than numbers at the beginning of the scale. Not until after a couple of years of formal education in mathematics will the scale become linear according to Lundberg and Sterner (2009)!

The following examples are recited from Lundberg, Sterner (2009), pp. 20-21.

Example 1. This example (*cf.* Lonnemann et al. (2008)!) is presenting a test, where three numbers, *e.g.* 26, 37, 82, are shown to the student, and where the problem is to, as fast as possible, tell which distance, with respect to quantity, is the largest; the distance between 26 and 37 or the distance between 37 and 82? The physical distance between the three numbers is altered in a variety of ways, where, in one condition, a small distance between 26 and 37 and a large distance between 37 and 82 is marked

(26,.....37,.....82).

Here, we thus have congruence between the differences in size between the numbers and the physical distance between them. In another condition, there is incongruence:

(26,.....37,.....82).

Those with a mental, linear number axis apparently will be encountering with more difficulties in the incongruent condition, yielding longer reaction time and more errors, while the

difficulties in the two conditions above will be of more equal degree for those, who have as yet not developed a functionable number axis, *i.e.*, students with claimed reduced counting abilities do not encounter with the same amount of difficulties in the incongruent case.

Example 2. In another example, the character of the number axis is determined by letting the student, on a horizontal line, mark where a given number should be placed. The line can *e.g.* have the numbers 0 and 100 marked at the two endpoints. For each marking conducted by the student, the distance to the correct position is measured. Thereupon, the students markings are plotted in a diagram, and they are compared with the linear position, whereupon one can calculate how much of the variation, which is due to a linear function, and to what extent a logarithmic function can characterize the students markings. The more linear the function, the more mature the sense of numbers and the less the risk of difficulties with numbers and calculations (*cf.* Opfer and Siegler (2007)!).

Example 3. Another interesting attempt in pinpointing the absolute form of dyscalculia has been performed by Butterworth (2003). He has developed a computer based screening instrument, where he is trying to isolate the basic lacking in ability in telling numbers (*'how many'*) (*core systems*). In a partial test the student is shown a picture of a number of filled circles or other subjects in the left part of the computer screen, while the right part shows a figure. The mission is simple. Which one of the filled circle or the figure does represent the largest number? The student answers by pressing one of two buttons, whereupon the lapse of time between the appearing of the picture and the pressing of the button is registered with the tolerance of 0,001 s. Butterworth testifies to that a correct answer alone is not sufficient, but the sense of numbers is inherent in the ability of rapidly discerning the correct number, when viewing the picture. Butterworth is of the opinion that a person with absolute dyscalculia is incapable of this.

For further details, *cf.* Lonnemann et al. (2008)! The common factor of all of these studies - and all the theories - is however the focus on the fact that people with dyscalculia are having difficulties with the structure of the aforementioned natural number system, where the starting point thus is that the natural numbers are totally connected with 0,1,2,... like we with our - for communication purposes - developed language have chosen to denote them.

An excellent and detailed overview of essentially the entire material from all of our other references is offered by Sjöberg (2006). In this thesis, he mentions more or less all aspects of dyscalculia, which have been the toehold for take-off of pretty much all research studies performed within this area. Sjöberg emphasizes that whichever model you choose in order to

pinpoint down the nature of dyscalculia, almost all research literature can be derived from the *medical-neurological*- and the *neuropsychological* area. Sjöberg also testifies to the fact that there is big confusion around the very concept of dyscalculia. He believes that one explanation for this is that the area of expertise engages a lot of experts from different professions. There are physicians, neuropsychologists, pedagogues/educationalists and persons from students' welfare. Last but not least there are of course parents of the children with unsatisfactory results in school mathematics. Ginsburg (1997) certifies to the fact that a lot of children back then receive a diagnosis - *i.a.*, *dyscalculia* – while those establishing the diagnosis are lacking in adequate competences within the area.

Although it is apparent that the essence of the problem is mathematics as science – the axiomatic presentation of the natural number system $\mathbb{N}$ - it is, in the light of the facts having emerged in this section, safe to say that not one single time has a *mathematician* been asked advice within this area, and not one single article has a mathematical approach to this set of problems. Mathematics being the only science 100% based on causality with proofs watertight down to the axiomatic level; while all other sciences have a certain – more or less – degree of correlation/empiricism involved, and hence the truth of their scientific results is dependent on whether the researchers have focused on the correct parameters or not.

5.1. Dyscalculia statistics and world view.

Worldwide and according to UNESCO dyscalculia, which is often referred to as developmental dyscalculia is defined as a condition resulting from neurological differences in the brain. The fact that dyscalculia and dyslexia are two different independent conditions, is a worldwide common belief. Children and adults with dyscalculia struggle in understanding numbers and encoding or transforming them into useful information. Also, in dyscalculia the person cannot see any logic meaning or coherence between numbers, including right sequences and logical order of smallest to biggest. Since the struggle is in translating the numbers, the difficulties do not derive from the “number sense” like many studies argue, but in language and understanding/comprehending language. In other words, it is not about numbers in themselves but about how numbers in terms of symbols are encoded and processed in the brain. The struggles to understand the language is due to different malfunctioning of different cognitive

abilities such as short- and long term memory, placing the numbers/information in cognitive schemas and being able to retrieve the information in a coherent manner.

UNESCO (2014-2019) has proposed many studies that have shown that dyscalculia can be detected in a very early age, where babies lack this “number sense”. Szucs et al., (2013) reported that 3-6 percent of the world’s population suffers from dyscalculia. Devine et al., (2013) conducted a study showing that there was no significant gender differences in the prevalence of dyscalculia. However, it is near impossible to reach a conclusive estimation of the how many individuals worldwide suffer from dyscalculia, since each country has its own assessment system. Some countries measure the IQ and/or administer standard tests such as the American SAT (a test examining the student’s general knowledge), which is a mainly mathematics and language test, on a single occasion. Other countries, such as Sweden has various assessment strategies, like for instance observing the individual in different settings besides the standardized tests. In addition, since one’s cognitive abilities vary from day to day statistics may be inaccurate in testing. Results from one single testing occasion should not be defining a general diagnose for the person.

As countries differ in the explanation for dyscalculia, the main responsible and decision making actors also vary. In Norway and Sweden for instance, the responsible actors is the school and other highly educated professionals within the educational discipline. In contrast, in Germany, a medical doctors and psychologist are the main decision makers in the assessment process as well as deciding for the appropriate helping-aids (Vårdgivarguiden (2015)). Regardless of who is responsible for assessing and deciding for suitable helping-aids, all interventions must be child/adult centred, meaning that the individual’s needs are the deciding and guiding factors.

5.2. Is dyscalculia a single independent condition?

Yet today, research is controversial regarding the causes of dyscalculia and dyslexia. Träff et al., (2017) have discussed the different biological, environmental, cognitive and neurological origins of dyscalculia and argues further that because of the different causes, treatment and helping-aids cannot be generalized, but must be individualized to each person’s origin of disability and thereby needs. Träff et al., (2017) distinguish between two categories of dyscalculia, primary and secondary dyscalculia. The primary is originated from inborn biological and neurological impairments whereas the secondary dyscalculia is due to memory,

attention and/or concentration difficulties, which further cause struggles to solve mathematical problems. This also explains why many children suffering from ADHD also get diagnosed for dyscalculia, which in this case is a secondary effect of the ADHD where the mathematical struggles arise from lack of attention and motivation for making the effort to solve mathematical tasks. That's why, once again, it is very important to pinpoint at the origin for the mathematical struggles in order to apply appropriate helping-aids.

However, Vårdgivarguiden (2015) presented a study where children suffering from dyscalculia were prescribed with a medication for ADHD and the result showed that their mathematical abilities and skills did not significantly improve, which supports the fact that dyscalculia is mainly originated from neurological differences in the brain and is a form of dyslexia. Even if the individual's attention and concentration is improved, the learning disability of reading, encoding and processing information still exists. Therefore, dyscalculia is not a single independent condition, but as this paper argues for, a form of dyslexia as it arises as a secondary effect from the dyslexic disability.

Vårdgivarguiden (2015) further argues that children with dyscalculia do not show consistent symptoms, in other words, do not struggle continuously as the magnitude and ability in perceiving, encoding and processing numbers vary significantly from one time to another. This further supports the argument that as dyscalculia is resulted as a secondary condition, it is a subcategory in dyslexia which alters itself according to the individual's primary cognitive conditions and abilities that determines the grade and magnitude of dyslexic struggles. When human beings are in stressful situations, one's cognitive abilities are affected in many different ways, such as concentration difficulties, memory losses and disability to process and retrieve information.

5.3. What are the cognitive psychological explanations behind students' difficulties in mathematical problem solving?

In cognitive psychology human behaviour is explained due to our mental process of perceiving, processing, storing and retrieving information. Cognitive psychologists argue that when change of behaviour is desired, focus should be upon the person's mental/cognitive process of

perceiving, encoding, translating and relating information to one's already existing information stored in our cognitive schemas. A cognitive schema is the where we store information and knowledge, which guides us in life and makes our different abilities and skills automated. For example, when we ride a bike, we can still talk and think about other unrelated things; since "riding a bike" has been done so many times that it has become automatic in our cognitive schemas. When applying this concept of cognitive schemas to problem solving in mathematics, students should start changing their negative thoughts of not being able to solve math, in other words altering the automatic negative perception to encoding the information in a positive way of thinking. This can be hard since encoding mathematics to a positive way of thinking does not match our already existing cognitive schemas.

In learning, humans have different ways and abilities of perceiving, encoding, storing and retrieving information. According to Tambychik and Thamby (2010) the main reason for experiencing difficulties in mathematical problem solving is due to the students' different cognitive abilities in learning which results in struggles when learning different mathematical problem-solving abilities. Studies showed that some students could solve mathematical problems that do not require transformation of information, which means encoding and processing the information. When relating this to dyscalculia, the problems in mathematics thus appear in transformation of the information, in other words to linguistically understand and encode the problem. This problem of encoding symbols in itself tells us that there is no such phenomena as dyscalculia on its own, but a type or form of dyslexia as it is all about encoding symbols whether they are numbers or letters. When the information through language is unclear, students get confused and find difficulties in decision making of how to solve the mathematical problem. Also, the disability of understanding or encoding the linguistic information, result in struggles when trying to recognize and relate the new incoming information to previous knowledge which is stored in the persons cognitive schemas.

Wilson et al., (2015) conducted a study showing that dyscalculia and dyslexia in adults, co-exists together in a very complex matter accompanied by neural and cognitive deficiencies. This study supports our purpose for this research that argues that dyscalculia and dyslexia is one category as an individual cannot suffer from dyscalculia alone, as it's a form or subcategory in dyslexia. Furthermore, considering dyslexia, research suggests that dyslexics has a malfunctioning phonological processing ability, which means that they cannot match letters and words with sounds to recognize the words at another time. Letters and words are symbols that our brain translates into useful information, therefore if one would have difficulties encoding and processing letters the same would go for numbers since numbers are also symbols. As single

letters causes same struggles as single numbers, words cause same difficulties as longer numbers (many digits) do. All interviewed informants for this study, stressed on the fact that the longer/greater the numbers are, the more difficulties, likewise longer words they experience. This confirmation, argues that the problems in dyscalculia and dyslexia arises from the perceiving, understanding and thereby encoding information of *all* kinds of symbols.

Tambychik and Thamby's (2010) study mentioned above, examines the students' own experience, where the researchers have measured how the students' cognitive abilities operate. Firstly, Tambychik and Thamby explain how problem solving is manifested and how the students experience difficulties. There are two aspects of the problem-solving issue, where the first is how the student understand/translate the problem linguistically (words). The second problems-solving issue is the non-linguistic aspect, which refers to numbers and graphs. Along with these two aspects, the problem-solving difficulties can result in how the linguistic information is translated to mathematical terms (numbers) and/or how the numbers should be solved/computed. Students showed more difficulties in the first aspect in which they transform the words into numbers and graphs (Tambychik and Thamby, 2010). The results from Tambychik and Thamby's study again supports the fact that difficulties in the mathematical problem-solving is due to only dyslexia, explaining that dyscalculia is a *form* or *way* in which of dyslexia is showing itself. If one cannot read the text then knowing which learning technique and problem-solving method they should use is impossible. Since, the information cannot be understood while our working-memory is not functioning, the knowledge will never go to our long-term memory and/or be stored in our cognitive schemas for later retrieval and use.

Bugden and Ansari (2014) argues that children and adults with dyscalculia suffer from different cognitive disabilities like for instance, both long- and short-term memory struggles as well as difficulties in recognizing and translating the numbers to meaningful and useful information in the brain. In the case of dyslexia, the cognitive disabilities are similar, as one's ability to understand and transform letters into useful information which fits with our memorized cognitive schemas is limited. Bugden and Ansari (2014) have like other researchers showed how the non-diagnosed children brains work differently from the dyscalculia brain and stresses on the fact the dyscalculia brain do not have any brain damage, it just works differently. As mentioned in earlier section, when dyscalculia is caused by cognitive disabilities, it is categorized as a secondary dyscalculia which also explains that dyscalculia and dyslexia should be in one category since the cognitive origin of the struggles effect reading all types of symbols and does not distinguish between letters or numbers.

The same study by Bugden and Ansari (2014) mentioned above, suggests that the struggling in encoding linguistic information, can affect different cognitive abilities and vary in magnitude. For instance, when recognizing the information, understanding, associating, processing and/or retrieving and explaining it, dyscalculics differ from each other in both the cognitive abilities as well as grade of difficulties. Therefore, it is vital that teachers and other educational responsible authorities are well aware of where the struggles starts and in which stage the problem is most critical in order to adjust the structure of the lesson and the helping techniques. According to the informants for this research likewise in most cases of learning disabilities, it is very effective to provide a lot of pictures with the text, where the pictures resembles and explains the information in the text. When the students are exposed to pictures, they grasp a great deal of information meanwhile they save a lot of energy for the reading part, in which becomes much less to understand due to the helping pictures. In addition, besides the helping pictures, the text and numbers should be in larger font with more space between the lines.

5.4. The brain activity of dyscalculic- and dyslectic individuals.

A study conducted by Petersa et al., (2010) analyses similarities versus differences in dyscalculic and dyslexic children in comparison to a control group of non-diagnosed children. Results show that children with both learning disabilities, if we faulty distinguish between them, have similar brain activities and neural process but differed significantly from children without any learning disability at all. In brain screenings of individuals suffering from dyscalculia, images show that the brain's parietal lobes function and cerebral hemispheres development differ from a non-dyscalculic person, as they do not present same kinds of activities. When the right hemisphere is working poorly, the person suffers in understanding the assets in quantities, cannot learn sequences in space and have difficulties in solving daily "automated" problem solving math. When the left hemisphere is not working properly, the individual faces struggles in transforming the numbers into useful information and putting the numbers in a logic meaning (Sudha and Shalini, 2014). Screen images have also shown that dyscalculic and dyslexic individuals have similar brain activities as well as neurological processes in their brains and

therefore, a dyslexic person experience same struggles, but with letters instead of numbers which can be explained by the same neural processes when encoding symbols of any kind.

The findings above supports our argument that dyscalculia is a form of dyslexia due to the similar brain activities and neurological processes in the dyscalculic and dyslexic brain. Furthermore, in supporting that argument of this paper, the biological processes and psychological cognitive disabilities are both significant causing factors in dyslexia and thereby dyscalculia. The neurological structure and processes is affected by the cognitive abilities and vice versa. Both disabilities are caused and strengthened from the different working cognitive abilities, which means that when the individual start experiencing difficulties in math due to for instance a short-memory or lack of attention, the neurological processes is affected and thereby altered or not trained, which inhibits the actions of mathematical skills to become automatic. When an action is performed several times, the neurological actions for that process are strengthened and eventually become automatic and is stored in our cognitive schemas. In addition, next section will discuss when the individual experiences anxiety and negative feelings towards mathematics, the cognitive abilities are affected which furthermore alters the neural processes in the brain.

5.5. Is dyscalculia inherited?

Butterworth (2003) suggests that dyscalculia may be inherited due to a study conducted on twins. The results of this study argue for a particular position on the X- chromosome, although this dislocation is not present at all cases of dyscalculic individuals. SPSM (2016) claims that the disposition of developing dyslexia is inherited and since dyscalculia is form a dyslexia, it may also be a relevant factor for dyscalculic individuals. In this case, one can argue that this disposition arises from environmental factors and not biological, as the child sees the parent having struggles and negative thought towards mathematics and therefore acquire similar feelings to even try in understanding mathematics. Likewise, when it comes to mathematical anxiety, Butterworth (2003) presents a study of nine year old children who claim that they have dyscalculia, showing that they are very anxious regarding mathematical problem solving. This anxiety can result from the parent's negative feeling towards mathematics and is therefore an inherited psychological disposition of developing mathematical anxiety, which is a secondary kind of dyscalculia.

In the secondary category of dyscalculia where anxiety is the origin of the problem, such negative and demotivating feelings towards mathematics in turn alters the cognitive abilities, such as encoding, processing and retrieving information. If the child has a deep and strong belief that he or she lacks the mathematical skills like their parents, the trying to solve in itself will be hard since it does not fit the automated cognitive schema of “I cannot do math”.

Kaufmann and Von Aster’s research (2012) and other studies suggest that environmental factors such as drinking alcohol during pregnancy and premature birth, can cause dyscalculia and other learning disabilities. In this case such environmental factors results in biological differences in the exposed child’s neurological system, in other words, the environmental factors are transmitted and become biologically manifested. Furthermore, Kaufmann and Von Aster (2012) suggest that dyscalculia frequently occurs in children suffering from disorders in the metabolic system, various genetic alterations such as fragile X-syndromes, William Beuren syndrome and Velocardiofacial syndrome. These disorders can be inherited in a way that the child develop a disposition for the development of dyscalculia, or in other words for the struggles of encoding, understanding, processing and retrieving information. However, there has not been enough data supporting these biological causes for dyscalculia, which again argues for the fact that dyscalculia is a form of dyslexia having the same causal factors for developing.

5.6. In what way is dyscalculia a comorbidity of cognitive disabilities and different neurological processes?

As mentioned above, some studies suggest that dyscalculia is a comorbidity which arises in hand with other learning disabilities and/or improper cognitive abilities such as short- and long-term memory, attention disabilities, concentration struggles and/or information processing difficulties. Also, ADHD has been associated with children suffering from dyscalculia as in this case dyscalculia is a comorbidity and secondary condition of the ADHD effects from the lack of concentration and short attention span. However, as shown in previous section, when students were administered with ADHD medication to improve the concentration and attention, the struggles in mathematical problem solving did not improve.

Kaufmann and Von Aster (2012) conclude their study that dyscalculia is a comorbidity of many different learning disabilities, neurological differences and/or mental illnesses. Such results clearly show that dyscalculia cannot be defined as one independent disability, but a

“side-effect” of other disabilities and/or illnesses. Mathematical skills and tasks usually requires more thinking, understanding and mental processing in order to become automated in our learning systems and that is why it may be the very first affected skill that suffers when the individual is smitten by a mental illness and/or cognitive disability. In addition, as Adler (2001) argues, the two of the basic cognitive processing methods in solving mathematical tasks is the ability to recognize and to see logic patterns. Therefore if the individual has a malfunctioning short- and long-term memory, it may be very hard to recognize symbols (number and letters) and to see logic patterns. The same happens in the case of dyslexia, when the person fails to recognize letters and words, he or she cannot understand a sentence. In this case, again dyscalculia is a secondary effect (side-effect) and a part of dyslexia in which the individual cannot recognize and associate symbols in general.

To sum up, dyscalculia is a comorbidity of a vicious cycle in which every process affects each other. First we feel anxious and worried about mathematical tasks which affects our cognitive abilities for short-term memory, concentration and attention which finally causes our neurological process to not be trained and work as they should and therefore the mathematical skills do not become automated. The alteration of the neurological process which inhibits the strengthening of the involved brain cells causes the mathematical skills to not become automated and that is why dyscalculic and dyslexic individuals cannot recognize, associate and see logic patterns.

5.7. The assessment of dyscalculia.

Firstly, the assessment section in this paper is extensive due to the fact that it is the main key for realizing and handling dyscalculia. Furthermore, we must be well acquainted with the origins of dyscalculia in order design and implement appropriate interventions and further evaluating the results and thereby altering and updating the interventions. As Adler (2001) argues, the assessment of dyscalculia is vital as soon as possible on both an individual level as well as for developing more knowledge in our society.

Some teachers and other educational authorities believe that giving the individual a dyscalculia diagnose may inhibit the person from trying to learn mathematics while depending on his or her diagnose. However, Adler (2001) showed that diagnosing an individual reinforces the person to handle mathematics in a different and more positive manner, as the person does not feel stupid anymore as before. In such case the dyscalculic individual's self-esteem

improves due to the explaining diagnose of earlier failure. In the assessment procedure, the responsible assessor must map out what the person should not work with as well as pointing out the individual's effective abilities and strengths. All subject teachers must be involved in the assessment map, where the assessor and the student together fills out a scale document of how well the individual works in various subjects. The student's theoretical versus practical abilities in all subjects should also be analysed in addition to the student's self-perception of his or her own abilities, strengths and weaknesses. This document should be updated continuously and simultaneously with the evaluation of the interventions. The child's parents and/or caregivers should be well informed and updated with this document. It is also up to each student to decide whether the rest of the class should be informed about the learning disability or not.

According to Runström Nilsson (2012), in Sweden today the assessment map-documents usually identifies the individual's strengths versus weakness on three levels:

1. The school level: the school personal, teachers and school system of education are assessed.
2. The classroom level: the interaction and socialization with the classmates in class and during the breaks, teamwork, daily schedule and the physical appearance of the classroom are analysed.
3. The individual level: the individual's own experience and feelings towards each subject in terms of strengths and/or weakness are identified. On the individual level, the person engages in a self-evaluation process to assess the individual capacity in different situations and time of the day. For instance, the dyscalculic student may experience greater or less struggles in the morning or after the lunchbreak.

In addition, due to a diagnose and thereby pinpointing at the particular mathematical problems as well as incorporating suitable interventions, the individual receives the appropriate helping-aids and a tailor made mathematical program. By incorporating the appropriate helping techniques, the child and/or adult can finally experience the feeling of success and victory (Adler, 2001).

As for the societal benefits, by providing more diagnoses and not neglecting mathematical struggles, diagnoses opens up the way for more extensive research and knowledge, which in turn can help develop more effective helping strategies. When an individual struggles in mathematics, it is too often neglected and blamed for as laziness or dislike of mathematics and this is one of many reasons why dyscalculia is not researched enough and explained

conclusively. However, it can still be tricky to assess dyscalculia as the individual's cognitive abilities vary from day to day. The change in cognitive abilities, depend on both personal factors such as motivation and stress level as well as due to environmental factors like for instance the classroom condition, the teacher and the representation of the mathematical task itself (Sudha and Shalini, 2014). When the individual and environmental conditions are controlled for and ruled out, there are three basic and very broad types of assessment aspects:

- 1- Standard test which includes different reading, solving and understanding information on a limited and measured time span.
- 2- Observation in different settings and environments, such as in the classroom, in the breaks and outside school. In the assessment process, the students work in classroom should also be evaluated, such as scores and capabilities on various tests. It is vital to also assess why and how does the student fail, attendance, concentration in class, interaction and participation in different subjects and social interaction with the classmates.
- 3- Measuring the neural brain activities (dyscalculia screening) in comparison to a non-diagnosed individual (Sudha & Shalini, 2014)

According to Vårdgivarguiden (2015) it is important to exclude different individual psychological factors and “outside” environmental aspects (see below list) that can cause the secondary dyscalculia and/or misdiagnose the individual:

- Young age and is yet not ready for mathematical problem solving.
- Difficulties in theoretical studies
- Concentration problems
- Lack of motivation
- A limited working and short term memory
- Missing school which causes missing the mathematical lessons and explanation of problem solving
- Speaking multiple languages at a young age can cause education delays
- General difficulties in mathematics in which the person feels “blocked” and cannot move on since the previous problem is unsolved
- Linguistic difficulties
- Dyslexia, writing and reading disabilities
- Low self-esteem due to earlier failure
- Change of teachers and/or school and therefore missing explanations.

When assessing dyscalculia in young children, the assessing person whether it is a teacher with special education or a paediatrician, must conduct extensive interviews with the child's parents or caregivers. In early detection of any learning disability the assessment cannot mainly rely on the child's stories and experiences, as the parents/caregivers must be well attentive how the child counts, measures quantities and the ability to see the sequence of for instance the daily routine. The assessment should also analyse if the child has any learning and/or any neurological disability, mathematical anxiety, school phobia or have general negative thoughts and feelings towards the school. It is also very important to assess the family history of any learning disability, cognitive and/or neurological disorder as well as the general attitude towards mathematics.

Statistics in Sweden show that diagnoses for dyscalculia increases as with age, although a great deal of science argues that dyscalculia is a chronic and inborn condition which shows itself in early ages. This statistics supports the fact that dyscalculia is a form of dyslexic since children may claim that they can read but not do math for the fact that they may consider mathematics boring and energy consuming since mathematical tasks require more concentration, thinking and being present in the teaching lesson. Furthermore, statistics indicates that a great deal of the dyscalculia diagnoses are accompanied by other learning and cognitive disabilities which in itself effects the concentration and motivation to make the effort of solving mathematical tasks (Vårdgivarguiden (2015)).

5.8. Is there a treatment for dyscalculia?

Any intervention and treatment should always be individually tailor made for the person. The assessment results should determine the nature and structure of the treatment, which also tells us that any treatment cannot be generalized to a whole group of dyscalculic children or adults (Kaufmann & Von Aster (2012)). There are no conclusive treatments for dyscalculia likewise for dyslexia, but research suggests that in order to improve in mathematical skills children must learn how to automatically associate numbers and symbols in their cognitive schemas and/or brain representations. This practice is the same for dyscalculic as for dyslexics, where dyscalculic children train themselves to associate numbers with their amounts and values, dyslexics practice on associating letters with their sounds. Such practices should speed up the reading, encoding and understanding process for becoming more automatic (Rubinsten &

Henik (2006)). The presented treatment strengthens the argument for this paper that dyscalculia is a form of dyslexia and not two separate conditions as the principal for treatment is the same.

Price and Ansari (2013) have suggested two mathematical programs to improve children's mathematical skills and performance. The first one, called "The number race" is a practice in which the children are asked to distinguish between two different collections with different amount of dots. The children should identify which collection has the smallest versus largest amount of dots. After the answer is submitted, the computer tells the right answer as well as shows how the amount of dots differs in the two collections. The other mathematical practice "Graphogame" is very similar to "The number race", but is more exact and focuses on combining numbers with mathematical symbols. Though, results showed children's development in comparing number amounts, the studied children still experienced struggles in other mathematical tasks, which again tells us that dyscalculia has its root in encoding, processing and retrieving information likewise in dyslexia.

Short, according to Kaufmann and Von Aster (2012) any treatment should try to achieve the following results:

1. Knowing the basic numbers from 1-10 and how to combine them,
2. The ability to create and associate the representation of numbers in 3- dimensions,
3. Developing skills for mathematical reasoning,
4. Knowing the procedures of how to perform different calculations and
5. Reaching the point where basic mathematical facts become automated.

However, to make optimal use of any possible helping-aid, the psychological part in treatment is vital before starting the computerized training programs. If math anxiety is present and not treated, various helping-aids will not function as desired due to the person's cognitive processes that is directly affecting the individual's performance.

Adler (2001) argues that dyscalculia can be treated and the dyscalculic individual has the capability of improving significantly. However, based on interviews and a personal experience of working with dyscalculic students, the learning disability never disappears but with age the individual learns how to handle and accept the disability which can faulty convey the message that the disability is treatable. With practice, motivation and patience one may learn the appropriate strategies to solve basic mathematical tasks and become less anxious about the disability. The same goes for dyslexia, as there is no conclusive treatment that diminishes the disability, there are several ways and strategies to make the reading and daily life tasks easier.

However, such patience and practices are related to age and one's biological development which explains why improving results are not as obvious in children like in adults.

Still there are significantly more studies concerning dyslexia than dyscalculia and in terms of helping-aids, the interventions are the same. Students with dyscalculia as well as dyslexia must be provided various computerized programs with dictator functions and guessing the word options. When solving mathematical problems and reading text, the interviewed students (subjects) argued that it is easier on the computer since the letters and numbers become clearer. Besides the digital helping-aids, students suffering from any learning disability much be provided shorter tasks (breaking up long task into many steps), short goals, larger text (font 14, New Times Roman) and space between the lines (1.5). Each responsible teacher must be well aware of the students' abilities in order to adjust and provide suitable requirements and tasks for the student to reach the desired goal for passing and achieving the deserved grades. As the helping-aids for dyslexia helps in dyscalculia, it strengthens the aim of this paper which argues that there is no single learning disability called dyscalculia, but it is a form of dyslexia

5.9. How effective are the interventions?

As mentioned in earlier section, the tailor made intervention plan must clearly state what the dyscalculic person not should do as well as which areas in mathematics needs and are possible to develop. The purpose of the interventions are not only to directly "cure" the dyscalculic person, but to also act as an evaluator and designer for both the individual's further interventions as well as for the teacher and other educational authorities. As Timperley (2013) argues, the implementation of any intervention for students' learning disabilities should always be designed after the individual needs and thereby continuously modified and updated due to the individual's results. Timperley (2013) suggests that assessment, interventions and the results, all together act in a cycle in which the interventions, development and outcomes design the next coming cycle. Timperley's argument also discuss the ineffectiveness of "all around" generalized interventions that are usually unfortunately too often administered due to lack of time, uninterested authorities and a shortage of resources.

According to Kaufmann and Von Aster (2012), interventions in English speaking countries showed to be most effective when they involve easy mathematical tasks which did not include any problem-solving skills. In addition, long-term interventions did not show as positive results as the weekly and day to day goals. Thirdly, individuals claiming to have dyscalculia solved

better mathematical tasks when they worked in small groups with direct instructions from the teacher. Surprisingly, when students in the English speaking countries went through interventions, the personal contact helping aids (person to person) were much more effective than the computerized intervention options. This result supports the fact that the nature of dyscalculia cannot be generalized and is very different from one individual to another, since all the students interviewed for this research argued that they could solve mathematical problem solving tasks “normally” when performed on the computer.

5.10. Who is decides for the assessment and helping-aids for dyscalculic students in school?

The process of including students with learning disabilities and/or special needs into the normal education/class varies drastically between the countries, but in case of dyscalculia, students are usually present in the ordinary classroom. Overall, it is the school principle that holds the main responsibility for financing the helping aids and assuring their effectiveness. In Sweden, each school has a team called “Elevhälsoteam”, consisting of the principal, a teacher, nurse, social worker, psychologist, teacher with special education for children with learning disabilities and if possible a speech educator (Hjörne et al., (2013)). This team strives towards providing a well- adjusted learning environment for the dyscalculic student and is responsible for reviewing and evaluating the implementation plan as well as the helping aids techniques. The “Elevhälsoteam” must always be well acknowledged with the student’s educational level in accordance to the grade requirements and incorporate help and assistance for the student if needed besides the helping aids. It is important that the school have regular meetings with the parents or caretaker of the student so the students can get the same helping-aids in the home like in the classroom. If the student feels excluded, ignored, anxiety and/or does not benefit from the planned interventions, the school principle is responsible for providing more extensive assessment and thereby interventions.

Occasionally, some students get ignored and are not diagnosed due to confusion with the school personal about whom is responsible for the assessment process. Furthermore, if the student has experienced learning difficulties, he or she tends to dislike and show no interest in school which leads to regular absents. The absent and attitude towards the school do in turn give the responsible teacher and authorities a reason to not engage for an assessment claiming

that the low grades are due to the school absents and low academic interest. To not misunderstand any student's behaviour and/or attitude towards school and thereby neglect any learning disability, a teacher with special education should be immediately incorporated for evaluation when there is the least suspect of any learning struggles.

5.11. Inclusion or not?

Bhatia and Kapur (2018) have studied the concept of social inclusion in Indian schools and argues that in order to implement an educational inclusion system, the teachers' personal values and attitudes must believe in inclusion. The teachers' beliefs are vital since it affects the student's behaviour and performance. The student acts according to the teacher's expectations and develop a self-perceived picture that fits well with the teachers beliefs and expectations, which is called "A self-fulfilling prophecy". This means that if the teacher believes in inclusion, his or her attitude will shine through and affects the student's behaviour and feelings in either inclusion or exclusion.

Although Bhatia and Kapur's study does not directly measure the educational inclusion system for students with learning disabilities, the concept is the same and can be implemented for any differences between the students in the same classroom and school. Regardless of the kind of differences between the students, whether it is of social and/or functional abilities, the teachers must internalize the concept of inclusion. When a belief is internalized, it means that the person acts accordingly without thinking or doubting it before the action is performed, in other words the action becomes automatic due to our cognitive schemas. Therefore, teachers' education and training must include special education and the science for inclusion. In addition, besides the teachers' values and beliefs, the overall school educational system must function towards an including educational system (Bhatia and Kapur, 2018).

However, Lundgren et al., (2017) discusses the problems of inclusion in the classroom, which can cause the learning disabled student to feel him or herself "primitive excluded" where the student is excluded within the including system. Today, most of the research conducted discussing interventions for learning disabilities, suggests that inclusion in the school and classroom is the most optimal system in which the students are in the "normal" classroom with the non-disabled students. However, in order to limit the student to feel him or herself primitive excluded, it is the teacher's main responsibility to confirm the student's existence in the

classroom in a positive and non-discriminative way. It may be a dilemma for the teacher where he or she must include the dyscalculic student to “fit” in the rest of the class which in turn may require more extensive attention to the student which further is a way of segregating and/or distinguishing the student from the rest of the class. The extensive attention, which is needed so that the student can master the mathematical tasks, can seem as pointing out the student as less “good” who requires a specialized teaching method

5.12. Summary.

Research has shown that dyscalculics and dyslexics have alterations in activity in their parietal lobes in comparison to individuals with no learning disability. As stated earlier in this paper, dyscalculia, likewise dyslexia, is a comorbidity of cognitive disabilities, neural processes as well as it may appear as a secondary side-effect from various mental illnesses.

Many children are misdiagnosed, whereas there are many environmental and “outside” factors that can cause difficulties in mathematics. In ADHD children, it is very common that children have difficulties in concentrating, and thereby find difficulties in mathematics. Also, in mental illnesses such as depression and anxiety people do not have the effort or “clear” cognitive ability to concentrate, grasp and process new information which can be misdiagnosed as dyscalculia. If people suffering from mental health can read normally, than they do not have dyscalculia, since they are able to understand and process letters/characters. The problem arise in mathematics only because it requires extra concentration and putting additional effort in solving it and that is why the person loses interest. Finally, if the student is not interested in school, dislikes the teacher and teaching structure in mathematics, learning struggles are likely to appear, since as mentioned the learning process usually requires paying attention to the teaching class. In other words, it is not about inability to grasp number, but lack of interest. If there would be a single definition called dyscalculia, as Sudha and Shalini (2014) argue the individual would be unable to see sequences in time and space. This argument supports this papers discussion that dyscalculia is a form and “side-effect” of dyslexia, since as also the students who were interviewed for this study implies that they have dyscalculia but yet could they tell different sequences of their daily life and routine.

6. The natural number system $\mathbb{N}$ - The mathematical/scientific perspective.

The fundament of the structure of the entire axis of the real numbers is the structure of its smallest subset with respect to the set inclusions

$$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R},$$

i.e., the natural numbers $\mathbb{N}$. The axiomatic representation of this number system is commonly known as *Peano's axioms*, which are constituted by the following five postulates, in which the natural numbers $\mathbb{N}$ is an **unspecified** set, where the existence of *one* natural number 0 (a *first* element) is postulated and a function $s: \mathbb{N} \rightarrow \mathbb{N}$ which too is considered **undefined**. The axioms are the following.

- (i) $0 \in \mathbb{N}$;
- (ii) $s(n) \in \mathbb{N}$ if $n \in \mathbb{N}$;
- (iii) There is no element $n \in \mathbb{N}$, such that $0 = s(n)$; (1)
- (iv) For all $m, n \in \mathbb{N}$, we have that $s(m) = s(n) \Rightarrow m = n$;
- (v) If $S \subseteq \mathbb{N}$, $0 \in S$ and if $s(n) \in S$, $\forall n \in S$, we have that $S = \mathbb{N}$.

Here, we thus particularly note that the natural number system $\mathbb{N}$ in mathematical - scientific - sense is constituted by an **unspecified** set, which has a first element - a 'zero' - a function s which to each and every element $n \in \mathbb{N}$ designates a successor $s(n) \in \mathbb{N}$, which also, initially, is *unspecified*, and where two elements having the same successor by necessity must be the same. (v) is commonly referred to as the *axiom of induction*.

To be able to understand the mathematical meaning of *number*, we introduce the following definition.

Definition 1. A function $f: A \rightarrow B$ is constituted by three objects. Two sets A och B and a rule f , which to each element $a \in A$ assigns a fully determined element $f(a) = b \in B$. These three objects determine the function f uniquely. The function f is said to be

- (i) one-to-one if for each element $b \in B$, there is at most one element $a \in A$, such that $b = f(a)$;

- (ii) *onto if for each element $b \in B$, there is at least one element $a \in A$, such that $b = f(a)$;*
- (iii) *bijective if it is both onto and one-to-one, i.e., if for each element $b \in B$, there is exactly one element $a \in A$, such that $f(a) = b$.*

Remark 1. In view of the above, we conclude that the mathematical meaning of a set consisting of a certain number of objects, e.g. 5 persons, is that there is a bijective function between the natural numbers 1,2,3,4,5 and the set in question. This means that we can pair the set of people together, one by one, with the numbers 1,2,3,4,5 and the rule f in *Definition 1* is the order, in which this is done; the order in which we count the persons. If we replace the natural numbers 1,2,3,4,5 by another, initially **unspecified**, set, e.g. a set consisting of 5 rocks, this new set will serve equally well in keeping track of the number of persons in the crowd. One rock for each person and 'which' rock given to each person is the rule f in *Definition 1*. This also yields another representation of the initially **unspecified** set in *Peano's axioms* and a new function s yielding the successor of each element of the set, i.e., the order in which we distribute the rocks among the persons one by one. Hence, we have created a way of telling numbers, which is in no way connected to the written characters 0,1,2,3,4,... which we generally use as representation of the natural number system.

Though not of dire need for this article, we will also introduce the following definition of the binary relation of addition, which requires a little more than basic treatment of numbers (i.e., telling 'how many'), and hence lies beyond the scope of dyscalculia.

Definition 2. We define addition recursively by $m + 0 = m$ and $m + s(n) = s(m + n)$, $m, n \in \mathbb{N}$.

Example 4.

$$\begin{aligned}
 2 + 3 &= 2 + s(2) = \\
 &= s(2 + 2) = s(2 + s(1)) = s(s(2 + 1)) = s(s(2 + s(0))) = \\
 &= s(s(s(2 + 0))) = s(s(s(2))) = s(s(3)) = \\
 &= s(4) = 5.
 \end{aligned}$$

7. Analysis and conclusion.

Once again, the notations $0, 1, 2, \dots$ of the natural numbers rest completely on the nature of the characters, which we have made into standard in our development of *the language* as means of communication in writing. In view of *Section 6*, this is thus merely a *representation*, a *presentation*, of the *unspecified* set constituting the natural numbers $\mathbb{N}$ according to the axiomatic representation of the same mathematical structure; and our way of counting, *i.e.*, the order relation $0, 1, 2, \dots$ is merely a *representation* of the *unspecified* function (*cf. Peano's axioms* (1) in *Section 6*!) $s : \mathbb{N} \rightarrow \mathbb{N}$, which to each element $n \in \mathbb{N}$ assigns its successor $s(n) \in \mathbb{N}$. Thus, in order for dyscalculia to be able to exist in the sense that the experts of today claim it does (*cf. i.a.* Lundberg and Sterner (2009), pp. 17--21, and *Section 5*!), the arisen difficulties in the fundamental ability of counting, the ability of counting and telling numbers (*i.e.*, 'how many'), *i.e.* the epitome of the *natural number system* $\mathbb{N}$ must rest *neither* on the choice of representation of the *unspecified* set constituting the set of natural numbers $\mathbb{N}$ *nor* on the choice of representation of the *unspecified* function $s : \mathbb{N} \rightarrow \mathbb{N}$ (*cf. (1) Section 6*!), which thus assigns to each element $n \in \mathbb{N}$ its successor $s(n) \in \mathbb{N}$ and hence, according to the *axiom of induction* (axiom (v) in (1)!), constitutes the order between the elements of $\mathbb{N}$. The persons with claimed dyscalculia *must*, by necessity, encounter with the same kind of difficulties independently of which ever representation we choose to adopt to concretize the mathematical structure of $\mathbb{N}$. Hence, in order to disprove the existence of dyscalculia as an own concept, it thus suffices to produce *one* representation of $\mathbb{N}$ *one* counter example, where the persons with claimed dyscalculia will not experience any problems like with the representation given by the characters $0, 1, 2, \dots$. Our unambiguous conclusion will then be that the phenomenon of dyscalculia, its problems, are inherent in the nature of the representation of $\mathbb{N}$ given by our developed characters $0, 1, 2, \dots$. This phenomenon is commonly known as *dyslexia*. Note that people with dyscalculia, as well as dyslexia, are assumed to have an, in other parts, well-behaved intellect.

Theorem 1. *Distinguishing between different natural numbers, i.e., determining 'how many', is mathematically equivalent to orientating oneself in time, i.e., keeping track of the chronological order of events in a certain time sequence.*

Proof. Let us first recall that, according to *Remark 1*, the distinguishing between different natural numbers, i.e., e.g. 'Which pile of stones represents the largest natural number?', is in no way in general mathematically connected to our *representation* of the natural number system $\mathbb{N}$ by means of the written characters $0, 1, 2, \dots$, which we have developed as a means of communicating in writing. If we, in view of *Peano's axioms* (cf. (1) of *Section 6*!) consider each pile of stones as a subset of the initially *unspecified* set $\mathbb{N}$, and if we line up the stones of each pile in parallel lines, from left to right, the stones together with the order, in which we line them up will represent a substructure of the natural number system $\mathbb{N}$. The pile of stones yielding the line with last stone ending up furthest to the right, thus represents the largest natural number.

Now, assume that the *unspecified* set (cf. (1) of *Section 6*!) constituting the natural numbers $\mathbb{N}$ is the set of all every day events - with the first element, the element 0 as our waking up in the morning - and the function s is the order in which we perform them during the day. This will yield another representation of the mathematical/scientific structure of $\mathbb{N}$ as described in (1) of *Section 6*! This representation is *not* connected with a good ability to read and to write. The successor of 0 might be *getting out of bed*, which with our original representation would be denoted by 1 . Hence, the telling of differences in 'size' - as described in *Section 6* - would consist in telling when, during the day, we carry out a certain chore, e.g. is the distance between *getting out of bed* and *having morning coffee* bigger than the distance between *having morning coffee* and *having dinner*, i.e., between which ordered pair of these events is the lapse of time the biggest, i.e., between which pair of events will the biggest number of other chores take place?

Thus, we conclude that distinguishing between different natural numbers is mathematically equivalent to orientating oneself in time, meaning the ability to chronologically keeping track of the different events in a certain time sequence. Thus, the theorem is completely proven.

Remark. Sudha and Shalini (2014) establish that brain screening reveals that when your right brain hemisphere is malfunctioning, the person suffers from lacking in understanding the concept of *quantities* as well as lacking in ability of learning sequences in space (time) (cf. *Theorem 1* above!). Thus, these two abilities apparently emanate from the same brain area, which is substantiated by *Theorem 1* above!

Remark 2. The test described in *Example 1* would then look something like the following, where we are to tell between which of the chores *having breakfast*, *going to work* and *going to*

bed at night we perform the most number of other chores, *i.e.*, between which of the aforementioned chores is the lapse of time the biggest?

The congruent and the incongruent presentation of the problem in the test would then look as follows:

(having breakfast,.....going to work,.....going to bed)

(having breakfast,.....going to work,.....going to bed),

respectively. The fact that, in the original test, as described in *Example 1*, where the test was conducted with digits/characters with the outcome that the test persons with claimed dyscalculia, and which are claimed not having developed a mental number axis, make less mistakes than the persons without claimed dyscalculia, would be a very strong indication that most people experiencing problems in counting and telling numbers are in fact having problems with the *representation* of the natural numbers $\mathbb{N}$ (*cf. Section 6!*), which we have obtained by developing our written language and the inherent characters we use, *i.e.*, 0,1,2,... to *represent* the *unspecified* set in *Peano's axioms* (1), *Section 6*. More precisely, persons with a mechanical, represented with characters and drummed into their heads, model of the number axis are very likely to make more mistakes in the test in *Example 1* as a consequence of their mechanical knowledge leading to spontaneous answers without consideration and any deeper understanding of the subject. The person with claimed dyscalculia, however, who, for some reason has failed to develop a mental number axis is making less mistakes in this test... Let us establish that without a well developed, mechanically drummed into your head, mental number axis, you are constrained to *think* before yielding the answers to this test. This implies that the persons with claimed dyscalculia make fewer errors and have a higher frequency of correct answers. This would, without a too intrinsic, further analysis, bear testimony of a good understanding of and a similarly good ability to treat the mathematical structure (*cf. Section 6!*) of the system of the natural numbers.

In the light of the discussion in *Theorem 1* and *Remark 2* above, we come to the unequivocal conclusion that a person with dyscalculia - according to studies conducted within the area of expertise - would have enormous difficulties in keeping track of the order in which he/she is carrying out the everyday chores of his/her life. This means that these persons would have excessive problems of orienting themselves in time and space and in structuring their day. This, however, does occur with some people having brain damages or suffering from some kinds of

mental diseases, but these diagnoses are not compatible with *in other respect having a well-behaved intellect*. For further problems inherent in 'dyscalculia', cf. Lundberg and Sterner (2009), pp. 16-19!

Remark 3. Moreover, *Remark 1* gives the exact mathematical meaning of two sets having the same number of elements; they can be paired together two by two and no one will be left odd. Mathematically telling which one of two sets 1 and 2 contains the most elements is playing a party game. Two players take one set each. Player 1 starts by throwing one element from set 1 into the ocean. Player 2 answers by throwing one element from set 2 into the ocean. The player, who first gets rid of all the elements is the winner; his set thus contained the least number of elements from the beginning. If we reconnect to *Example 3* – Butterworth (2003) - this test consists of, apart from the ability of telling *how many*, yet another moment, another potentially difficult step - the ability of reading (and reading fast!) the figure in the circle to the right; and after that combining this exercise in reading with telling *how many*. That makes this test not a test of pure dyscalculia but more like a test of reading capabilities, and therefore a poor result on this test is inconclusive for telling that a person has difficulties in assimilating the essence of the scientific structure of mathematics, which would be dyscalculia. Instead, this test is more likely to measure the ability of handling the extra problems inherent in our elaborated *representation* of the system of natural numbers, which is a product of the characters we use for this purpose. That would be dyslexia.

Remark 4. Note that the recursive definition of addition from *Definition 2* would work equally well with the set representing the natural numbers in *Theorem 1*, which would give us a completely analogous theory of the natural number system!

In the light of *Theorem 1* and *Remark 2* above, dyscalculia - as commonly studied and defined - cannot be considered existent. Dyscalculia in the mathematical sense would imply substantial restrictions in a person's concept of reality and intellect, which is not the assumed in the prevailing theories of dyscalculia. These theories are intimately connected with our *representation* of the system of the natural numbers, *i.e.*, our expressing the natural numbers by means of the characters available, *i.e.*, $0, 1, 2, \dots$. In the process *reading and understanding a mathematical problem/elaborating/formulating/writing/presenting a solution* a person with dyscalculia would encounter with problems in the subsequence *elaborating/formulating a solution to a mathematical problem* (cf. *Section 1*. 'When I have read the problem many times, then I understand and can solve the problem.'). Therefore, the phenomenon called 'dyscalculia'

does not mean problems in handling the *mathematical* structure (*cf. Section 6!*) - the scientific quintessence - of the natural numbers, but in the way we communicate them, the way we *represent* the natural numbers, by means of the characters we use in communicating in writing. Thus, the phenomenon called 'dyscalculia' is instead intimately connected with the extra set of difficulties inherent in our *representation* of the natural numbers by means of reading and writing them 0,1,2,....

In Tambychika and Thamby (2010), the authors have performed a study reported from the students' own experiences, where the researchers have measured how the students' cognitive abilities operate. Firstly, they explain how problem solving is manifested and how the students experience difficulties. There are two aspects of problem solving - according to Tambychika and Thamby (2010) (*cf. Section 1!*) – where the first is how the student understands/translates the problem linguistically (words) (*cf. Section 4!*). The second problem solving issue is the non-linguistic aspect (*cf. Section 5!*), which refers to numbers and graphs. Along with these two aspects, the problem solving difficulties can result in how the linguistic information is translated to mathematical terms (*cf. Section 6!*) – numbers – and/or how the numbers should be solved/calculated. Students showed more difficulties in the first aspect, in which they transform the words into numbers and graphs. Again, this supports the fact that difficulties in mathematical problem solving are due only to dyslexia, and dyscalculia is a mere guise of dyslexia.

Bugden and Ansari (2014), argue that children and adults with dyscalculia suffer from different cognitive disabilities like for instance, both long- and short term memory struggles as well as difficulties in recognizing and translating the numbers to meaningful and useful information in the brain. In the case of dyslexia, the cognitive disabilities are similar as one's ability to understand and transform letters into useful information which fits with our memorized cognitive schemas. Bugden and Ansari (2014), have - like other researchers - shown how the non-diagnosed children brains work differently from the dyscalculia brain and stresses on the fact the dyscalculia brain do not have any brain damage, it just works differently. Prominence is also given to the fact that, when dyscalculia is caused by cognitive disabilities, it is categorized as a secondary dyscalculia which also explains that dyscalculia and dyslexia should be in one category since the cognitive origin of the struggles effect reading all types of symbols and does not distinguish between letters or numbers.

Regarding developmental dyscalculia, almost all the experts in this area give prominence to the fact that an unambiguous definition of this phenomenon is nowhere to be found; they say that it is even dependent of in which age – and at which mathematical level – the different persons

are. Szűcs and Goswami (2013) list a number of different phenomena (Dyscalculia, Developmental Dyscalculia (DD), Arithmetic-related learning disabilities (AD), Arithmetical disability (ARITHD), Arithmetic learning disability (ALD), Mathematical Disability (MD), Mathematics Learning Disabilities (MLD), Mathematical Learning Difficulty (MLD)), which they state might or might not be equivalent to DD. However, the common treat of all experts opinions is that the primeval state of what they believe to be DD is the problem of telling 'how many', *i.e.*, problems in handling the *axiom of well-order*, which is equivalent to axiom (ii) in equation (1) in Section 5. Thus, this is ultimately the core of what is called *dyscalculia*. Should this primeval state of the current learning disability give rise to more general problems in handling mathematics and arithmetics later on in life, the persons in question would also have problems in handling the *axiom of well-order*, *i.e.*, telling 'how many', which thus is mathematically equivalent to orientate oneself in time and space. Moreover, Center for Neuroscience in Education (2019) explains that there are two predominant theories in this area. The first one believes dyscalculia to be the result of an impairment of the *Approximate Number System*, ANS, which equip humans with the capacity of quickly and instinctively understand, estimate and manipulate non-symbolic quantities. This impairment would then, according to *Theorem 1* and *Remark 2*, unequivocally affect the ability to orientate oneself in time and space, and this is *not* compatible with possessing an in other parts well-behaved intellect. The second theory states that dyscalculia impairment is not linked to the ANS per se, but that dyscalculics' ability to automatically map symbols to their corresponding magnitude is disrupted. This would be encountering with problems with our chosen representation of the natural number system $\mathbb{N}$ and is thus linked to the ability of reading and writing the numbers $0, 1, 2, \dots$ and decoding the information in the brain. This would be dyslexia.

Although there certainly and undoubtedly exists people with genuine problems in handling mathematics and arithmetic, the origin of their problems is the crux of the matter. Their problems could be due to brain damages, poor teaching, the fact that we, simply, are good at different things, etc., etc., ... However, in order for the phenomena dyscalculia and developmental dyscalculia to exist as concepts of their own, they have to arise from the most primeval learning disability in this line of categorization; and this would be difficulties in telling 'how many', *i.e.*, problems with the *axiom of well-order*, which in turn - according to what we just have established (*cf. Theorem 1* and *Remark 2*) would be mathematically equivalent to problems in orientating themselves in time and space, which is *not* compatible with possessing an in other parts well-behaved intellect.

In a comparison with previous research in this area, we find that our mathematical approach in order to find a proof of that *dyscalculia* is not a concept of its own - but merely yet another one of the sad guises of *dyslexia* - is completely new, and as all mathematics didactics on the basic level of dealing with the natural number system, where '*dyscalculia*' first manifests itself, is completely based on our representation of the natural numbers by means of our characters $0,1,2,\dots$, this approach (*cf. Theorem 1* and *Remark 2!*) could almost certainly be used to elaborate new theories in the area of *mathematics didactics*.

Altogether, we conclude that the quintessence of the phenomenon called 'dyscalculia' is not inherent in the nature of mathematics/science (*Section 5!*), but in our way of communicating mathematics/science, our *representation* of the system of the natural numbers by means of characters (*cf. Remark 1, Section 6, Theorem 1* and *Remark 2-4, Section 7!*). Altogether, 'dyscalculia' - as today classified and defined - cannot be considered existent but is merely just another one of the sad guises of dyslexia (*cf. Section 5.12. and Section 1!*)! This completely answers the question of *Section 1*.

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