CHAPTER 7

"Mathematics Is Bad for Society"

Reasoning about Mathematics as Part of Society in a Language Diverse Middle School Classroom

Ulrika Ryan, Annica Andersson and Anna Chronaki

Abstract

In this chapter, we report on a small-scale critical mathematics education project in a Swedish classroom with students of varied language backgrounds. The project departed from the student Arvid's statement "Mathematics is bad for society." Our research interest was twofold. On the one hand, we wanted to explore what knowledge is being (re)produced by students as they try to connect and reason with a statement like "Mathematics is bad for society." And on the other hand, we were also interested in how the students in this classroom, in which they do not have shared mother tongues, can express and (dis)acknowledge knowledge when reasoning about mathematics in society. We found that when the students (and their teacher) grappled with unpacking critical aspects such as "mathematics in society," their reciprocal assessment of claims was based on their individual ways of knowing and talking, and tended to shape both their actions and the outcome of their efforts. We show that the discussion around critical aspects of mathematics in society that came to the fore was intertwined with both students' and the teacher's (lack of) meta-understanding of language diversity.

Keywords

language diversity – mathematics and society – inferentialism – knowledge bases – critical mathematics education

• • •

"Mathematics is bad for society," said Arvid, a grade five (11 years old) student with Albanian background, in a somewhat challenging tone when we were awaiting the teacher for the math class to start. I, Ulrika, did not have the opportunity at that moment to reply or ask him why he claimed mathematics to be bad for society. It was one of the first days that I met

DOI: 10.1163/9789004465800_007

> the class as a participant-observer and my relations with the students had not yet developed into closer friendships. I had introduced myself as a researcher interested in reasoning in math class but had not mentioned societal aspects of mathematics. Arvid's claim got stuck in my mind and I became curious about his reasons for making the statement. I found it surprising for such a young student to say that "mathematics is bad for society," or even to position mathematics as being connected to societal matters. In my mind, Arvid's claim raised questions about middle school students' ideas on mathematics and society. (Vignette, based on Ulrika's field notes, 2017-05-08)

To the first author,[1] Arvid's statement brought forward the idea of conducting a small-scale project to explore the theme "Mathematics is bad for society" in his class. Arvid's statement seems to exemplify concerns in critical mathematics education about how societal issues of mathematics can be addressed at school, a task that remains a challenge for mathematics educators.

Critical mathematical literacy is shaped through "constructing knowledge of particular concepts, ideas, skills and facts … to help [students] recognise oppressive aspects of society" (Gutstein, 2006, p. 6). In this line of pedagogical interventions, students often engage in discussing social issues, formulated as mathematical problems by their teachers (see for example Andersson, 2011; Barwell, 2013; Frankenstein, 1990; Gutstein, 2006, 2016). Teachers assist students in translating "real world" problems into mathematics, which in a sense creates a challenge as argued by Gutstein (2016). Drawing on the ideas of Paolo Freire, he argues that critical mathematical literacy contains knowledge of three types: classical academic, communal, and critical. However, Gutstein (2016) identifies challenges in interconnecting the different knowledge bases and asks: "How does one connect and synthesize all three knowledge bases?" (p. 458). He proposes the idea of a pedagogical "dance" that means moving between the three knowledge bases, and that "signifies the braided interconnections that people make between mathematics and socio-political reality" (p. 469). To us, embedded in the statement "Mathematics is bad for society" we see classical academic knowledge about mathematics as well as communal knowledge about society and mathematics in society. Moreover, the word *bad* opens up a value-laden space that allows for critique.

Our endeavour in this chapter is to explore how the students in Arvid's classroom struggle to make value-laden critical interconnections between mathematics and society. The classroom can be characterised as language diverse yet monolingual. It is language diverse because a third of the students belong to a wide diversity of mother-tongue backgrounds and it is also monolingual since the curriculum language is Swedish. As such, our small-scale project explored more deeply how students from diverse language backgrounds engage in making inferences and interconnections when they reason about "Mathematics is bad for society," since these inferences allow us to unpack implicit meanings in such statements (Brandom, 1994, 2000).

As we examine the outcome of this small-scale project, our interest is twofold; it is directed towards grappling with *what* knowledge the students' use, produce, and inferentially interconnect in their claims to articulate meanings of Arvid's statement, and we look at *how* that knowledge is expressed and (dis) acknowledged[2] in the language diverse yet monolingual classroom. These interests mean that we anticipate that, to achieve an understanding of knowledge, attention to its dialogical articulation is vital (Derry, 2013). Theoretically, we focus our analysis on the *language game of giving and asking for reasons* (GoGAR) (Brandom, 1994, 2000) to examine the what and the how as elements in flux in face-to-face reasoning.

The research questions explored in this chapter are:

- *What* knowledge do students use, produce, and interconnect as they reason about the statement "Mathematics is bad for society"?
- *How* do students express and (dis)acknowledge knowledge when reasoning about mathematics in society in a language diverse yet monolingual classroom?

The three knowledge bases to which Gutstein (2016) refers contain aspects of functional mathematics as well as critical knowledge about mathematics and society. Therefore, we start with some brief comments on mathematical literacy and socio-political aspects of mathematics in society. We then give a theoretical account of the GoGAR, which we use to analyse what inferential interconnections between knowledge claims about mathematics and society the students in Arvid's classroom make, as well as how they express and treat those claims as they reason about his statement "Mathematics is bad for society." Thereafter we describe the context and methodology of the present study, followed by our analysis of specific episodes and student claims. The chapter ends with some concluding remarks, which address the complexity of students' reasoning about mathematics in society in a language diverse classroom.

1 Mathematics and Society

Functional mathematical literacy refers to the capacity of creating and applying mathematical knowledge when required (Jablonka, 2003). Such conceptions of mathematical literacy are closely linked to the social and economic needs of the market, as well as the individual's participation in an advanced technologised democratic society (Jablonka, 2003; Skovsmose, 2007). At the extreme end of functional mathematical literacy, mathematics is value-free and merely technically related to societal demands (Jablonka, 2003). However, articulated in Arvid's claim "Mathematics is bad for society," mathematics is connected to society in a value-laden sense.

Critical mathematical literacy recognises mathematics as a distributor of power in society. Mathematics acts and can be made to act (Skovsmose, 2007; Chronaki, 2010). Kollosche (2014) claims mathematics to be "a body of knowledge and techniques which has served the interests of power from its very beginnings" (p. 1062), with the purpose of shaping citizens' behaviour. Being critically mathematically literate is, for instance to hold the capacity of understanding and shaping the world using mathematics to make it socially just (Frankenstein, 1990, 2014; Gutstein, 2006, 2016), environmentally sustainable (Barwell, 2013; Hauge & Barwell, 2017), or to critically understand the political role of mathematics in the world (Andersson & Wagner, 2017; Gellert & Jablonka, 2009; Kollosche, 2014; Skovsmose, 2007).

According to Skovsmose (2007), the distribution of power through the use of mathematics "refers to processes of construction, operating, consuming, marginalising which [can] be addressed functionally or critically" (p. 17). Processes of construction apply to advanced systems of knowledge and techniques used by, for example, engineers, economists, and computer scientists. Operating processes conducted by operators bring constructors' advanced use of mathematics into operation. Operators are not necessarily aware of the specific mathematics behind their performances. Citizens consume the mathematical objects, developed by constructors, which are fed mathematical information by operators. In high technological societies, mathematics has become a necessity because "if the citizens were not able to read information put into numbers, the society would not be able to operate" (Skovsmose, 2007, p. 14). However, technology blackboxes mathematics. The more technological artefacts that societal life demands, the more opaque and thus powerful the constructors' mathematics behind them becomes (Gellert & Jablonka, 2009). Hence, for citizens, mathematics in society is double-edged. On the one hand, it is a necessity or demand; on the other, mathematics is concealed by technology.

It is reasonable to problematise how students develop critical mathematical literacies as a matter of interconnecting knowledge of different kinds (Gutstein, 2016) as they reason about both mathematics per se and about how mathematics can be used. Although the perspective of critical mathematics education has much to offer in terms of reconsidering curriculum design for critical mathematical literacy (e.g., Frankenstein, 1983; Gutstein, 2006; Skovsmose, 1994; Skovsmose & Borba, 2004), particularly highlighting pedagogical moves for formatting interconnections between knowledge bases (Gutstein, 2016), our focus here will not primarily be on pedagogical issues. Nor is our focus on understanding the kinds (e.g., academic and communal, Gutstein, 2016) or funds (González, Andrade, Civil, & Moll, 2001) of knowledge that students might draw on when reasoning. Rather, our focus is on the students' articulation of the interconnections themselves and on how these suggested interconnections are treated in their reasoning, while they allow us to think about how the students in Arvid's class see mathematics and society as critically connected. To grapple with the complexity in the students' reasoning, we use the *language game of giving and asking for reasons* which is at heart of the philosophical theory of inferentialism (Brandom, 1994, 2000). Drawing on Wittgensteinian ideas, inferentialism approaches meaning and knowledge from a social pragmatic perspective emphasising that meaning has to be located in real-life practices of language use, so-called *language games*. While inferentialism attends to how people make, treat, and interconnect each other's claims in face-to-face conversations, it allows us to think about the social and epistemological complexities and dynamics in students' reasoning in a language diverse classroom. In the next section, we discuss some aspects of inferentialism in relation to the small-scale project on Arvid's statement "Mathematics is bad for society" further.

2 Reasoning as a Language Game

To grapple with how students in a language diverse yet monolingual classroom reason to interconnect claims to unpack the meaning of the statement "Mathematics is bad for society," we argue that inferentialism is fruitful because, (i) it sees reasoning as a language game in which implicit interconnections between knowledge claims can be articulated and made explicit; (ii) it argues that such articulation is first and foremost dynamic and dialogical, rather than formally logical; and that (iii) norms regulate how interconnections are made, treated and used. Conceptualising reasoning as playing the *game of giving and asking for reasons*, allows for a type of analysis that is not concerned with efficiency or correctness in students' reasoning. Analysis is not geared towards eliciting individual knowledge from social reasoning, or towards modelling individual mental representations, rather it is concerned with reasoning itself. Inferentialism asks for inferences and their origins: "An inferentialist epistemology does not prioritize students' mathematical reasoning over out-of-school reasoning; nor does it view mathematics and out-of-school reasoning as separated from one another, but rather as inferentially connected to one another" (Schindler et al., 2017, p. 7). By inferences, we do not mean formal logical ones, but inferences that are inherent in language itself. For instance, from the statement "Stockholm is north of Copenhagen" it can be inferred that Copenhagen is to the south of Stockholm provided that the social and cultural norms that regulate the concept's use allow that. Alternatively, from a person saying "I am hungry," it can be inferred that she or he wants something to eat. Hence, being hungry and eating as well as north and south are inferentially related to each other under particular socially and culturally normative circumstances that are inherent in the concept's use.

The scope of the GoGAR is not to produce "a canon or standard of right reasoning. Rather, it can help us make explicit (and hence available for criticism and transformation) the inferential commitments that govern the use of all our vocabulary, and hence articulate the contents of all our concepts" (Brandom, 2000, p. 30). That is, when we ask and give each other reasons for our claims we unpack the content of our concepts and open up a space in which concept use can be troubled, critiqued, and altered. The theory of inferentialism does not argue that the content of concepts primarily represents some object or state of affairs; rather, concepts are caught up in statements that mean something by virtue of their inferential interconnections to other statements (Bransen, 2002). Hence, Arvid's statement "Mathematics is bad for society" means something in the students' reasoning due to the inferential interconnections they make to other statements.

As Arvid and his peers ask for and give reasons for Arvid's statement (i.e. play the GoGAR), they position the statement within a network of inferential interconnections to other statements. Inferential interconnections are usually implicit, but as the GoGAR is played they are made explicit. Premises, consequences, and incompatibilities that follow from uttering a statement or making a claim come to the fore. For instance, a premise for claiming "Mathematics is bad for society" is that mathematics operates in society and that it is not value-free. A consequence is that mathematics may cause some kind of harm in society. Mathematics as disconnected from society is incompatible with the claim. This dimension of the GoGAR concerns the *what*-questions of our endeavour as presented in the introduction.

When we play the GoGAR we help each other out with discerning what follows implicitly from statements by normatively assessing and using each other's claims, which means that "playing [the GoGAR] well [is] more than a matter of an individual player's competence: to play the game of giving and asking for reasons well, one needs a team" (Bransen, 2002, p. 387). To play the GoGAR well means that the team manages to discern (at least some of) what implicitly follows from a claim and that the team has a set of reasons that could be used to back up or question other claims. For instance, if player A questions player B's claim, she might ask B for reasons for it to find out if B has good reasons for her claim. Giving reasons for a particular claim is to interconnect it to other claims or statements. If player A thinks that player B has good reasons for her claim, player A endorses it and uses it in her own reasoning. Players assess interconnections between claims according to what they find to be good, appropriate ones. Which interconnections players know of and find appropriate depends on their experiences of participating in various social practices and previously played GoGARs. This dimension of the GoGAR concerns the *how*-questions of our endeavour presented in the introduction.

Children in a language diverse classroom participate in multiple, sometimes blurred social practices, which means that "a critical perspective [on mathematics] cannot develop without espousing children's social and cultural backgrounds as well as their personal ways of knowing and learning as they develop and grow through experiencing the particular political contexts of living and work" (Chronaki et al., 2015, p. 150). The formation of students' critical reasoning about mathematics and society, in a language diverse classroom embracing students' personal ways of knowing is intertwined with the socially and culturally normative inferences inherent in language(s). Radford (2012) claims that "in the students of multicultural classrooms we … find mathematics (as a plural noun) that speak about different worlds, even if the mathematics is expressed in the same official language. We do not produce accents when talking only. We also produce accents when thinking" (p. 341). Students' reasoning about Arvid's statement, "Mathematics is bad for society" draw on their personal ways of talking and thinking using their own particular "accents" which are influenced by the inferences they make from concepts to other concepts. As students in language diverse classrooms play the GoGAR and assess interconnections between claims, their "accents" guide their assessments as well as the interconnections they make.

3 The Present Study: Classroom Context, Participants, and Methods

Embedded in Arvid's claim "Mathematics is bad for society" lies the idea that mathematics acts in society and that it is not value-free, as Skovsmose (2007)

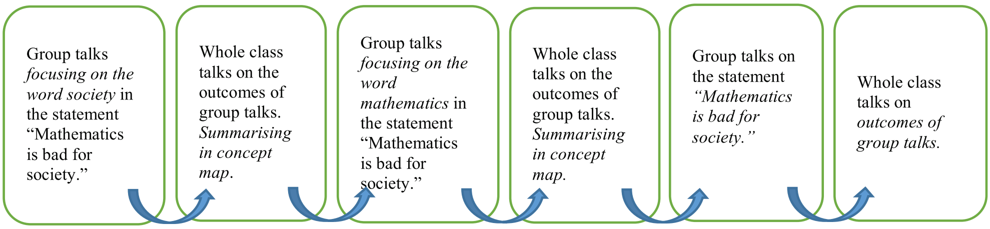

FIGURE 7.1 Flow chart of lesson orchestration

points out. Therefore, this very statement was taken as a starting point for group and whole class explorative talks (Mercer et al., 2004). In groups of four or five, students were asked to talk and reason about the statement. The outcome of each group discussion was elaborated on in whole class talk orchestrated by Ulrika. She drew a concept map on the board to summarise the students' findings. The orchestration of this discussion in one 40-minute lesson plus one 20-minute lesson,[3] organised by Ulrika, is shown in the flow chart in Figure 7.1.

The whole class talks were inspired by interactive, talk-based teaching (Hufferd-Ackles et al., 2004) in which the practices of explorative talks are characterised in a way that:

- all relevant information is shared;
- all members of the group are invited to contribute to the discussion;
- opinions and ideas are respected and considered;
- everyone is asked to make their reasons clear;
- challenges and alternatives are made explicit and are negotiated;
- the group seeks to reach agreement before taking a decision or acting (Mercer et al., 2004, p. 362).

The lesson orchestration was designed to provide good opportunities for students to make, question, use, and interconnect each other's claims while synthesising knowledge bases to unpack the meaning of "Mathematics is bad for society."

Arvid's classroom is situated in the south of Sweden in a suburban community close to two cities, to which most residents commute for work. The school districts of the municipality incorporate both middle class and socio-economically deprived areas. In this particular Grade 5 class,[4] some students travel to school from homes located in the deprived areas, a choice made by their parents to avoid as one student said, "the rowdy local schools." Most students are Swedish-only speakers but over a third of them (8) claim to have various degrees of access to languages other than Swedish, including Albanian, Arabic, Bosnian, Hebrew, Kurdish, Norwegian, Persian, Polish, and Serbian.

The formal language of teaching and learning is Swedish. Despite the fact that about 20% of the students at the school have one or two parents born abroad, the Swedish-only-discourse (Norén, 2010; Norén & Andersson, 2016; Skog & Andersson, 2014) is strongly operating. For instance, there are no signs or books in languages other than Swedish and languages other than Swedish are seldom heard in the corridors, classrooms, or schoolyard.

Two students, *Samir* and *Elsa* exemplify how several of the students in Arvid's class have cultural, social, and linguistic backgrounds that are complex and blurred. Samir speaks Arabic, Hebrew, and sometimes Kurdish at home. His parents are well-educated refugees who arrived from the Palestine about 2.5 years ago. Elsa, a girl with divorced parents, alters on a weekly basis between living with her well-off CEO father, who speaks Serbian and came to Sweden during the Balkan war, and her Swedish working-class mother, who lives in a socio-economically deprived area. The students were used to group and whole class talks during mathematics class, since that is part of their usual teacher's pedagogy. However, from individual interviews, Ulrika learned that they were not used to talking critically about mathematics.

To grapple with the students' reasoning about critical interconnections between mathematics and society, we attended to *what* they inferred from "mathematics" and "society" in the claim "Mathematics is bad for society." We also analysed inferential interconnections, which were used to articulate critical aspects of mathematics in society. Critical instances chosen for analysis were when students made explicit aspects of mathematics in society following from "Mathematics is bad for society." Analysis of the critical instances pivoted around the following questions:

- What inferential interconnections are discerned when the students play the GoGAR about the statement, "Mathematics is bad for society"?
  - How are these inferential interconnections used as premises and consequences in claims about mathematics in society?
  - How do these interconnections articulate critical aspects about mathematics in society?

Moreover, we analysed interaction that illustrates *how* the students expressed, treated, and (dis)acknowledged claims as they reasoned in the language diverse classroom, to address the following questions:

- How are attempts to play the GoGAR well made; e.g., (how) do students help each other out with discerning what follows implicitly from statements by normatively assessing and using each other's claims?
  - What are the outcomes of these attempts?

In the following section, we present the results of our analysis.

4 Reasoning about "Mathematics in Society"

When Ulrika wrote, "Mathematics is bad for society" on the board there was a little surprised and curious fuss as the students were not used to talking about mathematics as something bad. However, the students quickly engaged in the group and whole class talks with curious interest.

4.1 *Students' Inferential Interconnections between Mathematics and Society*

In this section, we elaborate on how the students reasoned about society and mathematics and how they located mathematics within their reasoning about society and vice versa. Figure 7.2 shows an image of the concept map that summarises the students' reasoning. This concept map provided the empirical material for our analysis.

To make explicit what follows implicitly from using the concept "society" in the statement, the students discerned inferential connections focusing on their environment; "school, house, trees, where we live, pollution." They positioned "society" as inferentially related to humans as social beings: "us," "humans shape it," "where we live"; and to humans' power distributing actions: "we change it together," "what we do," "we affect society with our pollution."

When reasoning about "mathematics" as used in "Mathematics is bad for society," the students discerned connections to do with mathematics as actions:

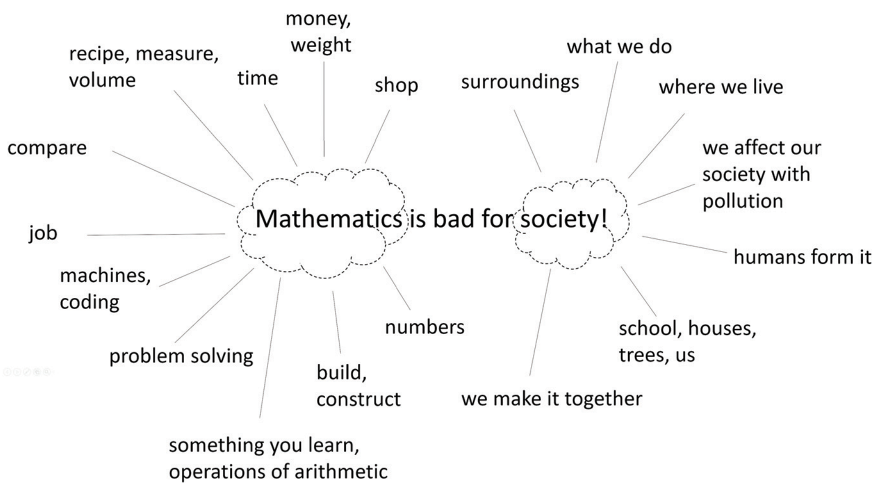

FIGURE 7.2 Image of the concept map showing the inferential interconnections that the students made

"problem-solving," "measure," "learning," "comparing," "coding," "building"; and as conceptual constructs: "time," "money," "weight," "volume," "numbers." They also located mathematics in physical objects: "workplaces," "shops," "machines," "recipes."

The students' inferential interconnections, as written on the whiteboard, revealed a stereotypical middle-class discourse of society as something that an undefined "we" (the students?) participate in, have the agency to impact, and are obliged to care for environmentally. Students made inferential interconnections showing that mathematics operates as a tool for the working, consuming, and problem-solving citizen in such a society. Hence, their reasoning reflected and reproduced functional mathematical literacy, but lacked critical aspects of mathematics in society. This is precisely why Arvid's claim "Mathematics is bad for society" is crucial for the students to elaborate on while it troubles mathematics as merely functional.

Further, the students articulated prevailing ideas about society and mathematics that did not appear to accommodate their linguistic, cultural, and social diversity; instead, diversity seemed to be silenced. One of Ulrika's hopes when designing the mini-project was that talking about "mathematics as bad for society" might disrupt and trouble stereotyped images and allow for complexity and plurality to emerge in the students' talk, as suggested by Chronaki et al. (2015). The mind map on the board did not show conflicting ideas or culturally diverse images speaking about different worlds using various "accents" (Radford, 2012).

There could be several reasons for this lack of diversity. Small group talks preceded the whole class discussion. From these group talks, a group spokesperson summarised their group's contribution to the mind map. In the small group talks, ways of applying concepts, in this case society and mathematics, were normatively assessed by the students. It is likely that the Swedish-only-discourse excluded linguistic and cultural troubling, while the students assessed them as normatively inappropriate in the group talks. Hence, the group spokesperson did not report them in the whole class talk. Language, social, and cultural diversity did not appear to be norms that regulated the talks. The fact that Ulrika is a middle-class native Swedish speaker and a former mathematics teacher with many years of teaching experience shapes the "accent" she used to understand and account for the students' summaries, which adds to the complexity of interpreting the concept mapmaking. The episode below illustrates some of the complexity that emerged in the classroom as Ulrika grappled with making the concept map.

5 Attempts to Play the GoGAR Well—Episode 1: Overbridging Different "Accents"

Episode 1 illustrates how Ulrika struggled to grasp what Aldrin, a speaker of Persian and Swedish, wanted her to add to the map about the concept "mathematics." Darko, who speaks Serbian and Swedish, assisted their conversation by using Ulrika's formal Swedish mathematical "accent" (turn 3; "equations," turn 10; "assignments." turn 13; "problem-solving") to interconnect or overbridge Aldrin's claims and Ulrika's grasping of them. With the help of Darko, the three interlocutors formed a team that managed to play the GoGAR so that they, at least seemed to discern enough of what follows from Aldrin's claim in turn one for Ulrika to grasp it, namely the fact that we have numbers at all is due to mathematics.

While this interaction was in progress, Ulrika found herself in a dilemma. On the one hand, to grasp Aldrin's claim, which she thought might be unclear, not only to herself but to other students in the classroom as well, she needed to find reasons for it. Specifically in turns 2, 9, 11, and 17 she implicitly tried to elicit what reasons Aldrin had for his initial claim (turn 1). On the other hand, while doing so she realised that she was assessing the inferential interconnections Aldrin brought forward using her own middle-class mathematics teacher "accent," such as, for example, when she claimed not to have understood what Aldrin meant in turns 9 and 17.

When Ulrika asked the students for reasons for their statements, in order to grasp them, her assessment of the claims shaped what was included in the concept map. This resulted in Ulrika acting in a way that was quite the opposite of her aspiration of allowing the students to disrupt and trouble stereotyped images about mathematics and society and to let complexity and plurality emerge. Hence, teachers' good intentions concerning students' diverse "accents" or personal ways of knowing and talking about mathematics (Radford, 2012) and society appears to be a complex matter. However, students' interruptions in solidarity with one another, such as Darko's overbridging actions, can underpin these good intentions.

5.1 *Students' Derived Inferences from "Mathematics Is Bad for Society"*

In this section, we show how four students discerned what could follow implicitly from the statement "Mathematics is bad for society" to articulate critical aspects of mathematics as a power distributor in society.

– *Felix's claim:* Felix said "Man kan ha programmerat fel på trafikljusen så att det blir (OHÖRBART) krsch" ["Someone could have made a coding mistake with the traffic lights so it gets (UNINTELLIGIBLE) krsch"]. Felix used

EPISODE 1 Overbridging different "accents"

1	Aldrin:	Ja ett tal med ett tal det är lika med någonting. Det är matematik.	Yes one number together with another number equals something. That is mathematics.
2	Ulrika:	Räknesätt menar du?	An operation of arithmetic you mean?
3	Darko [INTERRUPTS]:	Ja det är ekvationer.	Yes it is equations.
4	Aldrin:	Räknesätt är ju plus minus och sådant.	Operations of arithmetic are plus and minus and such stuff.
5	Ulrika:	Ja, det var det du menade?	Yes is that what you meant?
6	Aldrin:	Att man tar ett tal och ett annat tal … ja typ som ett räknesätt	That you take one number and another number … yes well kind of like operations.
7	Ulrika:	Eller är det inte det du menar?	Or is that not what you meant?
8	Aldrin [hesitant]:	Ja …	Yes …
9	Ulrika:	Nu förstår jag inte riktigt vad du menar.	Now I do not quite understand what you mean.
11	Ulrika [to Aldrin]:	Du löser uppgifter …?	You do assignments …?
12	Aldrin:	Det är som det …	It is that which it …
13	Darko:	Problemlösning	Problem-solving
14	Aldrin:	Det som … (OHÖRBART)	That which … (UNINTELLIGIBLE)
15	Ulrika:	Var det det du menade?	Is that what you meant?
16	Aldrin:	Typ	Kind of
17	Ulrika:	Typ fast inte riktigt. Det är jag som inte fattar vad du menar. Säg en gång till så ska jag försöka se om jag förstår hur du menar.	Kind of but not quite. It is I who do not get what you mean. Say it once more and I will try to see if I can get what you mean.
18	Aldrin:	Typ att man tar ett tal med ett annat tal så lägger man ihop det och så …	Kind of that you take one number and add it to another number and then …
19	Ulrika:	Alltså *att* det finns tal som man kan använda och lägga ihop och så?	That there *are* numbers which we can use and add and so on?
20	Aldrin:	Ja!	Yes!

inferential interconnections, conceptualised as humans in action driving cars, and mathematics as actions materialised as coding located in physical objects i.e. traffic lights. In making these interconnections, Felix showed an awareness of the mathematical algorithms operating behind the switching traffic lights. Algorithms are designed by constructors who intentionally command the traffic and, therefore, hold power over people's lives. Thus, Felix made inferential interconnections between the statement "Mathematics is bad for society" and critical aspects of black boxed mathematics (Gellert & Jablonka, 2009).

- *Edin's claim:* Edin said "Man kan räkna ut något fel och då kan det bli dåligt för samhället" ["You can make a calculus mistake and then it can be bad for society"]. As premises for his claim, Edin used inferences interconnected to humans as social beings whose actions affect each other. Mathematics in his claim is related to systems of thought, such as calculus.
- *Elsa's claim:* Elsa said "på sjukhusen finns maskiner och om de blir felprogrammerade då så kanske de gör något som är farligt för människorna eller typ skadar dem" ["At the hospitals there are machines and if they get wrongly programmed they might do something which is hazardous to people or kind of hurt them"]. We interpret Elsa's claim as meaning that erroneous input data ("wrongly programmed") put into the hospital machines by operators that causes the harm, and not the algorithms themselves (as in the case with the traffic light). Elsa recognised that operators put information into machines and from that information, the machine "makes decisions." Thus, there is human action on two levels (constructors and operators) involved in the decisions of the machines. Elsa used inferences interconnected to society as physical places (hospitals) and human actions (giving care). She related mathematics as inferentially located in physical objects (machines) and conceptual constructs (data to be put into the machines).
- *Aldrin's claim:* Aldrin said, "Jag tänker så här att om man kör bil och sen så 20 (OHÖRBART) 20 på den där vägen och så får man köra 70 då måste man kunna matte" [I think like this that if you drive a car and then 20 (UNINTELLIGIBLE) 20 on that road the speed limit is 20 and you may drive 70 you need to know maths"]. Aldrin's claim places mathematics inferentially interconnected to regulatory power that shapes citizen behaviour. He recognised that mathematics is "useful" for conforming to the speed limit. Mathematics is helpful to regulate ourselves within societal norms and rules. Precisely how his claim is given as a reason for "mathematics being bad for society" is unclear. Perhaps he would prefer no speed limits. If so, his claim could be interpreted as an objection to the regulatory powers of mathematics. He inferentially interconnected mathematics to conceptual constructs related to time and distance or, in other words, speed.

Our analysis reveals how students provided reasons for why "mathematics is bad for society" by articulating inferential interconnections which showed how mathematics acts and is made to act in society. For instance, in Felix's and Aldrin's claims, mathematics operates as a regulating power in people's lives. In Edin's and Elsa's claims, mathematics was made to act when hospital employees use machines or when calculus is used in general. Edin's and Elsa's claims also showed that mathematics in society can distribute responsibility. It was implicit in Elsa's claim that hospital machines make decisions rather than humans, and hence that the machines will be held responsible in case of an accident. Edin's claim showed that humans rely on mathematics to tell the "truth"; that is, unless a calculus mistake has been made. In Edin's case, mathematics was a trusted "truth" teller (who is responsible) when things go right. Should things go wrong, i.e. should mathematics not tell the "truth," then it is a human calculator who is responsible for the "mistake."

The students used their own observations of the world to reason about mathematics in society, in contrast to teachers providing students with *their* observations of the world for the students to reason about. We argue that critical mathematics education not only means that students discuss social issues formulated as mathematical problems or approach these issues mathematically with the help of their teachers (see for example, Andersson, 2011; Barwell, 2013; Frankenstein, 1990; Gutstein, 2006, 2016). We think that critical mathematics education also means providing classroom activities where students put to the fore their own observations about the world and reason about them as a matter of interconnecting them to knowledge on mathematics in action in society. Moreover, as shown in the students' claims discussed above, such activities provide opportunities for students to disrupt discourses in and about mathematics as a value-free functional tool. At the same time, such activities may highlight students' individual "accents," shaping and sharing ways of talking and thinking about critical aspects of mathematics in society.

6 Attempts to Play the GoGAR Well—Episode 2: Peers Struggling to Grasp Each Other's Claims

As the students started the group talks focusing on "society" in the statement "mathematics is bad for society," Samir, an emergent Swedish speaker, claimed, "I know what society is. It is my brother's school." None of the three other students in the group responded to Samir's claim or used their own claims. For a while, Samir was silent. Then he re-entered the group talk telling the other students that he does not understand what "society" is. Episode 2 shows how the group talk proceeded thereafter.

EPISODE 2 Samir and his peers struggling to grasp each other's claims

9	Samir:	Jag fattar inte vad samhälle är.	I don't get what society is
10	Darko	Samhälle det är Fältvidda … det är ett samhälle … samhälle är som en ort.	Society it is Fältvidda [the community where the school is located] … that is a samhälle [in Swedish the same word, samhälle, is used both for society and community] … samhälle is a place.
11	Greta:	Jag tycker att det är människorna runt omkring en som innebär ett samhälle.	I think that it is the people around you that means a society.
12	Alla/All	Ja, ja.	Yes, yes.
13	Samir:	Kan ni förklara för mig?	Can you explain to me?
14	Greta:	Samhälle det är alltså typ så här jag tänker att det är människorna runtomkring en som bildar ett samhälle som är en del av samhället.	Society it is kind of I picture it as the people around you together make a society which is part of the society.
15	Samir:	Jag tycker samhälle är en skola som min bror går.	I think that society is a school that my brother attends.
16	Greta:	Ok [SLÅR UT MED HÄNDERNA] Tror du att det där är samhälle?	Ok [PUTS HER HANDS OUT] Do you think that is society?
17	Darko:	[SKAKAR PÅ HUVUDET]	[SHAKES HIS HEAD]
18	Samir:	Det är gymnasiet.	It is upper secondary school.
19	Darko:	Solajmon [SLANG?] TYSTNAD	Solajmon [SLANG?] SILENCE
20	Oscar:	Ja men ja det Greta sa.	Yes, but yes that what Greta said.
21	Samir:	Vad sa du? [TITTAR PÅ GRETA]	What did you say? [LOOKING AT GRETA]
22	Greta:	Att människorna är … [BLIR DISTRAHERAD AV DARKO]	That the people are … [GETS DISTRACTED BY DARKO]
23	Samir:	Människorna är …?	The people are …?
24	Greta:	Det är människorna runt omkring en som bildar ett samhälle.	It is the people around you who form a society.
25	Samir:	Jag fattar inte.	I don't get it.
26	Darko:	Samir Samir du ser de husen där borta?	Samir Samir do you see the houses over there?
27	Samir:	Jepp, jepp.	Yeah, yeah.
28	Darko:	Det är samhället … Fältvidda är ett samhälle.	That is society/community … Fältvidda is a society/community
29	Greta:	För det är människor i Fältvidda så bildar de ett samhälle.	Because there are people in Fältvidda they form a society.
30	Samir:	Fältvidda är ett samhälle. [HOPPAR SITTANDE UPP OCH NER PÅ SIN STOL]	Fältvidda is a society. [JUMPS UP AND DOWN IN HIS CHAIR]

We find in this episode an attempt to play the GoGAR well, so that Samir and his peers can discern what follows implicitly from each other's claims in order to grasp them.

In lines 9, 13, 21, 23, and 25, Samir urged his classmates to give reasons for their claims and in lines 10, 11, 14, 24, 26, and 28 they try to do so. The problem is that they did not manage to use Samir's claim about his brother's school as premises or conclusions in their own claims and vice versa. Samir's claim is inferentially related to the upper secondary programme that Samir's brother attends, which is called "Samhällsprogrammet" [Social science programme]. Literally, this translates as "Society programme," which is how Samir attempted to inferentially interconnect his brother's school programme with his peers' claims about "society." However, the other group members did not find it appropriate to make inferential interconnections between somebody's school programme and what can be inferred from "society," according to their previously played GoGARs. Samir, on the other hand, does not appear to assess the other group members' claims as appropriate. In line 30, Samir said "Fältvidda is a society" which could indicate that he had assessed Darko's claims in lines 10 and 28 as appropriate. However, Samir jumps up and down in his chair uttering the words in a chanting style, indicating that he is simply repeating Darko's words.

The missing inferential interconnections, the inferences from claims in lines 10, 11, 14, 24, 26, and 28 to Samir's claim about his brother's school were not made explicit. This meant that the group was unable to provide Samir with inferential interconnections which would have been useful for him in order to position his own, and his peers' claims as interconnected to each other and vice versa. For the students to play the GoGAR well, in this case to make explicit the interconnections between society and the upper secondary school programme "Samhällsprogrammet" [Social science programme] someone who can make the interconnection needs to enter the conversation. Such a GoGAR player could provide the missing inference to overbridge Darko and Greta's claims about what society is with Samir's claims about his brother's school. In the previous episode, Darko's "accent" comprised the inferences needed to overbridge Aldrin's and Ulrika's claims. In this episode, such an "accent" is missing, which could be a reason for the unsuccessful outcome of the GoGAR.

7 Attempts to Play the GoGAR Well—Episode 3: A Peer Gives Reasons for an Emergent Swedish-Speaker's Claim

In line with the notion of explorative talk (Mercer et al., 2004), Ulrika invited the students to challenge each other's claims. Samir disagreed with a peer who

EPISODE 3 Samir makes a claim and gets entitlements to it through support from a peer

1 Ulrika:	Jag frågar om det är någon som vill protestera jag frågar vad ni tycker.	I ask you if somebody would like to object I ask what you think.
2 Samir:	Det här som vi sa jag tycker inte att alls det är så.	This what we said. I do not think that is how it is.
3 Darko:	Affär för där har man ju en dator som räknar ut åt en man räknar ju ut på en dator man scannar ju så får man talet där så plussar man så så har man ju en dator som räknar ut det istället	Shops, because there you have a computer that counts for you. You count on the computer. You scan and you get the number and you add so so you have a computer to do that for you instead.
4 Ulrika:	Och då är det inte matematik menar du eller?	And then that is not mathematics you mean?
5 Darko:	Ja.	Yes.

claimed that cashiers who operate tills use mathematics. Darko, sitting next to Samir in the classroom, entered the whole-class talk right after Samir's objection. In the video recording, it is possible to hear the two of them engaging quietly in an intense conversation right before Samir makes his objection. What they say is unfortunately inaudible in the recording. It is likely that they were discussing reasons for Samir's objection.

Darko, who knew more Swedish than Samir, immediately gave reasons for Samir's claim using previously articulated inferential interconnections between mathematics and shops and computers. While doing so, Darko acted as a carrier of reasons for Samir's claim and as such he made it possible for Samir to provide good reasons for his initial claim. If Darko had refrained from giving reasons for Samir's claim, it could have been questioned and Samir would have risked not being able to give good reasons for it (in Swedish). Furthermore, the GoGAR would not have been played well, since the inferential interconnections revealed by Samir would have been left unarticulated. As it turned out, Samir's claim, the reasons for which were articulated by Darko, was questioned by other students. As a result, even more reasons for it were given, and additional inferential interconnections between Samir's and other students' claims were discerned.

In this episode, students discerned conflicting reasons for locating mathematics either in people's head or in machines. Finally, they were able to discern good reasons for claiming both locations. The analysis shows attempts to play the GoGAR well when a peer acted as a carrier of reasons, which to some extent overbridged the skewed access to language caused by the "Swedish only" discourse (Norén, 2010; Norén & Andersson, 2016; Skog & Andersson, 2014).

8 Closing Remarks

Of course, the small-scale project analysed in this chapter does not equip the students with all means necessary to fully establish the interconnections between knowledge bases that Gutstein (2016) discusses. However, we find that it did allow the students to see mathematics and society as interconnected and to grasp the political role of mathematics in society, i.e. to be critically mathematical literate. If students cannot see mathematics and society as interconnected, teachers' efforts to engage students with critical mathematics activities risk becoming just another mathematics assignment to students. For the students in Arvid's classroom to make inferences connecting mathematics and society, they had not only to unpack implicit meanings of mathematics in society, but also to make displacements from prevailing discourses about mathematics as being neutral or beneficial in society towards disturbing these same discourses. It requires courage, confidence, and language proficiency to articulate and unpack such ideas—a complex challenge, particularly in a language diverse classroom. In our study, Darko appeared to be aware of this challenge as he displayed a meta-understanding of language diversity and acted accordingly, by for instance, overbridging Ulrika's and Aldrin's "accents" and by giving reasons for Samir's claim.

As students and teachers grapple with unpacking critical aspects of mathematics in society in a language diverse but monolingual classroom, their reciprocal assessment of claims based on their "accents" of knowing and talking about mathematics and society shape both their actions and the outcome of their efforts. Hence, the critical aspects of mathematics in society that come to the fore in a language diverse mathematics classroom are intertwined with students' and teachers' (lack of) meta-understanding of language diversity and attempts to make their ideas explicit in order to put forward and grasp each other's claims. As normative assessments guide the outcome of these attempts, power is present in the reasoning. In other words, the semantic *whats* are dialectically entangled with the social, power-related *hows* in the complexity of addressing and enhancing critical mathematical literacy in a language diverse yet monolingual classroom. Further research to address this complexity is needed in order to deepen and widen the understanding of how critical mathematical literacy can be established in classrooms like Arvid's.

Notes

1 A version of this chapter appears in Ulrika Ryan's doctoral dissertation (Ryan, 2019).
2 In this chapter we use the word "acknowledge" with its colloquial meaning.

3 The lessons were video recorded using a Lessonbox©, i.e. a set of three cameras and three microphones. Unfortunately, one of the Lessonbox© cameras did not function. Therefore, a handheld camera was used in addition.

4 For ethical reasons school location and students' names in the chapter are pseudonyms.

References

Andersson, A. (2011). A "curling teacher" in mathematics education: Teacher identities and pedagogy development. *Mathematics Education Research Journal, 23*(4), 437–454. https://doi.org/10.1007/s13394-011-0025-0

Andersson, A., & Wagner, D. (2017). Numbers for truth and reconciliation: Mathematical choices in ethically rich texts. *Journal of Mathematics and Culture, 11*(3), 18–35.

Barwell, R. (2013). The mathematical formatting of climate change: Critical mathematics education and post-normal science. *Research in Mathematics Education*, *15*(1), 1–16. https://doi.org/10.1080/14794802.2012.756633

Brandom, R. B. (1998). *Making it explicit: Reasoning, representing, and discursive commitment*. Harvard University Press.

Brandom, R. B. (2001). *Articulating reasons: An introduction to inferentialism*. Harvard University Press.

Bransen, J. (2002). Normativity as the key to objectivity: An exploration of Robert Brandom's articulating reasons. *Inquiry*, *45*(3), 373–391. https://doi.org/10.1080/002017402760258204

Chronaki, A. (2010). Revisiting mathemacy: A process-reading of critical mathematics education. In H. Alrø, O. Ravn, & P. Valero (Eds.), *Critical mathematics education: Past, present and future* (pp. 31–49). Sense. https://doi.org/10.1163/9789460911644_005

Chronaki, A., Moutzouri, G., & Magos, K. (2015). "Number in cultures" as a playful outdoor activity: Making space for critical mathematics education in the early years. In U. Gellert, J. Giménez Rodríguez, C. Hahn, & S. Kafoussi (Eds.), *Educational paths to mathematics: A C.I.E.A.E.M. Sourcebook* (pp. 143–159). Springer. https://doi.org/10.1007/978-3-319-15410-7_8

Derry, J. (2013). Can inferentialism contribute to social epistemology? *Journal of Philosophy of Education*, *47*(2), 222–235. https://doi.org/10.1111/1467-9752.12032

Frankenstein, M. (1983). Critical mathematics education: An application of Paulo Freire's epistemology. *Journal of Education, 165*(4), 315–339. https://doi.org/10.1177/002205748316500403

Frankenstein, M. (1990). Incorporating race, gender, and class issues into a critical mathematical literacy curriculum. *The Journal of Negro Education*, *59*(3), 336–347. https://doi.org/10.2307/2295568

Frankenstein, M. (2014). A different third R: Radical math. *Radical Teacher*, *100*, 77–82. https://doi.org/10.5195/rt.2014.160

Gellert, U., & Jablonka, E. (2009). The demathematising effect of technology: Calling for critical competence. In P. Ernest, B. Greer, & B. Sriraman (Eds.), *Critical issues in mathematics education* (pp. 19–24). Information Age.

González, N., Andrade, R., Civil, M., & Moll, L. (2001). Bridging funds of distributed knowledge: Creating zones of practice in mathematics. *Journal of Education of Students Placed at Risk, 6*(1–2), 115–132. https://doi.org/10.1207/S15327671ESPR0601-2_7

Gutstein, E. (2006). *Reading and writing the world with mathematics: Toward a pedagogy for social justice*. Routledge. https://doi.org/10.4324/9780203112946

Gutstein, E. (2016). "Our issues, our people—Math as our weapon": Critical mathematics in a Chicago neighborhood high school. *Journal for Research in Mathematics Education, 47*(5), 454–504. https://doi.org/10.5951/jresematheduc.47.5.0454

Hauge, K. H., & Barwell, R. (2017). Post-normal science and mathematics education in uncertain times: Educating future citizens for extended peer communities. *Futures, 91*, 25–34. https://doi.org/10.1016/j.futures.2016.11.013

Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education, 35*(2), 81–116. https://doi.org/10.2307/30034933

Jablonka, E. (2003). Mathematical Literacy. In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.), *Second international handbook of mathematics education* (pp. 75–102). Springer. https://doi.org/10.1007/978-94-010-0273-8_4

Kollosche, D. (2014). Mathematics and power: An alliance in the foundations of mathematics and its teaching. *ZDM Mathematics Education*, *46*(7), 1061–1072. https://doi.org/10.1007/s11858-014-0584-0

Mercer, N., Dawes, L., Wegerif, R., & Sams, C. (2004). Reasoning as a scientist: Ways of helping children to use language to learn science. *British Educational Research Journal, 30*(3), 359–377. https://doi.org/10.1080/01411920410001689689

Norén, E. (2010). *Flerspråkiga matematikklassrum: Diskurser i grundskolans matematikundervisning* [*Multilingual mathematics classrooms: Discourses in compulsory school in Sweden*] (Doctoral dissertation). Stockholms universitet.

Norén, E., & Andersson, A. (2016). Multilingual students' agency in mathematics classrooms. In A. Halai & P. Clarkson (Eds.), *Teaching and learning mathematics in multilingual classrooms: Issues for policy, practice and teacher education* (pp. 109–124). Sense. https://doi.org/10.1007/978-94-6300-229-5_8

Radford, L. (2012). Commentary on the chapter by Richard Barwell, "Heteroglossia in multilingual mathematics classrooms." In H. Forgasz & F. Rivera (Eds.), *Towards equity in mathematics education: Advances in mathematics education* (pp. 339–342). Springer. https://doi.org/10.1007/978-3-642-27702-3_30

Ryan, U. (2019). *Mathematics classroom talk in a migrating world: Synthesizing epistemological dimensions* (Doctoral dissertation). Malmö University.

Schindler, M., Hußmann, S., Nilsson, P., & Bakker, A. (2017). Sixth-grade students' reasoning on the order relation of integers as influenced by prior experience: An inferentialist analysis. *Mathematics Education Research Journal, 29*(4), 471–492. https://doi.org/10.1007/s13394-017-0202-x

Skog, K., & Andersson, A. (2014). Exploring positioning as an analytical tool for understanding becoming mathematics teachers' identities. *Mathematics Education Research Journal, 27*(1), 65–82. https://doi.org/10.1007/s13394-014-0124-9

Skovsmose, O. (1994). *Towards a philosophy of critical mathematics education.* Kluwer.

Skovsmose, O. (2007). Mathematical literacy and globalisation. In B. Atweh, A. C. Barton, M. C. Borba, N. Gough, C. Keitel, C. Vistro-Yu, & R. Vithal (Eds.), *Internationalisation and globalisation in mathematics and science education* (pp. 3–18). Springer. https://doi.org/10.1007/978-1-4020-5908-7_1

Skovsmose, O., & Borba, M. (2004). Research methodology and critical mathematics education. In P. Valero & R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology* (pp. 207–226). Kluwer. https://doi.org/10.1007/1-4020-7914-1_17