COORDINATING THE IC-MODEL WITH A FRAMEWORK ON COMMUNICATION IN ANALYSING STUDENT-TO-STUDENT INTERACTIONS IN MATHEMATICS

Marie Sjöblom

Malmö University

The aim of this paper is to investigate possibilities for coordinating two theories in order to analyse interactions in a group of four students completing a mathematics task in upper secondary school. The investigation suggests that the theories can be coordinated, but their interpretations of the interactions provide information on different levels. Alrø and Skovsmose's IC-model gives a more general picture of what is happening in the interaction between students, whereas Fuentes's framework gives details to what is happening in the different dialogic acts in the IC-model.

Keywords: student-to-student interaction, student communication, coordinating theories, educational design research, communication frameworks

INTRODUCTION

In mathematics education research many studies have been conducted on giving students opportunities to talk mathematics, both in whole-class situations and in small group work. Alrø and Skovsmose (2004) claimed that "the qualities of communication in the classroom influence the qualities of learning mathematics" (p. 11). This paper focuses how quality and quantity in students' mathematical communication can be studied with help of two theories on student interaction, namely the IC-model (Alrø & Skovsmose, 2004) and Fuentes's (2013) framework for analysing student communication. Both theories have previously been used for examining students' interactions, but the theories have not been connected to each other. In a previous paper (Sjöblom, 2014), I described how the IC-model (Alrø & Skovsmose, 2004) was used to design mathematical tasks with the aim of increasing student-to-student interaction in an upper secondary class. However, the IC-model (Alrø & Skovsmose, 2004) was not developed as an analytical tool for understanding small group work, so when analysing the student communication, a need for more theory on student interaction occurred (Sjöblom, 2014). Therefore, in this paper, possibilities for coordinating the IC-model (Alrø & Skovsmose, 2004) with Fuentes's (2013) framework are investigated in order to understand how capable the theories are and if coordinating them can help studying the quality and quantity in students' mathematical reasoning. If the theories can be coordinated, it will help to answer the research question: How do student interactions and perceptions change over time when different tasks are provided to increase student-to-student interaction?

There can be several reasons for connecting frameworks or theories in mathematics education research. According to Prediger, Bikner-Ahsbahs and Arzarello (2008), connections can contribute to gaining "increasing explanatory, descriptive, or prescriptive power" (p. 169). In my project, when designing different tasks to

increase student-to-student interaction, there is a need for a theoretical base to justify why the tasks are designed the way they are and also for how data is analysed. A connection of the IC-model (Alrø & Skovsmose, 2004) and the Fuentes's (2013) framework gives possibilities to explain and describe what happens in the communication between students from two different starting points and hence a possibility to evaluate improvements in the quality and quantity of the students' interactions. This can be done amongst other methods through looking at which mathematical questions that students ask each other and see how these questions are treated within the groups of students. One of the reasons for why the IC-model (Alrø & Skovsmose, 2004) and Fuentes's (2013) framework are chosen amongst other communication theories is that both theories address mathematical questions. They also have other equalities in their structures and can both be used for studying student-to-student interaction. A more detailed comparison of the structures is made after the theories are introduced.

ALRØ AND SKOVSMOSE'S IC-MODEL

The IC-model was developed by Alrø and Skovsmose (2004) to describe how teachers and students can work together to explore a landscape of investigation. This means that students instead of working individually with textbook exercises are introduced to a scene setting in which they can ask questions and choose directions for what they want to investigate (Alrø & Skovsmose, 2004). The students are active participants in an inquiry process with mathematical content. The IC-model can also be applied to students' inquiry processes in which a teacher not is present at all times (Alrø & Skovsmose, 2004). When students discuss mathematics in small groups, they are given possibilities to make use of each other's contributions assuming that they have different previous knowledge and different ways of interpreting and dealing with mathematical tasks. This can be connected to the students' *zones of proximal development*, that is the possibility students have to learn more together with a teacher or more capable peers compared to working alone (Vygotski, 1978). Also Goos, Galbraith and Renshaw (2002), have studied how zones of proximal development could be established through students' interactions in small group work, but they preferred to use the term *collaborative zone of proximal development* when describing student collaboration in order to differ it from what is happening when students talk to a teacher.

In the IC-model there are eight dialogic acts, namely: *getting in contact* (prepare for interaction), *locating* (understand the problem), *identifying* (find the mathematics in the problem), *advocating* (examine ideas), *thinking aloud* (make perspectives and thoughts visible), *reformulating* (clarify and rephrase), *challenging* (question) and *evaluating* (look back at the problem). The acts do not appear in linear order, and sometimes in conversations only some acts are present. Alrø and Skovsmose (2004) found that fully developed IC-models are rare. This could affect the quality as well as quantity of mathematical communication and learning as Alrø and Skovsmose (2004)

considered that "the IC-model not only represents qualities of communication, it also constitutes an important resource for learning" (p. 67).

In the IC-model, mathematical questions are important parts of the inquiry process. When exploring a landscape of investigation, questions about what to do, why and how need to be addressed. Some questions can be connected to acts in the IC-model. According to Alrø and Skovsmose (2004), tag-questions that students ask at the end of an utterance to reassure that other students are with them, are part of *getting in contact*. Further, when students are *locating* the problem, they need to ask what-if-questions and inquiring questions in order to explore different perspectives. Alrø and Skovsmose (2004) stated that when a what-if-question is followed by a why-question, then *locating* turns into *identifying*, and answers to why-questions when students give suggestions to how to solve problems, can be connected to *advocating*. Also the other acts can include questions, for instance, *reformulating*, can start with "do you mean…" and *challenging* is often an act of questioning mathematical statements. In all the acts in the IC-model, it is important that students are active in the interaction.

In order to be able to study the quality and quantity of what is happening in the different dialogic acts in the IC-model, one possibility can be to look at other theories concerning communication between students working together in groups. Therefore, in the next section, Fuentes's (2013) framework is presented.

FUENTES'S FRAMEWORK ON STUDENT COMMUNICATION

Fuentes (2013) conducted a series of interventions in a practitioner action research project on students' collaborative work. She claimed that it is important for students to evaluate each other's reasoning and found three categories of issues that hindered effective student communication: lack of communication between all students in a group, poor communication patterns and norms that impede student learning (Fuentes, 2013). In order to analyse what happened in the student communication, Fuentes (2013) extended a framework by Dekker and Elshout-Mohr (2004) to contain eight question/comment-response pairs used to analyse student interaction. Fuentes (2013) classified what happened in conversations between students with help of the question/comment-response pairs and then identified common interaction patterns between the students, which she used for promoting student-to-student discourse.

Question/Comment	**Response**
1. A asks B to show work	1. B shows own work
2. A asks B to explain work	2. B explains own work
3. A criticises B's work	3. B justifies own work
4. A rejects B's justification	4. B reconstructs own work
5. A asks B to evaluate work	5. B evaluates A's work
6. A suggests a strategy to the group	6. The group tries the strategy

7. A asks B a content question	7. B answers A's question
8. A asks B a clarification question	8. B answers A's question

Table 1: Framework for analysing student communication (from Fuentes, 2013, p. 54)

In Table 1, the first four question/comment-response pairs are from Dekker and Elshout-Mohr (1998), while question/comment-response pairs 5-8 are Fuentes's (2013) elaboration. Fuentes (2013) used the framework for analysing student communication in order to design teacher interventions that promoted student-to-student discourse, both for when the teacher was present and when the students worked by themselves in groups. The interventions were made to "model and foster activities such as asking questions, sharing and comparing strategies, listening to explanations, and assessing methods and solutions" (Fuentes, 2013, p. 69).

There are many similarities between this framework and the IC-model. Since a need to deeper analyse the dialogic acts in the IC-model was found in Sjöblom (2014), in the next section, an investigation of coordinating the theories is conducted.

COORDINATING THEORIES

Prediger et al. (2008) gave examples of different strategies to connect theories in mathematics education research, which stretched from *ignoring other theories* to *unifying globally*. Between these two extremes, there are different networking strategies: *understanding others, making understandable, contrasting, comparing, combining, coordinating, synthesizing* and *integrating locally*. In this paper focus is on *coordinating* theories, since this strategy enables "a networked understanding of an empirical phenomenon or a piece of data" (Prediger et al., 2008, p. 10). *Coordinating* also enables triangulation of data (Prediger et al., 2008). This is valuable in a study of students' communication, since then the communication could be looked at from two different perspectives, which gives more possibilities to study both the quality and quantity of students' interactions.

Wedege (2010) combined Prediger et al.'s (2008) networking strategies with Radford's (2008) morphology of theories in which theories in mathematics education are seen as triplets $T = (P, M, Q)$, where P is a system of principles, M is a methodology supported by P, and Q is a set of research questions. Wedege (2010) claimed that it is important when using Prediger et al.'s (2008) networking strategies, to look at how the triplets T of the theories are interrelated as well as the goal of the connection, so that these do not contradict each other.

Wedege (2010) claimed that Prediger et al.'s (2008), *coordinating* of theories was a connection strategy:

> The term *coordinating* is used when a conceptual framework is built by well fitting elements from different theories: elements e.g. from the basic principles P, are chosen and put together in a more or less harmonious way to investigate a certain research problem. (Wedege, 2010, p. 67)

In this paper, an investigation of the possibility to coordinate the IC-model and the Fuentes's framework is made, since a *coordinating* of them would help to analyse how students' interaction change when different tasks are introduced to increase the student-to-student interaction. Now the different elements in the triplets $T = (P, M, Q)$ for the two theories are discussed.

System of principles, *P*

The first step is to look at the basic principles, *P*, of the theories. *P* includes "implicit views and explicit statements that delineate the frontier of what will be the universe of discourse and the adopted research perspective" (Radford, 2008, p. 319). A first comparison of the system of principles show that they both build on sociocultural views that learning mathematics is connected to working together and using language to express thinking.

A second principle for both theories is that they can be applied to communication between students and that they intend to say something about the quality of the communication. Counting how often the acts in the IC-model or the question/ comment-response pairs occur also opens up for a qualitative analysis.

A third principle that is similar is that both theories assume that students are active participants in the communication and that students together build up the mathematical conversation.

Finally, a fourth principle is that both theories focus questions. In the IC-model, the different dialogic acts contain questions when students work together with the inquiry process and questions are also an important part of Fuentes's framework.

Methodology, *M*

The second step is to look at the methodologies *M* of the two theories. Both theories assume that there will be a communication about mathematics going on between students. In Fuentes's framework communication can be analysed by looking at question/comment-response pairs. In the IC-model, communication can be analysed by looking at the different dialogic acts. The data needed for these analyses is the same and the data collection can be made in similar ways, for instance through audio or video recordings.

Set of research questions, *Q*

The third step involves identifying the research questions *Q* that are possible to ask when working with Fuentes's framework or the IC-model. For both theories, questions can be asked concerning student communication and interaction. It appears that both theories aim at addressing issues to deepen the understanding for students' communication. In my study both theories could be used for answering the research question on how interactions and perceptions change over time when different tasks are provided to increase the student-to-student interaction.

Possibilities for coordinating theories

From looking at the triplets $T = (P, M, Q)$ for the IC-model and Fuentes's framework, many similarities in the structures for the theories can be found regarding all three parts P, M and Q, and the theories do not appear to contradict each other. It therefore seems plausible to coordinate them. Now an example of how a *coordinating* could be applied to a piece of data concerning student communication is given. First a short description to the background of the project from where the data originates is given.

METHOD AND AN EXAMPLE OF THE ANALYSIS PROCESS

Utilising educational design research methods (McKenney & Reeves, 2012), different tasks to increase student interaction are introduced in a first year, upper secondary mathematics class in a city in Sweden. Education design research is a cyclic method in which tasks are tried out and evaluated in cycles. The results from each cycle are incorporated into the planning of the next cycle. Each cycle has a theoretical base from which the choices of tasks and settings for the designs are made. The theories are used in both designing and evaluating the results of the tasks and thus provide an analysis of data that provides information about the research questions.

In the first cycle (Sjöblom, 2014), a mathematical group task was designed to promote some of the dialogic acts in the IC-model. The analysis of interactions between the students in the group was performed through categorizing student communication within the different acts. Results from the first cycle showed that students were active communicators in their small groups. However, the communication was not very structured. For instance, students did not seem to use mathematical questions deliberately as a part of the problem solving process, and a question list, given to the students as a support mean in the group work was not really utilised. Consequently it was decided to focus more on mathematical questions in the second cycle. Here Fuentes's framework for analysing students communication was considered an option and therefore in this paper, coordinating it with the IC-model is explored in order to understand how capable these theories were. A *coordinating* of the theories could provide a better theoretical base for answering the research questions, since a more detailed analysis of the interactions would be possible to conduct in the coming educational design research cycles.

Below is a transcript from a group with four students: Azad, Mohammed, Amal and Nour, who are working collaboratively on a mathematical task concerning loans and interests. The transcript was chosen to illustrate how communication can be related both to the IC-model and Fuentes's framework for analysing student communication. In the rightmost column in Table 2, connections are made to both these theories. When there is a number after a comment, question or response in Fuentes's framework, this refers to the corresponding line in Table 1. For instance response1 means that a student is showing another student his/her work (line 1 in Table 1).

The task that the students was given by their teacher Johannes was:

Marie and Johannes have one loan each. Marie's loan has an interest of 3% and Johannes's loan has an interest of 6%. Each month they pay the same amounts on their loans. How much money is it possible that they have borrowed?

When the students had read the task, they worked with it individually. Azad and Mohammed chose to discuss some possible solutions before the group discussion began. Azad saw the solution straight away, but Mohammed misinterpreted him or rephrased Azad's solution incorrectly when the following conversation began:

	Student	**Conversation**	**IC-act and/or question/ comment/response**
1	Moham-med	(*To Amal and Nour:*) Have you understood the task? Johannes has borrowed twice as much as Marie.	*getting-in-contact* comment6
2	Azad	Wait, let's start like this: Which questions do we need to ask to solve the task?	comment6
3	Nour	(*To Mohammed:*) But that will not be as much. If I take an example: 50 Swedish kronor and 100 Swedish kronor. But then she will pay 5… what was it?	*challenging* comment6, response6 comment3 question8
4	Amal	She pays 3 %...	response8
5	Nour	She pays 3%.	*rephrasing*
6	Amal	… but still just as much, but he pays 6%.	*locating*
7	Nour	Yes, and that will be…	
8	Amal	So it is she that has borrowed two times as much?!	*advocating* comment6
9	Azad	If they pay the same amount, then she must pay more than him, twice the amount, but I am not sure.	*advocating, reformulating* response6
10	Moham-med	Yes, yes, that is right.	
…	…	…	…
32	Nour	Wait a little; is it not Marie that pays more?	question2

33	Azad	Yes, Marie pays more since her percentage is less.	response2
34	Amal	But still… she pays the same amount as him?	*challenging* question3
35	Azad	Yes, she must pay more so that it can balance. Johannes pays more. No. They pay the same amount.	*thinking aloud* response3
36	Amal	But her loan is bigger, or, I don't know.	*advocating*
37	Azad	She has lower interest, but she has loaned more money. That's it. What do you think Mohammed?	*reformulating* response3, comment6 *evaluating* question5

Table 2: Transcript of interaction between Amal, Nour, Mohammed and Azad.

In the next section, results from the *coordinating* of theories are made.

RESULTS FROM THE COORDINATING OF THEORIES

In the short excerpt of the conversation between the students, several acts from the IC-model as well as questions that can be connected to Fuentes's framework were found. Alrø and Skovsmose (2004) stated that, the acts often do not appear in linear order, and all acts do not need to be present. This also seemed to be the case in this example. It also seemed that the questions related to Fuentes's framework did not occur the linear order that they appeared in Table 1. Even though a comment6 often is followed by a response6, it might be that while the students are *advocating* about how to solve the task, they might suggest a lot of strategies but do not carry all of them through. Then comment6 acts as a suggestion rather than a command.

In Table 2, the interactions could be analysed separately with the IC-model or Fuentes's framework. Both models give information to what is happening in the communication between students. *Coordinating* the theories, however, gives an opportunity to look at data from two different perspectives that complement one another. Thus connecting the theories gives a possibility to show in more detail how the quantity and quality of the student-to-student interaction could be expanded.

Starting with Fuentes's framework, it is possible to categorize what is happening in the different utterances between students. Adding the IC-model here gives a more general description about what the communication is about when categorizing it into different dialogic acts. For instance, an identification of questions with help of Fuentes's framework gives clear suggestions to what acts in the IC-model that the

students work with, which can facilitate understanding of what kind of *advocating* or *challenging* that is going on and the quality of the *advocating* or *challenging*.

It is also possible to start with the IC-model and look at what dialogic acts the students are working with. The IC-model then gives an opportunity to understand the context and reason for why different questions are asked. Adding Fuentes's framework here could give details to what type of question/comment-response pairs that students are using in the different dialogic acts.

Further analysis of the second cycle, indicates that some of the acts in the IC-model and Fuentes's framework are closely interrelated. For instance, *challenging* questions can be connected to question3/comment3; *advocating* often contains suggestions to how to solve the problem, which also is done in question6/comment6 and response6; and the act of *evaluating* can be connected to question5/comment5 and response5. Question7 and question8 are asked throughout the conversation and seem to be connected to the acts of *getting in contact*, *locating* or *identifying*.

These connections seems to provide a strong basis for identifying how both the quality and quantity of mathematical questions change over time when other tasks for promoting student-to-student interaction are introduced in further design cycles.

CONCLUSION

Analysing student-to-student communication is very complex, but yet an important and central question within mathematics education research. To deal with the complexity, Prediger et al. (2008) claimed that *coordinating* theories could provide "a networked understanding of an empirical phenomenon" (p. 174). In my study, a *coordinating* of the IC-model and Fuentes's framework gives opportunities for studying changes in the quality and quantity of students' interaction. Through the IC-model, the dialogic acts in the conversation can be found, while Fuentes's framework gives details to what is happening in the different dialogic acts in the IC-model. A *coordinating* of the theories can provide for instance answers to how questions are handled in the students' conversation with the goal to see how interactions change over time when different tasks are introduced to increase the student communication.

The results from *coordinating* the IC-model with Fuentes's framework will be used for the next cycle in my project. In the first cycle, when only the IC-model was used, it was sometimes difficult to find out what was problematic for instance when students tried to *advocate* a suggested strategy. With Fuentes's framework, it is more clear that when strategies are suggested (comment6) and that when questions are asked, it is not always that they follow the pattern in Fuentes's framework, that is a question/comment6 is not always followed by a response6. This was interpreted that students need to be more aware of the questions they are asking each other, why they are asked and how they are answered. In this way *coordinating* the theories do not only give a theoretical base for how to design tasks, but also a way to find strategies for promoting interaction and giving students support means for how to work together in their collaborative zone for proximal development (Goos et al., 2002).

A principle *P* that can be considered problematic with both the IC-model and Fuentes's framework is the assumption that students are active participants in the interaction. It will therefore not be possible to analyse all types of students' communication with these two theories, for instance it is difficult when students choose not to participate in the group work. More research is needed to find theories for analysing different kind of interaction between students and to see which theories that are best to connect in order to give a theoretical base for promoting, understanding and analysing student-to-student interaction.

REFERENCES

Alrø, H. & Skovsmose, O. (2004). *Dialogue and learning in mathematics education. Intention, reflection, critique*. London: Kluwer Academic Publishers.

Dekker, R. & Elshout-Mohr, M. (1998). A process model for interaction and mathematical level raising. *Educational studies in mathematics 35*(3), 303-314.

Dekker, R. & Elshout-Mohr, M. (2004). Teacher interventions aimed at mathematical level raising during collaborative learning. *Educational studies in mathematics 56*(1), 39-65.

Fuentes, S. (2013). Fostering communication between students working collaboratively: results from a practitioner action research study. *Mathematics teacher education and development 15*(1), 48-71.

Goos, M., Galbraith, P., & Renshaw, P. (2002). Socially mediated metacognition: creating collaborative zones of proximal development in small group problem solving. *Educational Studies in Mathematics, 49*(2), 193-223.

McKenney, S. & Reeves, T. (2012). *Conducting educational design research*. New York: Routledge.

Prediger, S. Bikner-Ahsbahs, A., & Arzarello, F. (2008). Networking strategies and methods for connecting theoretical approaches: first steps towards a conceptual framework. *ZDM 40*(2), 165-178.

Radford, L. (2008). Connecting theories in mathematics education: challenges and possibilities. *ZDM 40*(2), 317-327.

Sjöblom, M. (2014). Designing tasks and finding strategies for promoting student-to-student interaction in mathematics. Paper accepted for publication in the proceedings of the conference *MADIF9*.

Wedege, T. (2010). Connecting theories in mathematics education: from bricolage to professionalism. *Nordic studies in mathematics education 15*(1), 59-78.

Vygotsky, L. (1978). *Mind in society. The development of higher psychological processes*. London: Harvard University Press.