Doubly-magic character of $^{132}\text{Sn}$ studied via electromagnetic moments of $^{133}\text{Sn}$


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We report the first measurement of the magnetic dipole and electric quadrupole moment of the exotic nucleus $^{133}\text{Sn}$ by high-resolution laser spectroscopy at ISOLDE/CERN. These, in combination with state-of-the-art shell-model calculations, demonstrate the single-particle character of the ground state of this short-lived isotope and, hence, the doubly-magic character of its immediate neighbor $^{132}\text{Sn}$. The trend of the electromagnetic moments along the $N = 83$ isotonic chain, now enriched with the values of tin, are discussed on the basis of realistic shell-model calculations.

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In nuclear physics, certain numbers of protons ($Z$) or neutrons ($N$), such as 2, 8, 20, 28, 50, 82, and 126, are known as “magic.” These numbers endow the nucleus with a special stability analogous to the chemical stability associated with noble gases. Its existence led to the hypothesis that the nucleus contains shells of nucleons that are similar to the shells of electrons in an atom. About 250 species, of approximately 3000 discovered to date, have magic numbers of protons or neutrons, and only ten of them have magic numbers of both. Among this exclusive group, five nuclides are due to their radioactive nature notoriously difficult to access experimentally. However, thanks to state-of-the-art techniques, detailed spectroscopic information can nowadays be obtained for $^{132}\text{Sn}$ (50 protons and 82 neutrons), the heaviest radioactive doubly-magic nucleus.

During the past decade, many experimental studies have aimed to investigate whether $^{132}\text{Sn}$, eight neutrons away from the heaviest stable tin isotope, retains its doubly-magic character [1–7]. It is, in fact, well recognized, that the single-particle ordering which underlies nuclear shell structure may change in those nuclei with a large $N/Z$ ratio, leading to the disappearance of classic shell gaps and the appearance of new magic numbers. Clear evidence of this phenomenon has been
found for light- and medium-mass nuclei. For instance, it has been shown that in $^{32}\text{Si}$ $N = 28$ is no longer a magic number [8], whereas $N = 16$ does appear to be magic in neutron-rich oxygen isotopes [9–11] and the same is suggested for $N = 32$ and 34 in calcium isotopes [12–14] although the doubly-magic nature of $^{52}\text{Ca}$ is challenged by recent laser-spectroscopy work [15]. As for $^{132}\text{Sn}$, two transfer-reaction experiments have provided leading information through measurements of spectroscopic factors [16] and lifetimes [17] of ground and excited states in $^{133}\text{Sn}$. Both experiments have shown that, regardless of its large neutron-to-proton ratio, this nucleus can be considered a very robust doubly-magic core. Such a finding is of importance in current nuclear structure research as the persistence of (double-) magicity despite an unbalanced $N/Z$ ratio may shed light into the detailed mechanism causing the unexpected shell evolution in other areas of the nuclear landscape. Furthermore, it validates the choice of $^{132}\text{Sn}$ as a closed core in shell-model calculations, making them a reliable tool to describe this mass region, which is important for the rapid neutron-capture process creating elements in merging neutron stars [18,19].

In this Rapid Communication, we report new evidence of the doubly-magic character of $^{132}\text{Sn}$ through a measurement of the electromagnetic moments of $^{133}\text{Sn}$ using high-resolution collinear laser spectroscopy. The experimental data, in combination with state-of-the-art shell-model calculations, clearly show that $^{132}\text{Sn}$ plays a prominent role as a closed core and can, therefore, be used to describe more complex systems in this region. This is confirmed on higher-mass isotones ($N = 83$) for which experimental moments are found to be well described by theory.

The beam of $^{133}\text{Sn}$ was produced at ISOLDE/CERN. High-energy protons impinging on a tungsten rod generated spallation neutrons, which, in turn, induced fission in a uranium carbide target [20]. Following laser ionization [21], electrostatic acceleration to 40 or 50 keV and mass selection, the ions were injected into a linear Paul trap [22], which provided bunched beams with a temporal width of about 5 μs. Fast ion bunches were released to the collinear laser spectroscopy beam line, postaccelerated and neutralized by charge exchange with sodium vapor [23,24]. A continuous-wave laser beam was collinearly superimposed with the bunched atomic beam. The laser frequency was kept fixed whereas the Doppler-shifted frequency was scanned by varying the potential applied to the charge-exchange cell. The fluorescence emitted from the laser excited atomic beam was imaged by telescopes of aspheric lenses onto four photomultiplier tubes. To suppress background events a time gate corresponding to the photon signal. Details concerning the experimental setup can be found in the review by Neugart et al. [25]. A sketch of the collinear laser spectroscopy beamline is given in Ref. [26].

Hyperfine structures were measured in two complementary transitions of the neutral tin atom, shown in Fig. 1. The transition $5p^2 3S_0 \rightarrow 5p6s \, 1P_1$ at 453 nm offers a large quadrupole splitting whereas the transition $5p^2 3P_0 \rightarrow 5p6s \, 3P_1$ at 286 nm provides high sensitivity to magnetic moments. The laser light was produced by frequency doubling the fundamental light of a continuous-wave single-mode ring laser, operated either as titanium sapphire or dye.

Example hyperfine spectra of $^{133}\text{Sn}$ are presented in Fig. 1. Simultaneous analysis of the two transitions was conducted within the ROOT framework [27]. A combined $\chi^2$ was built and minimized using the WrappedMultiTF1 class and the MINUIT2 minimization package. The hyperfine $A$ and $B$ coefficients of the triplet state ($^3P_1$) and singlet state ($^1P_1$), respectively, were free parameters of the fit since these exhibit the larger response to the nuclear moments. The resonances were defined by

$$E_F - E_J = \begin{cases} c_1 R_A A(^3P_1) + c_2 B(^3P_1) & \text{for } 1P_1, \\ c_1 A(^1P_1) + c_2 R_B B(^1P_1) & \text{for } 3P_1, \end{cases}$$

where $c_1$ and $c_2$ are constants that depend on the nuclear, electronic, and total angular momentum quantum numbers [28]. The ratios of hyperfine coupling constants were defined with the aid of additional spectra, obtained in the same experimental run as explained below. $R_A = A(^1P_1)/A(^3P_1) = 0.0517(2)$ was determined with high accuracy from simultaneously fitting the $1/2^+$ states, which do not undergo quadrupole splitting in $^{115,117,119}\text{Sn}$ [26]. It was then used as a constraint in the fitting of the spectra of $^{133}\text{Sn}$ and $^{109}\text{Sn}$ also performed simultaneously. The addition of $^{109}\text{Sn}$ in the analysis aided the precision of the extracted $R_B = B(^3P_1)/B(^1P_1) = -0.25(2)$ which was then adopted as a constraint in the analysis of the odd-mass isotopes $^{117-131}\text{Sn}$ [26].

The line profiles for the fitting were described by a symmetric Voigt function [29]. The linewidth and background level were kept free and independent for each spectrum. The
TABLE I. Electromagnetic moments of the $I = 7/2^-$ ground state of $N = 83$ isotones from this work and from literature. Shell-model calculations using microscopic (Calc-M) as well as empirical (Calc-E) effective operators are included in the table.

<table>
<thead>
<tr>
<th>A</th>
<th>Exp.</th>
<th>Ref.</th>
<th>Calc-M</th>
<th>Calc-E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sn</td>
<td>133</td>
<td>$-1.410(1)$</td>
<td>This work</td>
<td>$-1.37$</td>
</tr>
<tr>
<td>Te</td>
<td>135</td>
<td>$-0.690(50)$</td>
<td>[35]</td>
<td>$-1.17$</td>
</tr>
<tr>
<td>Xe</td>
<td>137</td>
<td>$-0.968(8)$</td>
<td>[36]</td>
<td>$-1.13$</td>
</tr>
<tr>
<td>Ba</td>
<td>139</td>
<td>$-0.973(5)$</td>
<td>[37]</td>
<td>$-1.1$</td>
</tr>
<tr>
<td>Ce</td>
<td>141</td>
<td>$-1.090(40)$</td>
<td>[38]</td>
<td>$-1.1$</td>
</tr>
<tr>
<td>Nd</td>
<td>143</td>
<td>$-1.063(5)$</td>
<td>[39]</td>
<td>$-1.11$</td>
</tr>
<tr>
<td>Sm</td>
<td>145</td>
<td>$-1.123(11)$</td>
<td>[41]</td>
<td>$-1.12$</td>
</tr>
<tr>
<td>Gd</td>
<td>147</td>
<td>$-1.020(90)$</td>
<td>[42]</td>
<td>$-1.11$</td>
</tr>
<tr>
<td>Dy</td>
<td>149</td>
<td>$-1.119(9)$</td>
<td>[43]</td>
<td>$-1.1$</td>
</tr>
</tbody>
</table>

Quadrupole moment (b)

<table>
<thead>
<tr>
<th>A</th>
<th>Exp.</th>
<th>Ref.</th>
<th>Calc-M</th>
<th>Calc-E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sn</td>
<td>133</td>
<td>$-0.145(4)(10)^a$</td>
<td>This work</td>
<td>$-0.13$</td>
</tr>
<tr>
<td>Te</td>
<td>135</td>
<td>$+0.290(90)$</td>
<td>[35]</td>
<td>$-0.30$</td>
</tr>
<tr>
<td>Xe</td>
<td>137</td>
<td>$-0.480(20)$</td>
<td>[36]</td>
<td>$-0.36$</td>
</tr>
<tr>
<td>Ba</td>
<td>139</td>
<td>$-0.573(13)$</td>
<td>[37]</td>
<td>$-0.39$</td>
</tr>
<tr>
<td>Ce</td>
<td>141</td>
<td>$-0.610(21)$</td>
<td>[40]</td>
<td>$-0.43$</td>
</tr>
<tr>
<td>Nd</td>
<td>143</td>
<td>$-0.600(70)$</td>
<td>[41]</td>
<td>$-0.46$</td>
</tr>
<tr>
<td>Sm</td>
<td>145</td>
<td>$-0.640(13)$</td>
<td>[42]</td>
<td>$-0.47$</td>
</tr>
<tr>
<td>Gd</td>
<td>147</td>
<td>$-0.620(50)$</td>
<td>[43]</td>
<td>$-0.48$</td>
</tr>
<tr>
<td>Dy</td>
<td>149</td>
<td>$-0.620(50)$</td>
<td>[43]</td>
<td>$-0.51$</td>
</tr>
</tbody>
</table>

$^a$Statistical uncertainty is shown in a first set of parentheses and systematic uncertainty due to the electric-field gradient is shown in a second set of parentheses.

The effect of the hyperfine anomaly in $^{133}$Sn due to the extended distribution of the magnetization over the nuclear volume [31] and the extended nuclear charge distribution [32], was estimated using a developer version of the General Relativistic Atomic Structure Package GRASP2K [33]. The two-parameter Fermi model was used as the charge distribution and the magnetic distribution was approximated with the square of the harmonic-oscillator wave function of the last uncoupled neutron with $\hbar/(m_0) = A^{1/3}$. The resulting hyperfine anomaly,

$$119^A_{\Delta}^{133} = \frac{A^{119}_{133}}{g^{133}_{A} g^{119}} - 1 = 0.075\% \quad (1)$$

is smaller than the uncertainty of the magnetic moment. It was, therefore, neglected during the fit and further treated as a contribution to the experimental error.

By linking two independent measurements of the hyperfine structure in two $J: 0 \rightarrow 1$ transitions we were able to confirm the spin $I = 7/2$ for the ground state. The electromagnetic moments, presented in Table I, were evaluated from the measured hyperfine parameters $A(\hbar P_1) = -965.2(5)$ and $B(\hbar P_1) = -102(3)$ MHz, through the following expressions:

$$\frac{A}{\mu} = \text{const} = 2396.6(7) \text{MHz}/\mu_N, \quad (2)$$

$$\frac{B}{Q} = \text{const} = 706(50) \text{MHz}/b. \quad (3)$$

The constants above are taken from Ref. [26] and represent the average magnetic field per unit angular momentum and the electric field gradient generated by the electron cloud at the position of the nucleus, respectively.

This measurement completes the sequence of ground-state moments of $N = 83$ isotones from tin to dysprosium as shown in Fig. 2. In the extreme single-particle shell model, the $7/2^-$ ground state of all these isotones are expected to be dominated by configurations with one valence neutron in the $1f_{5/2}$ orbital which can be considered to be almost entirely responsible for the magnetic and quadrupole moment of the nucleus. Consistent with the expectation for a doubly-magic-plus-one-neutron nucleus, both the magnetic and the quadrupole moment of $^{133}$Sn are indeed very close to the single-particle estimates for a single neutron in the $1f_{5/2}$ orbital [34], indicated by the straight dotted lines in Fig. 2. On the other hand, for the higher-mass $N = 83$ isotones with an open proton shell, the single-particle shell model is a too crude approximation and the moments deviate from the dotted lines. These observations point to the single-particle character of $^{133}$Sn and, hence, confirm the robustness of the $^{132}$Sn core. In the following paragraphs, these qualitative findings will be supported by realistic shell model calculations. Note that the case of $^{135}$Te does not follow the general trend of the other isotones and will be discussed at the end of this section.

The experimental magnetic dipole and electric quadrupole moments shown in Fig. 2 are also summarized in Table I and compared with theoretical results obtained by performing a realistic shell-model calculation. An effective Hamiltonian was derived from the high-precision CD-Bonn NN potential
renormalized by means of the $V_{\text{low-k}}$ approach [45] with the addition of the Coulomb term for the proton-proton interaction. This Hamiltonian has already been adopted in several previous studies of neutron-rich nuclei beyond $^{132}\text{Sn}$ [46].

The doubly-magic $^{132}\text{Sn}$ was considered as a core and all the neutron orbits of the 82–126 shell (0$h_{9/2}$, 1$f_{7/2}$, 1$f_{5/2}$, 2$p_{3/2}$, 2$p_{1/2}$, 0$i_{13/2}$) and all the proton orbits of the 50–82 shell (0$g_{7/2}$, 1$d_{5/2}$, 1$d_{3/2}$, 2$s_{1/2}$, 0$h_{11/2}$) were included in the model space. The two-body matrix elements of the effective shell-model Hamiltonian for the chosen model space were derived using the $Q$ box-plus-folded diagram method for transition operators. Details on this procedure can be found in Ref. [52]. Shell-model calculations have been carried out using the shell-model code KHELL [53]. Within the adopted model space, with $^{132}\text{Sn}$ as a core, $^{133}\text{Sn}$ is a one-valence system. Therefore, the results for $^{133}\text{Sn}$ from Calc-E coincide with the single-particle estimates shown in this work.

The values predicted by both calculations are very close to each other. In fact, the renormalization of the bare one-body matrix elements of the $M1$ and $E2$ operators derived within the perturbative approach are consistent with the corrections introduced by using empirical effective charges and gyromagnetic factors. In particular, from Table I we see that the empirical and microscopic $M1/E2$ operator produces about the same diagonal single-particle matrix element for the 1$f_{7/2}$ neutron orbit, that is very close to the experimental value of $^{133}\text{Sn}$. From a quantitative point of view the predictions of both calculations are in good agreement with the experimental data also for the higher-mass isotones, except for $^{135}\text{Te}$ which will be discussed at the end of the paper. In fact, discrepancies for magnetic moments are less than 0.1$\mu_N$ in most of the cases, reaching the maximum value of 0.13$\mu_N$ in $^{137}\text{Xe}$, whereas for the electric moments the largest difference between theory and experiment is 0.14b in $^{139}\text{Ba}$.

It is worth noting that, starting from $^{137}\text{Xe}$ with four valence protons, the observed overall trends of the magnetic dipole and electric quadrupole moments are well reproduced by the theory as shown in Fig. 2. The behavior of the two curves essentially reflects the effects of valence protons, which give a positive contribution to the magnetic moments and a negative contribution to the quadrupole moments, determining their respective decrease and increase in magnitude as compared to the values of $^{133}\text{Sn}$. The ground state of a $N=83$ nucleus can be written in terms of a neutron coupled to a spin 0 or 1 neighbor. Since protons coupled to a spin 0 do not contribute to the magnetic or quadrupole moment, these proton contributions arise mainly due to $π_2^+ \otimes ν_f^+1/2$ configurations as shown by our calculations. Actually, we find that, whereas the $π_0^+ \otimes ν_f^+1/2$ component accounts for $\approx 85\%$ of the calculated wave functions of the $N=83$ ground states, a non-negligible percentage - ranging from 5 to 6\% - comes also from the $π_2^+ \otimes ν_f^+1/2$ component. This 5 to 6\% contribution in the wave function, indeed, results in an increase in magnetic moment and the amount depends on the magnetic moments of the yrast 2$^+$ state in the $N=82$ isotones. By using the experimental 2$^+$ magnetic moments, which are known from $^{136}\text{Xe}$ up to $^{144}\text{Sm}$ and range in value between $\pm 1.5$ and $\pm 2.0$ b.
Sensible, unfortunately, for the electric quadrupole moments. In calculation, which confirms the reliability of our predictions for their wave functions. Similar considerations are not possible, unfortunately, for the electric quadrupole moments. In fact, the required quadrupole moments of the 
\[ N = 83 \] ground states are reproduced quite well in a simple two-level mixing calculation, which confirms the reliability of our predictions for their wave functions. Similar considerations are not possible, unfortunately, for the electric quadrupole moments. In fact, the required quadrupole moments of the \[ N = 82 \] isonuclei have been measured only for \[ \text{Ba}^{138} \] and the sign of the \( \langle r^2 \rangle_1 |E2\rangle \langle 0 \rangle_1 \) matrix element, which also comes into play, is unknown.

In concluding, it is worth underlining that, although both microscopic and empirical calculations give a quite reasonable account of the experimental data, they are not able to reproduce the observed staggering for the magnetic moments, which may be related to changes in the structure of the proton wave functions not accounted by the adopted theoretical approach. Furthermore, the location of both theoretical curves for the quadrupole moment, which is slightly above the experimental one by 0.1b, suggests the need for a further small renormalization of the proton charge.

Regarding \( \text{Te}^{135} \), we observe that both experimental magnetic and quadrupole moments show a strong deviation from the experimental systematics and the calculated values. As suggested by the above discussion, the disagreement between theory and experiment implies that our calculations for the 
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We have presented the first measurement of the magnetic dipole and electric quadrupole moment of \( \text{Sn}^{133} \) by high-resolution laser spectroscopy. The obtained electromagnetic moments approach the single-particle estimates for a single neutron in the \( 1f_{7/2} \) orbital suggesting a single-particle behavior on top of a closed \( \text{Sn}^{132} \) core. Both magnetic and quadrupole moments are very well reproduced by theory, which gives also a good description of the moments of the higher-mass \( N = 83 \) isonuclei. We have also shown that the trend along the isotonic chain can be explained in simple terms by decomposing the ground-state wave functions of the \( N = 83 \) isonuclei as an \( 1f_{7/2} \) neutron coupled to the \( 0^+ \) and \( 2^+ \) states of the \( N = 82 \) neighbors. The perturbative approach used to derive the microscopic effective \( M1 \) and \( E2 \) operators, which does not need the introduction of adjustable parameters, induces the correct renormalizations.

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[49] Data extracted using the NNDC On-line Data Service from the ENDF database.


