
Beginning Early: Mathematical Exclusion

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In this paper, the distinction between Bernstein's horizontal and vertical discourse is used to show how two children are restricted in their possibilities to learn mathematics. The social relationships set up within contexts, both of the problems being solved, and between participants, contributed to the horizontal or vertical discourse being employed. In a circular motion, these discourses then reinforced the social relationships that could come into play. It is argued that mathematical exclusion can occur when social relationships, not only within problem contexts but also within interactions, miscue the kind of discourse which is foregrounded. Children can become confused over the sort of discourse that contributes to mathematics learning.

Mathematical Exclusion

It has been known for some time that certain groups of students become alienated from mathematics and thus are excluded from the opportunities that having mathematical qualifications might bring (Stinson, 2004). Often the cause for this exclusion is debated in regard to whether it is the lack of qualifications that exclude people from accessing further education and well-paid jobs or whether it is institutionalised racism or other discriminatory practices that ensures that some groups of students do not have opportunities to gain such qualifications or even if they do gain them, still find themselves excluded (Knijnik, 2002). Approaches to mathematics education may well differ according to which cause is accepted as the root of the problem. However, whatever the cause, groups of students will suffer from social exclusion.

Klasen (2001) suggested that social exclusion was “socially generated barriers that reduce the ability of the excluded individuals to

interact with society” (p. 416). Mathematics education, because of its role as a gatekeeper (Stinson, 2004), can be considered a socially generated barrier that reduces the ability of individuals to interact with society. Stinson (2004) discussed the issue of tracking of minority students as one example of how exclusion occurs while Lerman and Zevenbergen (2004) discussed how classroom practices act to exclude students from working-class backgrounds because of their unfamiliarity with those practices. Social exclusion, in which the way that mathematics is presented to children leads to them being excluded from it, we call mathematical exclusion.

Presently, there is much attention on the perceived lack of mathematics of young children from disadvantaged backgrounds who are beginning school. Reports such as those by Arnold, Fisher, Doctoroff, and Dobbs (2002) have found that the mathematics that children know on entering school will have an impact on their school learning. This has led to calls for more mathematics to be introduced to children in preschools (see Lange, Meaney, Riesbeck, & Wernberg, 2012) to overcome the likelihood of a disadvantaged school experience. These approaches have been critiqued for their underlying deficit assumptions about children (Meaney, 2014). Nevertheless, how mathematical exclusion operates in micro events of learning situations is not well identified, particularly in regard to young children.

In this paper, we compare and contrast two interactions which included the same two children to consider what characteristics of the situations contributed to them being mathematically excluded. To do this, we use Bernstein’s theory of vertical and horizontal discourse.

Theoretical Framework

Over several decades, Bernstein developed a systematic sociology of education which included the development of many different ideas. In his later years, he concentrated on the distinction between horizontal and vertical discourses (Knipping, Straehler-Pohl, & Reid, 2012). Bernstein (1999) defined these discourses by the forms of thought that they develop within individuals in regard to the kinds of knowledge that were valued within a wider society - “the structuring of the social relationships generates the forms of discourse but the discourse in turn is structuring a form of consciousness, its contextual mode of

orientation and realisation, and motivates forms of social solidarity” (p. 160).

Horizontal discourse is equated with common sense forms of knowing, essential for solving specific issues that are highly relevant to the solver, but not easily transferred to other situations (Bennett & Maton, 2010). On the other hand, vertical discourse, often equated with educational knowledge (Knipping et al., 2012), can be generalised to a range of situations. Thus, “the meaning of educational knowledge is given by its relations with other meanings rather than its social context” (Bennett & Maton, 2010, p. 327). Although Bennett and Maton (2010) suggest that social context or relationships are unimportant in regard to vertical discourse, Bernstein suggests that social relationships have a strong influence on the forms of discourse that operate. Therefore, it does not seem valuable to suggest that the social context of the vertical discourse should be ignored as unimportant in the development of meanings.

In this paper, we explore the tensions that arise when the social relationships, either through being maintained or disregarded, affect young children’s opportunities to engage in mathematics learning. Our argument is that one way that mathematical exclusion can occur is when social relationships, not only within problem contexts but also within interactions, miscue the kind of discourse which is foregrounded.

Data Collection and Analysis

In the first half of 2013, video recordings were made on four different occasions in one preschool class in Sweden, when most children were about six years old. The episodes used in this paper come from a free play interaction and a more formal teacher-led interaction. The first episode was analysed in depth in Helenius et al. (2014), whilst some of the second episode was used in Helenius, Johansson, Lange, Meaney and Wernberg (2015, forthcoming). Here, we focus on two children, known as Teo and Klara in Helenius et al. (2014) and as Teo and Lova in Helenius et al. (2015, forthcoming). The two children appear in both episodes and so we analyse their interactions from the perspective of vertical and horizontal discourses. Although these two children differ according to gender, we do not present them as examples of

how particular groups of children become mathematically excluded but rather use their interactions as examples of how the process of mathematical exclusion operates.

The interactions in which they participate seemed at first glance to be inclusive but the earlier analyses showed that this was not the case. In order to consider how mathematical exclusion operates, we examine the relationships between the participants in both episodes and the relationships set up within the problem contexts that the children are interacting around.

Episode 1—Buying a Popsicle

The problem at the heart of the free play episode is the cost of a popsicle, represented by a piece of LEGO. After some joint LEGO construction, Teo initiates this interaction by situating Tom as the seller and himself as the buyer (*Får man köpa nåt här?*). Tom first refused Teo's request to buy something but then demanded all of his money (play kroner notes). Figure 1 shows Tom indicating that Teo would need to hand over all of his money (*Den kostar alla dom*).

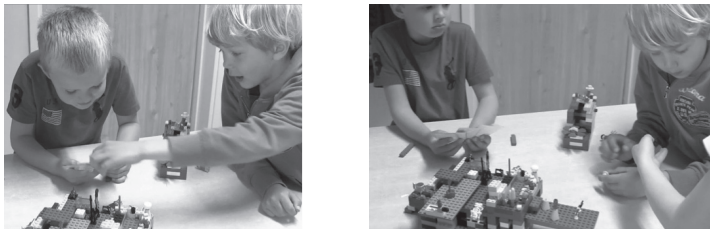


Figure 1: Free Play Exchange. Left to Right: Teo, Tom and Klara

Klara had been involved in the original LEGO construction, but left before Teo asked about buying the popsicle. She returned soon after Teo protested the need to hand over all of his money. At this point she tries to interrupt the discussion about the popsicle by indicating that her brown piece of LEGO was a piece of chocolate. Neither her first or second attempt to gain the boys' interest in the chocolate was successful (see Figure 1).

Later, Klara attempted to trade a car for some other constructions that she helped to build with the others (*Byter ni den här bilen mot alla de här och mitt bygge?*). There was some discussion around this

suggestion, but Tom rejected the possibility of a transaction which was made by both Klara and another child, Patrik. Tom's rejection was based again on them not having "enough" money (*Men var är pengarna då?*).

At the end of the episode, Teo makes another request to buy the popsicle but Tom again vetoes it, first outright and then by suggesting that Teo will die from eating the popsicle (*Varför måste du ha piggelinen, då dör du ju*). When Teo repeated that he wanted to buy the popsicle, Tom then said it would cost 40 000 kroner (*Det där är fyrtiotusen, I så fall får du betala, vänta de här också.*). After a further exchange about it costing too much, Teo stated "You are so mean, why does it have to cost that much?" (*Ni är så elaka varför måste det kosta så mycket?*). At this point, Tom capitulates and agrees to Teo buying it for only one of his play money notes.

Discussion

In our original analysis (Helenius et al., 2014), we considered that Tom and Teo had been involved in mathematical play as it included participation, creativity, and rule negotiation. On the other hand, it appeared that although Klara (and Patrik) tried to participate in similar ways, their suggestions were rejected, suggesting that they did not have the same opportunities to be involved in mathematical learning. By considering this episode using understandings about the differences between vertical and horizontal discourse that were in circulation, it is possible to clarify how Klara's mathematical exclusion occurred whereas Teo's did not.

Although the problem appeared to be an everyday one about buying something, for Tom and Teo the discussion focused on general understandings about ideas to do with enough and too much and how their meaning changed depending upon who used the terms. As both boys were only six years old, it is unlikely that either of them knew precisely how much 40 000 kroner was. Although Tom initially indicated that this is what he wanted Teo to pay, both boys recognised it as being more than what Teo could possibly have in play bank notes. Such considerations were generalizable to other situations and so can be seen as belonging to the vertical discourse.

It is not until Teo calls Tom "mean" at the end of the episode that

social relationships gain prominence in the discussion. They had been evident earlier, but implicitly, when Klara and Patrik tried to enter the discussion and Tom, and to a lesser extent Teo, did not respond. Tom was able to make such decisions because he was accepted as controlling the situation by the others. This may have been because, in the imaginary play situation, he had the role of seller, who had the power to determine the price of the things to be sold or exchanged. The context of the problem, set in a shopping scenario, seemed to set up a particular social relationship with someone in power. This kind of relationship contributed to the vertical discourse being used as it enabled one person to determine what knowledge was relevant, in a similar manner to teacher-class relationship. As is the case in this example, such a relationship within a play situation can contribute to generalizable concepts, rather than the specific problem, being discussed. Although Teo could overcome the power structure by appealing to his real-world friendship with Tom, in doing so he contributed to the ending of the play situation. In this case, the role of relationships within the problem context contributed to the vertical discourse being foregrounded with the everyday relationship between participants, contributing to the horizontal discourse becoming backgrounded.

Klara, in trying to navigate the vertical discourse within the play situation, choose not to bring in her real world friendship with Tom, but instead tried to participate in the same way as Teo. When her efforts to participate were rebuffed by Tom, she was unable to affect what knowledge was discussed and in what ways and so can be said to have been mathematically excluded. On the other hand, Teo was not mathematically excluded, even though he accepted Tom's right to make the decisions in the same way that Klara had. Whereas Tom did not accept any of Klara's attempts at contributing to the discussion, Tom accepted that Teo had a right to contribute and also to have a different opinion of what was enough and what was too much. Thus, although Teo's contributions were different to those of Tom's, he had a more active role in the vertical discourse than Klara. The negotiation of the meanings would have contributed to Teo's understandings of enough and too much and also to how to engage in negotiations within the vertical discourse, thus supporting his possibilities for learning mathematics.

Episode 2—Splitting 10 into 3 Groups

The second episode was a fairly typical mathematics lesson initiated by the teacher with a group of eight children. It began with a warm up activity to do with pairs of numbers that added to ten. Then a problem was posed about ten children in a small preschool class who were to be sent to three activities—woodwork, baking, and painting. The children were given time to determine individually how many children should be in each group, with the teacher stating that there are no wrong or right answers. She also said that it might be possible to distribute the class evenly or it could be that one group had more children or another group had no children (*Tre grupper och så de tio barn. Nu vill jag att ni tänker ut hur de delade sig. Hur delar man tio barn i tre grupper? Det finns inget rätt, inget fel. Ni bestämmer själva. Om det kanske bara är ett barn i en grupp, och det kanske är inget barn i någon grupp och det kanske delar något så att det blir lika i varje*). The children were told that they could record the distributions on paper in any way they liked. As the children worked, the teacher moved around the class asking them about their distribution. At the end of the session, the teacher had the children fold their papers and sit down in a horse-shoe so that they could explain the way they had distributed the class.

In the warm-up activity, Klara was the first to stand up and try to find who had the pair number for her 2. Nevertheless, it was her partner who told the teacher about their pair. On the other hand, Teo was one of the last to identify the child who had the other half of his pair, but it was he who took hold of both cards. Both children answered in chorus when the teacher asked for their answer (seven and three, *sju och tre*). The teacher wrote the solutions on the board using standard number symbols.

As the teacher described the problem of sharing ten children in the three groups, Klara could be seen using her fingers to work out

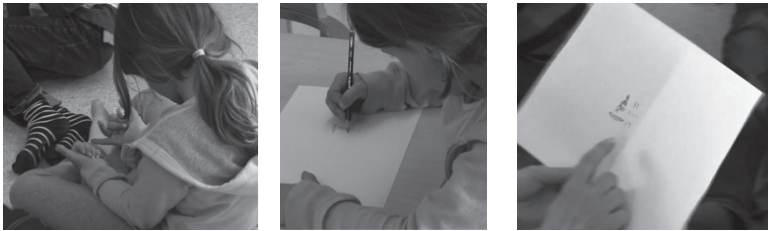


Figure 2: Klara's Problem Solving - 2 Painting, 5 Woodwork and 3 Baking

possible solutions (see Figure 2). Before she collected her paper, she shared her solution with another child who also used his fingers to find a solution. When working on the problem, Klara used first her fingers and then tally marks (see Figure 2). After she put one round of tally marks next to each group symbol, she counted them before putting down the next round.

Teo spread his hands and first counted the fingers on one hand before separating his fingers on both hands into groups so that he seemed to have found a 4, 3, 3 solution (see Figure 3). Still, his initial drawing showed that he had only 9 tally marks and he seemed stuck. Nevertheless, by the time the teacher talked with him, he had given the baking group, represented by a rolling pin, a fourth member.



Figure 3: Teo's Solution Process

Although the teacher usually presented each child's solution. Klara was asked to present a proposal (see Figure 3).

Teacher:	Klara, what proposal do you have? Oh, okay what is there? Can you tell me?	Klara, vad har du för förslag? Oj okej vad står där? Kan du berätta för mig?
Klara:	Three, five and one. Three, five and two.	Tre, fem och ett. Tre, fem och två.
Teacher:	Let me see, three, five and two, okay. It is, let's see here.	Jag får se, tre fem och två okej. Det är, ska vi se här.
Klara:	Three in one group, five in another.	Tre i en grupp, fem i en.
Teacher:	But where are the three, is it in the baking group?	Men var är de tre, är det i bak gruppen?
Klara:	Yes.	Ja.
Teacher:	And then it's five in the woodworking group and two in the painting group.	Och så är det fem i snickargruppen och två i målargruppen.
Klara:	Yes.	Ja.
Teacher:	Why did you decide this?	Varför bestämde du dig för det?

Klara:	Because the woodworking group, it's much more who like to do woodwork, less who like to paint and in the middle those who like baking.	För att snickargruppen, det är mycket mer som tycker om att snickra, mindre som tycker om att måla och mittemellan som tycker om att baka.
Teacher:	So they could choose for themselves in that class, okay.	Så dom fick välja själva i den klassen, okej.

In presenting what she had done, Klara focused on the numbers. However, the teacher shifted her attention, by asking which group had three children in it. Klara happily responded by discussing the popularity of activities and providing details about why she had split the ten children as she had. After Klara presented her proposal, Teo was asked to present a solution. His initial response was to deny that he had one.

Teacher:	Teo, do you have a solution?	Teo, har du någon lösning?
Teo:	I just do not know what the solution is.	Jag bara inte vet vad lösningen är.
Teacher:	Can you open it so I can see how you did it?	Kan du öppna den så jag ser hur gjorde du?
Teo:	I shared it so.	Jag delade den så.
Teacher:	Three, three and four, okay. Why was it four? You told me. Why are there four for baking? It was a little funny.	Tre tre och fyra, okej. Varför blev det fyra? Det berättade du för mig. Varför är det fyra på baket. Det var lite roligt.
Teo:	Nah, I changed.	Nä, jag ändrade.
Teacher:	You changed it. I thought it was a funny thing, you said about there being four in baking.	Du ändrade det. För jag tyckte det var en rolig sak du sa att de var fyra på baket.
Children:	What did he say?	Vad sa han?
Teo:	I changed.	Jag ändrade.
Teacher:	Then it would become many more buns.	Att det skulle bli mycket mycket bullar.
Teo:	I changed.	Jag ändrade.
Teacher:	But you decided three, three and four?	Men du bestämde tre tre och fyra?
Teo:	Yeah, I just wanted that.	Ja, jag bara ville det.
Teacher:	You just did that, okay. Thanks for that, okay.	Du bara gjorde det, okej. Tack för det, okej.

In this exchange, the focus is on finding a solution, whereas in Klara's case, it was about presenting her proposal. Teo's suggestion that he did not have a solution seemed to mystify the teacher who had previously talked with him. It may be that the word solution (*lösning*) suggested to Teo that she was looking for one specific solution. Having heard

several different suggestions, some of which the teacher had already stated were brilliantly solved (*strålande löst*), Teo seemed uncertain that his solution which was different could also be correct. Regardless of his reasoning, Teo was not interested in the teacher's questions about which group had which number of participants. Instead, he kept repeating that he had changed, which seemed to mean that he had changed the group with four members because he remained adamant about the 3, 3, 4 distribution.

Discussion

Although it could be expected that the social relationship between the teacher and the children would be similar, the type of problem that children were engaged in solving and the roles that they took up in presenting their solutions altered their social relationships with the teacher. The differences in social relationships affected and were affected by the discourses that were utilised in the interactions.

The problem itself allows for discussions about the different combinations of three numbers which add up to ten. As the teacher had begun the lesson with an activity based on two numbers that added to ten, it could have been expected that this would have been the teacher's focus. However, the setting of the problem within a preschool class and the distribution into three activities allowed the teacher and the children to become involved in a discussion about children's preferences for particular activities. In asking about children's preferences, the teacher moved from being an expert determining the correctness of the mathematics under discussion to an interested enquirer into what activities the children saw as the most interesting. By changing her role in the social relationship, she also changed those of the children. These changes led to the discussion being within the horizontal discourse where the focus was not on generalizable ideas but on the specific situation. In the original analysis of Klara's role (Helenius et al., 2015 forthcoming), we could see that Klara's opportunities to discuss mathematics were restricted by the discussion of preferences. However, the traditional use of a vertical and horizontal discourse analysis to focus on what was being discussed and in what ways did not provide information about how the social relationships contributed to this restriction.

Although Klara was happy to discuss her distribution of children into the three groups, Teo resisted this shift. He remained committed to his 4, 3, 3 distribution but refused to enter into a discussion about his preferences for the groups. It is unclear whether he was confused over the kind of discourse which he felt should have been in operation. If the teacher had asked him about his reasoning to do with the numbers adding to ten, perhaps he would have been more willing to engage as he had done in the warm-up activity around the pairs of numbers adding to ten. His resistance in responding to the teacher's questions about the distribution into groups meant that he adopted a different role in the exchange and this affected the social relationship with the teacher. Having taken on the role of resister, he was not drawn into the horizontal discourse of the teacher.

In this example, the teacher's role moved from being the expert in the warm-up activity to a gentler enquirer about the children's preferences in the presentation of solutions or proposals. Her shifting role affected the social relationships with the children and contributed to a move from the vertical discourse to a horizontal discourse. Although Klara moved with her, Teo did not. Nevertheless, as neither child had possibilities to engage in vertical discourse around how different combinations of numbers could be formed to make ten, both could be said to be mathematically excluded. It could be said that Teo in taking on the role of resister had the best opportunities of changing the discourse to a vertical one by refusing to consider that the teacher should have a different social relationship with him. In not accepting the teacher as a gentle enquirer who had the right to find out about his group distribution preferences, it might have been possible for him to force the teacher back into her role as expert on mathematics learning. Although in this case, this did not happen, in the earlier analysis (Helenius et al., 2015 forthcoming) when another child gave a combination of 5, 5 and 5 earlier in the interaction, the teacher had moved back into her role as expert. However, that child did not have the requisite mathematical knowledge to discuss the mathematics with her.

Conclusion

Watching her work on the solution of splitting ten into three groups, it appears that Klara had skills and interest in mathematics which could have been developed. Yet, she was excluded from mathematics in both interactions because she tried to act in accordance with the social relationships that those in control had put in place. As a result in the free play episode, she could be excluded from the vertical discourse by Tom, who being the seller and thus in charge, was not required to accept her suggestions. In the exchange, when the teacher moved to a horizontal discourse, Klara moved with her, and was then confined there.

On the other hand, Tom accepted Teo's right to be involved in the popsicle discussion, enabling Teo to take part in the vertical discourse and to learn about abstract notions such as enough and too much. In the episode with the teacher, Teo resisted being brought into the horizontal discourse around his preferences for the group distribution. As a result, he may have become more aware that mathematics was about being in the vertical discourse. This is in contrast to Klara who, if she does not gain experiences of being in the vertical discourse, is likely to associate mathematics learning only with the horizontal discourse.

Mathematical exclusion has been noted in many different situations, even if it has not been labelled as mathematical exclusion. However, insights about the impact of social relationships on the use of vertical and horizontal discourses provide a richer understanding of how mathematical exclusion comes into existence.

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