Deaf children's concept formation in mathematics

Deaf children's concept formation in mathematics is the name of the study I am presenting. It is also the title of my Ph.D. theses in Education accepted at Malmö University in Sweden in November 2003. My study (Foisack, 2003) is based on the question why deaf children have difficulties in learning mathematics. International research (Frostad, 1998; Magne, 1991; Moores, 2000) shows that deaf students achieve much lower results on tests in mathematics than hearing students do. On the other hand there is no research available today showing that the cognitive potential of deaf students differs from that of hearing students (Martin, 1991). If it is a fact that deaf students do not use their cognitive potential to a full extent in learning, it is of great importance to investigate why, and to find alternative ways of teaching. Other ways of assessing than by ordinary tests might show other results than those referred to.

The children in this study were students in a School for the Deaf and Hard of Hearing in Sweden. Going to school in a School for deaf and hard of hearing students in Sweden is in some ways different from going to a regular school. There are six such schools run by the government located throughout the country. The sign language environment is crucial as well as the bilingual approach with the two languages Swedish Sign Language and Swedish as two separate subjects (Skolverket, 2000/2002). But the basic objectives are the same as in all schools in Sweden. In mathematics the deaf students are taught on the basis of the same curriculum as all other students in Sweden (Lpo94, 2006) syllabi and criteria for grades (Skolverket, 2000) are the same.

In the curriculum for compulsory schools in Sweden the aim for learning mathematics is "to master basic mathematical thinking and be able to use it in everyday life". In the syllabus for the schools for deaf and hard of hearing in Sweden (Skolverket, 2000/2002) there is a general text on bilingualism from which the following quotation is taken:

> Learning occurs through both languages. Bilingualism is therefore important in all school subjects, not only in the two subjects Swedish and Swedish Sign Language. Every subject has its own subject-specific concepts and a terminology that the students do not meet in other subjects. As a consequence it is every teacher's obligation to make sure that the student masters these concepts in both languages (pp 95-96).

Concerning mathematics there are a great amount of well defined subject-specific concepts and related terminology in spoken/written languages that students in school need to master. The quotation above shows that there is a need for an established terminology in Swedish Sign Language for mathematics as well.

The aim of the study referred to is to illuminate deaf children's concept formation in mathematics by describing how some deaf children express themselves and act on their way towards understanding the concept of multiplication with whole numbers. In my theses I have a second study concerning the concept of length. But in this presentation I will talk about the multiplication study only. Questions of significance to the study:

How do deaf students, express themselves and act when confronted with a mathematical problem of this nature?

- those who have already understood the concept and
- those who are on their way towards understanding the concept

Does the way deaf students express themselves in sign language influence their concept formation?

From a mathematics education perspective two questions were then raised: What steps are needed for understanding the concept? And Are the steps the same for deaf students as for hearing students? They will however not be focused on in this presentation.

Theoretical framework

Two separate perspectives were used in the study, cognitive education and mathematics education. The cognitive education perspective was based on theories developed by Feuerstein (Feuerstein et al., 1991). Feuerstein has a constructivistic view of learning. Structural Cognitive Modifiability (SCM) is a theory closely related to theories by Piaget, but the uniqueness in Feuerstein's theory is the connection with the theory called Mediated Learning Experience (MLE). Feuerstein (Feuerstein et al., 1988) has been influenced by Vygotsky (1978) as to how learning is developed in a social context. The most important characteristics of Mediated Learning Experience are mediation of intention and reciprocity, there must be an agreement of the intention with the work between mediator and student, mediation of transcendency, development of thinking beyond here and now and mediation of

meaning. MLE is used in the method for assessing children's cognitive potential developed by Feuerstein, called Dynamic Assessment (Feuerstein et al., 1979) to reveal the learning potential of an individual. In dynamic assessment interaction is crucial.

From the mathematics education perspective insightful learning, problem solving and communication are considered to be crucial in developing mathematical knowledge (Verschaffel & De Corte, 1996; Hiebert & Lefevre, 1987; Ahlberg, 1992).

How deaf students' way of expressing themselves in sign language influenced their concept formation was analysed out of the four characteristics of sign language: iconicity, simultaneity, movement and spatiality (Bergman, 1979).

Empirical study

To get a group of students representing a variety as broad as possible the following group was chosen. The children were students in grade 4 in a Special School for the Deaf. There were seven 11-year-old students in the study. They were all the students in one of two parallel groups in the fourth grade. All seven had been taught in the same school since the first grade. Video recordings were made of student-teacher interactions in problem solving situations. The students met the teacher, who was I, one by one several times.

Problems to be solved were generated out of a given situation. The solution of a problem presented was discussed in four different ways: in sign language with no material available, with paper and pencil, with learning materials i.e. Centimo and Cuisenaire-rods, and with real objects.

The first lesson with each student and in each of the two parts of the study was regarded as an assessment lesson. After the lesson a brief analysis was made from the video recordings and an assessment was made of the student's ability to solve the problem and the level of understanding the concept. Assessment was also made of what else the student might need to know about how to learn and how to develop understanding of the concept. With students

who solved the problem on their own, attention was concentrated on how the student described his or her way of thinking in order to find new ways of helping other students to solve the problem. With students who needed help to solve the problem, analyses were made to find what was needed for the student to solve the problem and to develop better understanding of the concept. One or two lessons were consciously planned out of the needs of the student. How much mediation was needed and of what kind was registered.

A test was then given to the students in the study containing the same kind of problems as in the lessons. The tests were presented in written Swedish and were given to the students individually. For each student an assessment was made of what mathematical steps he or she mastered during the first lesson and in the test situation. The results from the two occasions were compared. The language of communication was Swedish sign language all through the study. Only in the final test, written Swedish was used.

In the multiplication study the problem was to find out how many apples are needed if three/four children are to have three apples each, if seven children are to have three apples each or if one-hundred-and-three children are to have three apples each. The reason why I chose the numbers of children to be 3 or 4, 7 and 103, was that they were numbers with a certain meaning to the students. They were the number of children in groups they belonged to themselves. There were 7 students in their class, there were 3 boys and 4 girls in the class and there were 103 students in the school altogether at the time, a matter that had recently been focused on in a speech by the principle of the school.

Findings

Here are some of my findings in analysing the material out of the question: How the students expressed themselves and acted, which was a central question of my study. I will begin with representations with paper and pencil, with manipulatives/learning material and with real things. I will then talk about how the students expressed themselves in sign language and in written language.

When the students were given paper and pencil, the only instruction they got was to explain the problem. Some of them started out drawing the children, some the apples and other by

writing numbers and symbols. They described the operation in different ways, like by drawing strait or curved lines, circles or arrows, by using the symbol for addition or multiplication. In the assessment lesson only one student used the symbol of multiplication. He was also the only one to solve the target task in the first lesson. Two of the students did not make any drawings of children or apples, but only numbers, lines and symbols. By closer analysis it was revealed that they had different reasons for doing so. One of them could explain her writing distinctly. She had no need of pictures or symbols for her own thinking or for explanation. The other one first wrote: 3+3+3+3=12. But when the teacher asked if she could right it in another way, she wrote 2+2+2+2+2+2=12, without connection to the problem. She had good help of using drawings of apples and children, which was mediated to her in the second lesson. The student who reached the target problem in the first lesson showed a big variety in explaining the problems by paper and pencil. He described the problem by drawings, by writing small stories or by writing with mathematical symbols in multiplication or addition. He was the student who showed the most concrete and the most abstract ways of representing the problems on paper.

When the students were asked to express themselves and act with manipulatives/learning material it was found to be helpful in understanding numbers since it is a content free way of representing the number and not an object as such. The material used in this study was the so called Centimo consisting of cubes with one centimetre long sides, rods and plates containing 10 and 100 cubes. Buttons in two colours were also used after suggestion of a student. It was used in mediation to students who mixed up representing children and apples.
When the students picked up cubes, some of them pick up one at a time, some three at a time grouping them. Putting the cubes to show the problem was done in different ways: one and one randomly, in groups, in columns, in rows or in rectangles. In describing the problem in sign language the situation was emphasised by pointing to the cubes, by touching with the index finger or with the hand formed to a number sign. Some students moved the cubes to show what happened, others made the cubes and buttons act, showing how the children ate the apples, while others put the cubes out and then explained the problem in sign language.

Advantages in using manipulatives compared to using paper and pencil or real things were i.e. that it was easy to visualize numbers. They were also easy handle and move around, numbers could be focused and not the objects and multiplication could be described as repeated addition as well as in two dimensions. In handling big numbers the rods of ten and the plates

of one hundred were of obvious advantage. That was of great help in explaining the commutative law explaining that 103 times 3 is equal to 3 times 103 in the target problem.

Now about expressing yourself and acting with real things. According to the design of the study it was important that the students could experience the problem and that it ended up in action in the group, each one of the students receiving three apples each. In the teaching situation the students acted by putting the apples in groups or in rows. They explained the problem by pointing to the apples, touching them with the index finger or with the hand formed to a number sign.

Solving the problems in many different settings and in using real things was helpful for the student to understand the numerousity and for the teacher to understand the student's comprehension of numerousity. Some students started out from scratch in counting when new material was brought out, while others used what they memorised from solving the problem earlier. To use known facts and to derive facts out of known facts are crucial competences in mathematics. Another advantage of using several ways of representing a problem is the opportunity to reason on a metacognitive level.

Now to how the students expressed themselves in sign language. I will show you some observations on strategies used by the students in expressing themselves:

1. For one-hand signs the students used either hand, not depending on what hand was their dominating hand, but out of the need or out of the situation to be explained.
2. To express multiplication, some students used both hands, one hand representing the number of children and the other the number of apples. The hand representing apples was showing the number of apples and was moved towards the other hand showing the number of children by turns. Students who consciously used both hands representing one factor each were successful in finding the solution, but students who used their hands arbitrarily easily failed.
3. Using localisation of the children and the apples in the problem was used by pointing on the table or in the air vertically or horizontally. It was also used by pointing in the direction towards a place that was localised earlier in the conversation. Students also used movements of the head or the eyes instead of pointing.
4. To express repeated addition distinct rhythmic movements were used. Head movements were also used. One of the students who used this, changed the time

intervals, and failed in getting the right answer over and over again. She was helped out through mediation of rhythmic counting.

5. The strategy of “double counting” was observed to be used by students in the study. It means adding by counting each unit not using any objects. You keep track of the number of units in the second addend to know when to stop, i.e. for 2+7=9 you count 1,2 3-1, 4-2, 5-3, 6-4, 7-5, 8-6, 9-7. It is a strategy often used by signers (Frostad, 1998), since it is easy to keep one number in each hand. It is however a strategy that needs attention, since if stigmatized, it may lead to difficulties in counting. An aim in using counting strategies is automatizing strategies in order to handle them as known facts. In this study it was obvious that understanding the commutative law was crucial in solving the target problem, $103 \cdot 3 = 3 \cdot 103$.

In the study you can find examples of how the students use the four characteristics of sign language: Iconicity, simultaneity, movement and spatiality. Iconicity is illustrated by the use of fingers for numbers. To be used as numbers in sign language it is crucial to use the right fingers. Showing the thumbs i.e. means one in Danish sign language but six in Swedish sign language. Simultaneity was illustrated by using one number in each hand. Movement was used in showing the action in the problem, representing the verb give. Spatiality was illustrated by localising the children in the space in front of you or on the table. In the study it is obvious that students using the characteristics of sign language consciously also succeeded better in solving the problems.

Not until in the final test, the students were exposed to written language in this study. It was obvious that the reading ability had impact on how the students succeeded to solve the problems on their own. In this case the students could ask for interpretation into sign language. But in general there is a need for the students to read and write. The teacher of the students was asked about their competence in Swedish. The students in this study represented a large variety in competence in Swedish. Two of them were good in Swedish and in mathematics and three of them were weak in both. One student was good in Swedish, was weak in solving the problems the way they were presented. One student was weak in Swedish was very much capable of remember and describing solutions of problems. This means that for the students in this study the assumption can not be drawn that there is a connection between their competence in Swedish and in mathematics.

One student fulfilled the target goal in the assessment lesson. He also showed full competence in solving the final test. After two teaching sessions for each of the other six students, four of them were capable of solving the problems in the final test. Two of the students were still on their way towards understanding the concept of multiplication.

Does the way deaf students express themselves in sign language influence their concept formation? That was the second question of my study, I wanted to focus on today.

Students using the characteristics of sign language consciously succeeded better in solving the problems – as I have illustrated before.

It was found that the structure of sign language could be of help but it could also be an obstacle in mathematics. Using the structure of sign language in mathematics was helpful when describing the problem.

On the other hand arguments have been raised whether the visual aspects of sign language may hamper concept formation in mathematics. In this study the possibilities of using the students' expressions in sign language to reveal their level of knowledge and to promote their concept development have been focused on. In constructing one's own knowledge it is essential that all possibilities are taken advantage of.

General conclusions

When analysing the data from the Cognitive Education perspective I found a large variety in the ability of the students to solve the problems. This variety was identified out of factors defined by Feuerstein (1988) to depend on i.e. self-confidence, looking for meaning and search of challenge, intention to finish the work, use of known facts. They are all factors of importance to communicative competence and to problem solving.

When analysing the data from the mathematics education perspective I found no difference in general concerning steps towards comprehension for the students in the study compared to those of hearing students as far as comparing spoken Swedish for hearing students and signed Swedish for deaf students. In the area of number sense, several students in the study did not master three-digit numbers, an ability usually automatized at an earlier age by hearing students. In accordance with earlier studies deaf students need more time to learn mathematics

than hearing students. As a consequence they may learn certain concepts at a later age and the pathways towards comprehension may vary compared to those of hearing students.

The importance of teaching mathematics by problem solving and by communication to deaf students as well as to hearing students has been emphasized in this study. For deaf students, a more developed terminology in sign language would make learning subject-specific concepts of mathematics less dependent on competence in spoken/written language. The students could then be offered better conditions to reach a more abstract level at an earlier age. The bilingual situation for deaf students is a reason for developing methods for teaching mathematics to deaf students, approaches differing from or supplementing methods used today.

References

Ahlberg, A. (1992). *Att möta matematiska problem - en belysning av barns lärande*. (The meeting with mathematical problems: an illumination of children's learning.) Göteborg: Acta Universitatis Gothoburgensis.

Bergman, B. (1979). Dövas teckenspråk – en inledning. (Sign language of the deaf – an introduction.) *Forskning om teckenspråk III.* Stockholms Universitet: Institutionen för lingvistik.

Feuerstein, R., Klein, P.S. & Tannenbaum, A. (Eds.). (1991). *Mediated Learning Experience.* London: Freund.

Feuerstein, R., Rand, Y. & Rynders, J.E. (1988). *Don't accept me as I am*. New York: Plenum.

Feuerstein, R., Rand, Y. & Hoffman, M.B. (1979). *The Dynamic Assessment of Retarded Performers: The Learning Potential Assessment Device*. Baltimore: University Park Press.

Foisack, E. (2003). *Döva barns begreppsbildning i matematik.* (Deaf children's concept formation in mathematics.) Malmö Studies in Educational Sciences, no. 7, 2003. Malmö: Lärarutbildningen, Malmö Högskola.

Frostad, P. (1996). Mathematical achievement of hearing impaired students in Norway. *European Journal of Special Needs Education, 11*(1), 66-80.

Hiebert, J. & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics.* Hillsdale, NJ: Lawrence Erlbaum.

Magne, O. (1991). *Dysmathematics. Facts and theories concerning mathematics learning for handicapped pupils.* (Educational and Psychological Interactions, 106). Malmö: School of Education.

Martin, D.S. (1991). Introduction. In Martin, D.S. (Ed.). *Advances in Cognition, Education and Deafness*. Washington D.C.: Gallaudet University Press.

Moores, D.F. (2000). *Educating the Deaf. Psychology, principles and practices.* Boston: Houghton Mifflin.

Skolverket. (2000). *Grundskolans kursplaner och betygskriterier*. (Syllabuses and criteria for grades for the compulsory school.) Stockholm: Fritzes.

Skolverket. (2006) *Läroplan för det obligatoriska skolväsendet, förskoleklassen och fritidshemmet, Lpo94*. (Curriculum for compulsory school system, the preschool class and the leisure-time centre) Stockholm: Fritzes.

Skolverket. (2000/2002). *Specialskolan. Kursplaner och betygskriterier.* (The special school. Syllabuses and criteria for grades.) Stockholm: Fritzes.

Verschaffel, L. & De Corte, E. (1996). Number and arithmetic. In A.J. Bishop et al. (Eds.), *International handbook of mathematics education.* Dordrecht, NL: Kluwer.

Vygotsky, L.S. (1978). *Mind in society.* Cambridge, MA: Harvard University Press.