CECILIA SEGERBY

SUPPORTING MATHEMATICAL REASONING THROUGH READING AND WRITING IN MATHEMATICS

Making the implicit explicit

MALMÖ UNIVERSITY

**SUPPORTING MATHEMATICAL REASONING THROUGH
READING AND WRITING IN MATHEMATICS**

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To gain knowledge in a subject area is more like getting to know a landscape than climbing a ladder (Hirst, 1974). The more you get to know a landscape, the more nuances and details you are able to distinguish. As you get to know the landscape it increases your possibilities to investigate it (Carlgren & Marton, 2000, p. 195, own translation).

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1. INTRODUCTION

“This is not mathematics.
In mathematics you write symbols not a lot of words”

This statement came up during a mathematics lesson when one of my students was asked to reason in mathematics. It reflects an opinion that I have heard several times from students during my fifteen years as a mathematics teacher and again when I visited a classroom for a year collecting data for this thesis. A common understanding is that mathematics is the same thing as performing tasks quickly, meaning the answers should be short and usually involve a number. This idea of what mathematics is (and is not) is often confirmed in the tasks given in the mathematics books in Sweden, which, in short, can be described as solely ‘places to perform tasks’ and nothing to do with reading, writing or reasoning about mathematics.

1.1 Reasoning competence in mathematics

Mathematics reasoning in school mathematics is the focus of this thesis. It is addressed as an important competence in learning school mathematics in research (see f. e. Kilpatrick, 2001; Lithner, 2008; Niss, 2003) and in several countries’ curricula (National council of teachers of mathematics, 2003; Skolverket, 2011). Reasoning in school mathematics is considered an essential practice, both to facilitate students’ meaning making in mathematics and to make the learning visible for students as well as for teachers (Baxter, Woodward & Olson, 2005). Earlier studies (Bieda, Drwencke & Picard; 2014; Sidenvall, 2015; Stylianides, 2009) have pointed out

that the opportunities for students' reasoning can be very limited when relying on the mathematics textbook. In many countries, the mathematics textbooks are extensively used in mathematics education (Haggarty & Pepin, 2002; Remillard, 2005; Thomson & Fleming, 2004), including Sweden (Boesen, Helenius, Bergqvist, Bergqvist, Lithner, Palm & Palmberg, 2014; Skolinspektionen, 2009). In short, one can say that it steers the teaching and learning, and as a result, the students' opportunities to develop their reasoning competence in mathematics may be limited and thus complementary activities may be needed. However, suitable activities to support students' reasoning competence when reading texts in school mathematics are seldom implemented, and mathematic teachers express that they lack knowledge about how to teach with these (Liberg, 2008; Ratekin, Simson, Alvermann & Dishner, 1985).

In addition, certain demands are put on the students' reasoning competence because the language of mathematics is a very specific language, which is often expressed in multimodal texts with various semiotic resources, such as illustrations, words and symbols. Thus, special demands are put on the students' reading skills (Barton & Heidema 2002; Shepherd, Selden & Selden, 2012). Also, the semiotic resources often need to be comprehended not only separately but also together (Lemke, 2003; Schleppegrell, 2007). In general, the language of mathematics differs from the other disciplinary languages that the students will have encountered, and therefore, if it is to serve as a resource for mathematical meaning making, it must be explicitly emphasized and taught in the classroom.

Another possible problem raised in earlier studies is the lack of an explicit definition of what reasoning in school mathematics means (Lithner, 2006). Some researchers (see Kilpatrick, 2001; Lithner, 2008; Niss, 2003) have tried to narrow the definition of reasoning to reflect the skill of being able to explain concepts and methods relating to mathematical solutions as well as the skill to predict, explain, ask questions and generalize the mathematical content. This last part of the definition builds on similar principles found in the Reciprocal Teaching model (RT) (Palinscar & Brown, 1984). The RT model emphasizes the four comprehension strategies of predict, clarify, question and summarize. In this study, the RT strategies are related to the question of how to improve the students' abilities to reasoning

in mathematics; thus, the model has been modified to fit and develop the specific linguistic genre of school mathematics.

The RT model has previously been used in mathematics education but mainly in connection with specific tasks that usually involve problem solving (Huber, 2010; Quirk, 2010). The approach in this thesis differs and instead involves RT for exploring the type of support needed for developing students' reasoning competences. In this study, mathematical reasoning refers to students' ways of explaining and describing mathematical phenomena such as the meaning of ideas, concepts, operations and processes as they evolve in specific mathematical tasks.

To further examine suitable activities to support students' reasoning in mathematics, an Educational Design Research (EDR) study was conducted. EDR involves the development of domain specific theory and instructional materials directed towards the improvement of teaching and learning. Central to EDR is the iterative process of designing (and re-designing), analysing and evaluating interventions (Cobb, diSessa, Lehrer & Schaube, 2003; Van den Akker, Gravemeijer, McKenney & Nieveen, 2006). In this process, collaboration between researcher and practitioners are deemed necessary, with the researchers being responsible for the theoretical parts and the teachers being responsible for the practical parts (Gravemeijer & Cobb, 2006).

In the present EDR study, the RT model was used to introduce a new way of fostering comprehension and comprehension monitoring activities into a Grade 4 mathematics class during one semester. The activities were developed from the coordination of two theoretical frameworks – the RT model and Systemic Functional Linguistics (SFL) (Halliday, 1973). The point is to look at these two as entangled and determine how they limit and make possible students' reasoning in mathematics. The first theoretical framework, the RT model, refers to instructional activities that take place between students and the teacher concerning the strategies of summarizing, questioning, clarifying and prediction. These four strategies are considered as a baseline for reasoning, which is adopted to this thesis as reasoning in school mathematics. Within the latter theoretical framework, SFL, language is viewed as a system of meaning which can be analysed at the clause level to understand how contexts, such as classrooms, are reflected in participants' linguistic choices. SFL has previously been

put forward as a suitable tool when studying spoken or written texts in mathematics classrooms (Herbel-Eisenmann, 2007; Morgan 2006; Wagner, 2012), but reading has not been an explicit focus, with the exception of Bergwall's (2016) study of TIMSS tasks.

Within SFL, the context of culture is an important aspect that influences students' meaning making and thus needs to be taken into consideration. In following section, the Swedish context is elaborated.

1.2 The Swedish context of school mathematics

The students' performance on national and international test in mathematics (for example TIMSS, PISA) and reading comprehension (for example PIRLS) has become more and more into focus by educational policy makers, parents and politicians in Sweden. This increasing engagement can be understood from the fact that Swedish students' performance in mathematics has decreased since 2002, according to the international large-scale assessments (Skolverket, 2012b). One way the government has responded to the decreased result of the students' performance in mathematics in Sweden was to explicitly define what mathematical learning should be in terms of five abilities in the curriculum for compulsory school, (Skolverket, 2011). These are:

- formulate and solve problems using mathematics and also assess selected strategies and methods,
 - use and analyse mathematical concepts and their interrelationships,
 - choose and use appropriate mathematical methods to perform calculations and solve routine tasks,
 - apply and follow mathematical reasoning, and
 - use mathematical forms of expression to discuss, reason and give an account of questions, calculations and conclusions.
- (Skolverket, 2011, p. 59-60)

The syllabus is based on previous curriculum Lpo 94 (Utbildningsdepartementet, 1994), and of two projects; the Adding Up report (Kilpatrick, 2001) and the KOM-project (Niss & Højgaard Jensen, 2002). Both of these projects had the intention to change teaching practice in schools by creating a broader view of what school mathematics means (Boesen et al., 2014). They both ended

up in explicit recommendations, which as stated above have served as a base for curriculum development. Kilpatrick (2001) raises five research-based recommendations, mathematical proficiency for “successful mathematics learning” (p. 105). These are conceptual learning, procedural fluency, strategic competences, adaptive reasoning and productive disposition (students’ appreciation of mathematics). Furthermore, Niss (Niss & Højgaard Jensen, 2002) refers to eight competences; thinking mathematically, posing and solving mathematical problems, modelling mathematically, reasoning mathematically, representing mathematical entities, handling mathematical symbols and formalism, communication in, with, and about mathematics and making aids of tools, that relate to the students. These recommendations concerning what mathematics learning consist of have influenced the current syllabus for mathematics in Sweden, which can be seen in the five abilities expressed above. This visualize that learning mathematics is not only about learning a mathematics content, instead the learning also consist of five abilities (abilities in international research expressed as competences, which also is used in this study further on) that are essential to embrace the mathematical content being expressed. In this thesis, the focus is on mathematical reasoning, which appeared in the Swedish curriculum 1969 (Skolöverstyrelsen, 1969), as a way to support the development of the students’ mathematical knowledge from concrete to abstract thinking but no further discussion what reasoning involves is provided. Further in the Swedish curriculum from 1994 (Utbildningsdepartementet, 1994) reasoning is stressed, as a goal to strive towards in school mathematics but how is not elaborated. In the current curriculum (Skolverket, 2011), reasoning is stressed as one of the five abilities that teaching in mathematics should essentially give students the opportunities to develop.

In previous research, it has been suggested that part of the reason for poor results in mathematics in Sweden is the reliance on textbooks (Bergqvist, Bergqvist, Boesen, Helenius, Lithner, Palm & Palmberg, 2010; Johansson, 2006; Löwing, 2004), which is also stated in the report from TIMSS 2011 (Mullis, Martin, Foy & Arora, 2012). Sweden was pointed out as being one of the countries that uses the textbook in mathematics education most. Moreover, in Sweden, among many parents, students, teachers and principals the mathematical textbook

is synonymous with mathematics education. The trust in the book is much more extended in mathematics than in any other school subject (Johansson, 2006).

A common practice during mathematics lessons is that students work individually in their mathematics textbooks (Bergqvist et al., 2010; Boesen et al, 2014; Johansson, 2006; Löwing 2004; Skolinspektionen, 2009). When students are expected to work individually in their mathematical textbooks, it is often up to each student to create their own understanding of mathematics from reading. However, to develop students' reading skills is not a part of the syllabus for mathematics (Skolverket, 2011), neither is it a common practice. Instead, reading is only presented in the current curriculum under core content in mother tongue tuition (p.84), and the subject of Swedish (p. 213) and Swedish as a second language (p. 229). Under the heading "Knowledge" in chapter 2 the following text is presented "Teachers should organize and carry out the work so that pupils: receive support in their language and communicative development" (Skolverket, 2011, p.16). This means that it is every teacher's responsibility to contribute to students' speaking, reading and writing skills in and through a certain subject. This is the policy but the current culture in the classroom might be different. However, the need for extra attention to so-called disciplinary literacy, e.g. being able to read and write in a certain discipline (Shanahan & Shanahan, 2008), differs from subject to subject and during the school years. Mathematics in Grade 4 in Sweden seems to be a time when disciplinary literacy becomes important, since the texts usually become longer and more complex, including new mathematical abstract concepts (Myndigheten för Skolutveckling, 2008; Skolverket, 2003). In Figure 1, an example of a common design of a textbook page in a Swedish textbook in Grade 4 is shown.

Välja och byta enheter

Hur lång är linjalen?

Linjalen är 1 meter lång i verkligheten.
 Linjalens längd kan också skrivas i **enheter**na dm, cm och mm.
 $1\text{ m} = 10\text{ dm}$ $1\text{ m} = 100\text{ cm}$ $1\text{ m} = 1\,000\text{ mm}$

21 Skriv som meter (m).
 a 10 dm b 100 cm c 200 cm d 2 000 mm

22 Tanja lägger 4 centikuber på en rad.
 Hur många centikuber behöver hon lägga för att få sträckan

a 1 dm b 5 dm c 1 m d 2 m

23 Filip mäter sin fot. Han markerar längden på en tallinje.
 Hur lång är Filips fot?
 Skriv som a millimeter b centimeter

24 Välj enhet (m, dm, cm, mm).
 a En penna har längden 1 ____ b En penna har tjockleken 1 ____
 c En bok har tjockleken 2 ____ d Samma bok har tjockleken 20 ____
 e Spagettins längd är 25 ____ f Spagettins tjocklek är 1 ____

25 Vilken elev är längst?

Kim: Jag är ettusen fyrahundra femtiotvå millimeter.

Linn: Jag är en och en halv meter.

Sam: Jag är etthundratrettionio centimeter.

Figure 1. An example of a textbook page (Sjöström & Sjöström, 2016, p. 41).

The structure with repeated cycles of exposition-examples-exercises is a common form also in other countries (Love & Pimm, 1996). If we take a closer look, we see that the heading is important for the readers' attention and is usually a summary of the content being presented on the page. The exposition is an introduction to the mathematical topic often involving a heading and an explanatory text with examples aiming to illustrate a generalised method of how to solve the exercises.

Furthermore, the purpose of the reading also changes in Grade 4 from learning to read to reading to learn (Skolverket, 2007). This puts new demands on the students' reading skills. We can also see that a correlation between students' reading skills and mathematical skills starts to appear in this Grade in Sweden (Möllehed, 2001). Yet, mathematical teachers talk about knowing how to work with language as a problem, since they state themselves that they lack knowledge to teach about the language in mathematics (Liberg, 2008).

The teacher often relies on the textbook to fulfil the written curriculum according to Jablonka and Johansson (2010). The report "Lusten att lära - med fokus på matematik" (Skolverket, 2003) and Johansson (2006) went as far as to suggest that mathematics in many classrooms in Sweden is simply what is written in the textbook. The textbook then plays an important role in instructions because it influences how many students learn and apply mathematical concepts (Bryant, Bryant, Ketheley, Kim, Pool & Seo, 2008).

The focus on textbook based education in mathematics has also been found in mathematics courses in mathematics teacher education in Sweden (Skog, 2014). In Skog's (2014) study, the mathematics content in the previous courses focused on teaching for understanding, on creativity and problem solving open for errors and different solutions. Thereby alternative ways to talk about and deal with mathematics were provided. However, as the tests approached, the students' aligned with mathematics with focus on high-speed calculations and superficial learning (textbook based learning), which also was found in Player-Koro's (2011) study among pre-service teachers in mathematics.

When considering that the textbook is one of the most important resources in teaching mathematics, the textbook needs to be of the highest quality. But, since 1992, there has been no government control or evaluation of textbooks in Sweden and consequently no guarantees that textbooks in mathematics are adapted to the current curriculum (Johansson, 2006). For instance, studies have shown that opportunities to reason are limited when working in the mathematical textbook (Bieda et al., 2014; Sidenvall, 2015; Stylianides, 2009).

1.3 The aim of the study

According to the literature review in chapter 2 and 1.2, previous research clearly states that reasoning is an important competence for meaning making in mathematics. However, what reasoning means and how students can develop this skill is less explored. Furthermore, it is not theorized in depth. The present study intends to contribute to the knowledge base on these issues. It departs from an EDR study which aimed to construct the theoretical based teaching activities and to analyse the effectiveness of these activities in practice in a Grade 4 class. More specifically, the aim of this thesis is to design and analyse strategies for mathematical reasoning with help from the RT model and SFL.

The following three research questions are addressed:

- How can mathematical reasoning be conceptualized into a local theory by operationalizing the Reciprocal Teaching Model coordinated with Systemic Functional Linguistics?
- Which reading and writing activities connected to the Reciprocal Teaching model and Systemic Functional Linguistic can support Grade 4 students' reasoning competence in school mathematics?
- In which ways, does the context of culture influence the development of the students' mathematical reasoning?

1.4 Outline of the thesis

In chapter two, a review of previous research is provided. Initially, I discuss literature on reasoning in mathematics in general. From this follows a section on literature about the strategies students use to embrace the mathematical content when reasoning and important aspects concerning the reading skills necessary for reading mathematics texts then follow. Finally, the chapter approaches how theory and practice interact to find suitable activities to develop students' reasoning competence in mathematics.

Chapter three introduces the conceptual framework, where the Reciprocal Teaching (RT) model and Systemic Functional Linguistics (SFL) are coordinated, and how it is used to inform this study's analysis and design. The RT model, consisting of the comprehension

strategies of prediction, clarification, questioning and summarization is considered as the baseline for how to construct possibilities for students to reason in mathematics. SFL provides an important analytical tool to analyse students' reasoning in relation to linguistic choices as well as the contexts of situation and culture.

In chapter four, the methodology is presented and the approach, Educational Design Research (EDR) is initially described in general followed by a presentation of how EDR is conducted in this study. Furthermore, the analytical tool is developed from the theoretical framework, and the limitations and ethical considerations in relation to the study are also discussed.

In chapter five, a summary of the five papers included in this thesis is presented. Conclusions and discussion are provided in chapter six. In the discussion, three aspects are highlighted: making the language explicit when reasoning, exercises to support students' reasoning competence and the context of culture. Finally a Swedish summary is provided in chapter seven. The papers that form the foundation for this thesis and where the analysis of the study is published are found as appendices at the end of this compilation thesis.

2. REASONING IN SCHOOL MATHEMATICS

The aims of this chapter are to provide both the background to the present study as well as a review of research relating to how reasoning has been described thus far in the curriculum in the context of what approaches to reasoning currently exist in education and how reasoning is closely related to reading, writing and language use. In this study, the focus is on educational ramifications not the philosophical and epistemological perspectives of reasoning. The reviewed research mainly falls within the context of elementary school, and in general, addresses reasoning and its relation to how reading and writing in school mathematics can support students' reasoning competence when engaging with mathematics texts. First, the overall framework of students' reasoning in mathematics education is discussed in relation to reading and writing, resulting in research where theory and practice interact, which is also presented and discussed. Finally, the relevance of this review is linked with the research aims and questions.

2.1 Reasoning in mathematics education

In several countries, reasoning has been emphasized in curricula guidelines as an important competence in mathematics, for example, in India (NCF, 2005), Sweden (Skolverket, 2011) and USA (NCTM, 2003). This could relate to globalization issues where curriculum standards are organised globally to suit large-scale studies, such as that of the PISA tests, which point out reasoning as an important competence for mastering mathematics (Turner, Blum & Niss, 2015). However, reasoning in mathematics education is also stressed as an

important competence in school mathematics by several researchers (Hanna, 2000; Kilpatrick, 2001; Lithner, 2008; Niss & Højgaard Jensen, 2002). For example, in direct relation to the mathematics curriculum and the school classroom, Baxter et al. (2005) and Cobb (2002) highlight that students' reasoning can potentially contribute to visualizing important aspects of how students have interpreted the specific mathematical content being processed. Further, it is stressed that by letting students argue and explain their thinking, the comprehension of mathematical content is being supported (Boaler & Staples, 2008). Ball and Bass (2003) give three reasons for the importance of reasoning in mathematics. First, mathematics is meaningless without reasoning; second, reasoning is considered a fundamental competence because simply memorizing and learning procedures and main ideas are not sufficient to apply to another context. And third, reasoning is crucial for reconstructing knowledge.

In Nunes, Bryant, Barros and Sylva's (2012) five-year longitudinal study with 1,000 eight-year-old students, the results show that the students' overall reasoning competence was a much stronger predictor of their future performance in mathematics compared to individual arithmetic competence. The result also indicates that schools must explicitly plan to improve mathematical reasoning in addition to arithmetic skills.

In the next section, reasoning is further discussed through approaching reasoning in school mathematics, the perspective of mathematical reasoning taken in this thesis is outlined, and finally, the culture of school mathematics concerning reasoning is discussed.

2.1.1 Approaching reasoning in school mathematics

Reasoning has been discussed as a core element when the function and nature of mathematical practices as historical and philosophical phenomena is being considered (see for example, Whitehead & Russel, 1962 or Toulmin, 2003). In research, Toulmin's model is extensively used in mathematics when examining arguments (Inglis, Mejia-Ramos & Simpson, 2007; Nordin, 2016). The model consists of six elements: claim (somebody is saying something), data (which information is used for the claim), warrant (how do you know that?), backing (facts), qualifier (when this is correct) and rebuttal (when it might not be correct). This pattern can be used in different situations,

not just a specific situation. Krummenheuer (1995) has reduced the model to involve four of the six elements: claim, data, warrant and backing, where several persons can contribute to the elements. Krummheuer considers argumentation as a basic communicative aspect of everyday activities, which also takes place in classrooms (Chronaki & Christensen, 2005). Krummenheuer's approach has been used in several studies in mathematics (Evens & Houssart, 2004; Knipping, 2003).

However, reasoning is also discussed in education, which is the baseline for this thesis. In mathematics education research, an elaborated and clarified notion of the concept of reasoning in mathematics is scant (Lithner, 2008; Yackel & Hanna, 2003). Some researchers have worked towards providing a more viable definition of reasoning in school mathematics. For example, Lithner (2008) defines reasoning as "The line of thought that is adopted to produce assertions and reach conclusions when solving tasks" (p. 257). However, reasoning does not necessary have to result in a correct outcome, as long as there are reasons available that support the thinking (Lithner, 2008). A similar definition is expressed in NCTM (2008), where reasoning is described as a cyclical process involving exploration, conjecture and justification.

One way to conceptualize reasoning is provided by Lithner (2008), which involves two categories of reasoning: imitative reasoning (IR) and creative reasoning (CR). *Imitative Reasoning* refers to remembering an answer or a whole solutions strategy and usually refers to routine tasks such as $2x + 3 = 9$. *Creative Reasoning* involves novelty, flexibility, plausibility and a mathematical foundation. Here, *novelty* refers to a new sequence of solutions where reasoning is created or when a forgotten sequence is re-created. *Flexibility* is defined as fluently admitting different adoptions and approaches to the situation. *Plausibility* refers to the arguments supporting the strategy choice and/or strategy implementation motivating why the conclusions are true. *Mathematical foundation* means that argumentation is founded in mathematical properties of the components involved in the reasoning process. IR and CR have previously been used to examine upper secondary students' reasoning competence in problem solving (Boesen, Lithner & Palm, 2010; Sidenvall, Lithner & Jäder, 2015; Sumpter, 2013) and university students' reasoning (Lithner, 2003), but also

as a framework for categorizing tasks in mathematics textbooks at these levels (see, for example, Boesen et al., 2010). At the present time, no studies have been conducted that explicitly examine these categories (IR and CR) further with students in middle school (email correspondence, Johan Lithner, October, 2016). Thereby, these categories have not been categorically expressed in what they involve for the lower Grades.

Other researchers (Hanna, 2000; Styliandies, 2009; Yackel & Hanna, 2003) conceptualize reasoning with proofs. One issue in this approach is that the focus tends to be on axioms, syntactic derivations and proofs, and explanatory text is usually missing (Yackel & Hanna, 2003). Explanations and generalizations are used by other researchers when conceptualizing mathematical reasoning to problem solving (Barret, Clements, Klanderma, Pennisi & Polaki, 2006; Bishop, Lamp, Philipp, Whitacre, Schappelle & Lewis', 2014; Bjurland, Cestari & Borgensen, 2008; Chen & Herbst, 2012; Jurdak & Mounhayar, 2014; Makar, 2014). This definition is similar to the definition used in this thesis and is further discussed in section 2.1.2.

In statistics, Maker (2014) reports on students in Grade 3 and their explorations in finding mathematical averages through reasoning; seven lessons were videotaped which showed that the students' understanding of average in problem solving tasks became more mathematically precise after asking the students to explain their thinking; further, they demonstrated that they were able to move from their own data to involve the population beyond their own classroom. Another study in statistics (Chen & Herbst, 2012) examines how high school students made conjectures and justifications through multimodal representations of diagrams during two lessons. The result showed that when limited information is given in a diagram, the students used verbal and gestural expressions to compensate for this limitation when reasoning. Similar findings were found in Bjurland et al.'s (2008) study of two groups of Grade 6 students who also used speech and gestures when explaining their thinking about diagrams where illustrations were included. The gestures related to their use of reasoning strategies played an important role in developing collaborative mathematical reasoning in the two groups.

In Bishop et al.'s (2014) interview study, forty-seven students aged 6–10 reasoned about negative numbers and this resulted in finding

three obstacles. The first obstacle concerned being asked to represent numbers that were “less than nothing”, the second obstacle involved removing something from nothing or removing more than one has, for example when solving $3 - 5$, and the third involved a contradiction to the generalization the children formed about the ways subtraction and addition function in whole numbers, namely, that addition does not make smaller and subtraction does not make larger. In Barret et al.’s (2006) interview study of 38 students in Grades 2–10, the researchers examined the development of their levels of understanding measurement by describing the coordination of geometric reasoning with measurement and numerical strategies in problem solving. The result showed that students in Grades 2–3 focused around the issue of units and iterating units when reasoning, Grades 5–6 focused on issues of generalization by using physical representations of units, and Grade 8–10 could move immediately to generalization without physical representations. In contrast, Jurdak & Mounhayar’s (2014) study with 1,232 students in Grades 4–11 showed an increasing trend in the level of reasoning in pattern generalization across the Grades rather than from one Grade to the next. These results indicate that teachers of all Grades need to adapt the teaching to suit their specific class, which means different materials and approaches may be needed. Consequently, these studies show that by prompting the students to reason by explaining their thinking, their understandings and obstacles could be shown in different topics in school mathematics.

To summarize, according to previous research, mathematical reasoning is approached in different ways. However, there is little discussion about how reasoning competence could potentially be developed through pedagogy and why and how such pedagogical practices may fall short in school classrooms (Sternier, 2015). For example, although the text on Principles and Standards in the U.S. curriculum for school mathematics (NCTM, 2000) gives several examples of high-quality reasoning, it does not provide a broader definition or a theoretical model of reasoning that could help suggest ways of developing such reasoning competences in the classroom. Jacobes, Franke, Carpenter and Battey’s (2007) professional development project involving 180 teachers and 3,735 students showed that the students in the project had developed a deeper understanding and a relational way of thinking about algebra than students in the

classes of non-participating teachers. The focus during the professional development was on three aspects concerning reasoning: students' thinking, mathematical conversations and noticing opportunities to extend arithmetic in problem solving.

In addition, previous research tends to focus on reasoning in problem solving tasks (Barret et al., 2006; Bishop et al., 2014; Lithner, 2008). Nevertheless, what was found in Nunes et al.'s (2012) study, is that schools must explicitly plan to improve mathematical reasoning in relation to all basic mathematical competences such as arithmetic and conceptual understanding and not just problem solving.

In the next section, the culture of reasoning in school mathematics is further discussed, as it may be closely related to why it has been so difficult to organize reasoning practices thus far.

2.1.2 The culture in school mathematics concerning reasoning

There are issues concerning the goal of mathematics education where some mathematicians want teachers to put more emphasis on the learning of definitions, rules and proofs while others want to concentrate on social and/or cultural issues connected to mathematics. Several researchers have tried to define mathematical knowledge (for example, Kilpatrick, 2001; Niss, 2002), while other researchers have instead concentrated on cultural and/or social issues to understand in a broader perspective how knowledge is developed within society (Wedeg, 2010). When considering that knowledge is developed in society, it becomes evident that context is important because it includes assumptions about how individuals learn. By using social perspectives in mathematics, specific insights can then be gained because language takes place in a social environment which is structured by the culture (Morgan, 2006). The latter perspective is supported in this research, where the culture is considered essential for students' learning. For the purposes of this thesis, the culture concerns opportunities for reasoning practices in the context of school mathematics where the focus is on language use where the teachers' and students' roles as well as materials such as textbooks are in focus. In this thesis, language is seen as a semiotic system, or resource, with which teachers, and students in the learning process, construct and share mathematical meanings; thus, learning mathematics involves learning its characteristic pattern of language. To visualize this, Halliday's

Systemic Functional Linguistics (SFL) is used (which is a part of the conceptual framework, see section 3.2) where the context of culture is considered essential (Halliday, 2007).

The context of culture concerning communication in mainstream mathematics classrooms is often characterized by teacher talk, which involves explaining procedures, giving directions, and explaining and correcting mistakes (see, for example, Hiebert & Grouws, 2007; Silver & Smith, 1996). This type of communication usually requires very little student-to-teacher talk or student-to-student-talk. Another common approach is the IRE pattern, initiation-response-evaluation (Gibbons, 2009; Stone, 1998), where the teacher initiates a question, the students respond, and the teacher evaluates something that they already know the answer to. The IRE sequence has been criticized for limiting students' possibilities to form collaborations, elaborate opportunities or develop other students' ideas (Wolf, Crosson & Resnick, 2005). This issue is also identified in Löwing's (2004) study where the teachers' questions mainly focused on asking *what?* (what the students should do) and not on *how?* (how the students think or how they used strategies). The students then focused on mainly providing the right answer without elaborating or expanding on their ideas (Cobb & Yackel, 1996) meaning, to engage in reasoning. This is an issue, as teachers' questions contribute to shaping the classroom environments and the cognitive opportunities offered to students (Boaler & Brodie, 2004).

Further, "Do it quick, and do it right" was a common assessment strategy used by teachers in four Grade 5 classrooms in Björklund Boistrup's (2010) case study, involving whether the answer was correct or not, and closed questions were usually asked.

Suggestions have been offered in NCTM (2000) curriculum documents about ways teachers can work with students and prompt classroom interaction. Some suggest that instead of being the transmitter of knowledge, the teacher should support and promote mathematics in a variety of settings, such as whole-class settings, small group work and investigative work (Lampert & Cobb, 2003; Jaworski & Potari, 2009). However, although teacher education programmes introduce student-focused and inquiry-based pedagogies, traditional teacher-directed learning using traditional textbooks is still widely used in the classroom (Lambert & Cobb, 2003; Nolan,

2012). Therefore, despite several efforts, the mathematics textbook in mathematics education is still extensively used (Boesen et al., 2014; Haggarty & Pepin, 2002; Remillard, 2005; Thomson & Fleming, 2004). According to Remillard (2005), this can relate to teachers' constructing their own unique version of the curriculum that relates to their understandings, needs and goals. Another aspect that might influence the culture is testing, which is a strong and engrained tradition in mathematics education worldwide; it indicates that learning and knowing mathematics involves the strict performance of knowledge in a given time frame performed in complete isolation after working with a specific topic or with several topics in the mathematics curriculum (Nolan, 2012). Testing mainly concerns the advance of procedures and rules with the purpose of showing how students can become more autonomous and work on their own.

Concerning textbook use, research has shown that the most common use of the textbook by teachers is to follow the same content and sequential order as the textbook pages as they are presented in its structure and rely on them to fulfil the required curriculum guidelines to ensure effective teaching (Thomson & Fleming, 2004; Vincent & Stacey, 2008). In Sweden, reliance on the textbook has been widespread for many years and, according to Jablonka and Johansson (2010), textbooks in mathematics education all too often replace the national steering documents in Sweden. The identification of what mathematical content is covered in a given set of curriculum materials is important because students do not learn content to which they are not exposed to (Hiebert & Grown, 2007). Further, the tasks with which students become engaged in the classroom form the basis of their opportunities to learn what mathematics is and how one accomplishes it. However, according to research, mathematical activity that involves developing students to articulate their reasoning competence seldom exists in elementary mathematics textbooks (Bieda, Drwencke & Picard; 2014; Sidenvall, 2015; Stylianides, 2009).

For example, in a study of a series of US mathematics textbooks on the elementary school level, Stylianides (2009) analysed the opportunities for students to be engaged in reasoning and proving. They found that out of 4,855 tasks, only about 40% of them offered at least one such opportunity for students and more than 50% offered no opportunity for the students to engage in processes that develop

mathematical reasoning. Similar results have also been found in Bieda et al.'s (2014) study where opportunities to reason was only 3.7% in seven upper-elementary (ages 9–11) mathematics textbooks published in the U.S. Instead, several other studies have shown that exercises in textbook materials mainly focus on developing students' procedural competence (Bergqvist et al., 2010; Boesen et al. 2014; Stylianides, 2009), which usually concerns answers with numbers. In Herbel-Eisenmann's (2007) study involving a linguistics analysis of a 64-page, middle school mathematics textbook, the "voice", or the interpersonal function of the text, was examined. This consisted of the main roles for the reader and how the relationship between the reader and author is constructed. This was conducted by looking at three linguistic forms: imperative, personal pronouns and modality. The analysis showed that the style of the text was authoritative through the use of commands (imperative) usually involving make, use, write, draw and find, and few opportunities for the students to explain their thinking which related to reasoning. To perform calculations was considered the students' preferred type of writing in Meaney, Trinick and Fairhall's (2009) study of students from Grades 1–11, and using mathematics vocabulary words was their least preferred type of writing. This probably relates to the similar context of school mathematics that the students were accustomed to, which also relates to the results in previous studies presented in this section.

Nevertheless, there is a widespread consensus in research endeavours concerning the view that language plays an important role in mathematical learning (see, for example, Pimm, 1987; Lemke, 2003). Teachers must be able to evaluate and create viable teaching practices to support students' language skills, which is essential for the development of knowledge in different subject areas (Alatalo, 2011; 2015; Reichenberg and Lundberg, 2013; Tjernberg, 2013). The teacher then needs to allocate time where the mathematical language that has been highlighted as important is discussed and elaborated further (Meaney & Flett, 2006). However, several mathematics teachers express that they lack knowledge in this kind of teaching (Liberg, 2008; Ratekin, Simson, Alvermann & Dishner, 1985; Teledahl, 2016). Similar findings were also found in Barton, Heidema & Jordan's (2002) study, where many teachers considered that they lacked adequate training in teaching reading strategies as well as the confidence to

integrate literacy instruction into their mathematics lessons. This is problematic because language use is closely related to students' reasoning competences, and apart from students' socioeconomic status and language factors, the knowledge and competence of teachers are the most important factors in supporting students' development and learning, (Darling-Hammond, 2000; Hattie, 2009). In Nunes, Bryant, Evans, Gottardis and Terlektsi (2015) and Rojas-Drummond and Zapata (2005) studies peer discussions where the students are asked to express, explain and share their ideas to problem solving were found successful for supporting middle Grade students' reasoning competence. Furthermore, in Larsson (2015) Stein et al.'s, model consisting of five practices were implemented to support teacher to orchestra whole-class discussions to support students' reasoning competence in problem solving in a successful way.

2.1.3 Approach of reasoning in this thesis

In Sweden, where the present study was conducted in 2013, reasoning was being argued at the time and still as having a very central role for school mathematics curriculum. It is being noted as one amongst the five main competences in primary school and it was stressed that students should develop reasoning skills in mathematical activity as part of the current curriculum requirements (Skolverket, 2011) and is more elaborated in section 1.2. However, a pedagogical definition of how reasoning could be employed in the school mathematics classroom reasoning in mathematics has not been explicitly expressed in the syllabus. Some description of reasoning was only partially mentioned as part of the curriculum document for upper secondary school with a commentary (see Skolverket, w.y.) of a more explicit description. This description was inspired from Niss' (2003) and Kilpatrick's (2001) assumptions and was expressed in following way:

“Reasoning competence means being able to bring mathematical reasoning involving mathematical concepts, methods and forms solutions to problems and modelling situations. Bringing reasoning also includes self and with others such as testing, propose, predict, guess, question, explain, find patterns, generalize, argue.” (p.2, Skolverket, u.å c, p. 2, own translation).

The second part of the definition, is similar to the definition that can be found concerning the reading comprehension strategies; prediction, clarification, questioning and summarization, in the Reciprocal Teaching (RT) (Palinscar & Brown, 1984) and in the Transactional Instruction Strategies (TSI). The national curriculum practices in Sweden is following Danish and US standard, but there is no adequate research in the Swedish context to support these choices and the aim of this thesis is to explore students' reasoning competence further. The focus is then on examining how the four comprehension strategies mentioned above which relate to deep level strategies that can contribute to support the students' comprehension skills on a deeper level, which is necessary for being able to reasoning. These four strategies have previously been used in mathematics education to support students' reasoning competence connected to specific problem-solving tasks with positive results (Borasi et al. 1998; Huber, 2010). But, the strategies have not been explored further connected to students' ways of explaining and describing mathematical phenomena such as the meaning of ideas, concepts, operations and processes as they evolve in specific mathematical tasks, where the textbook will be the baseline for this study. In Sweden, the students are usually asked to work individually in the mathematics textbook, which put special demand on students' reading skills since the text being presented in such mainstream school mathematics texts is radically different to other kinds of texts (Barton & Heidema 2002; Fuentes 1998; Shanahan & Shanahan, 2008; Shepherd et al., 2012). This refers to disciplinary literacy, which will be further discussed in section 2.2.

In next section, reading closely related to reasoning will also be further discussed which is essential for being able to use the reading comprehension strategies in the RT model in appropriate ways.

2.2 Reading skills for reasoning

To make meaning from a written text, reading comprehension is needed and within reading research there are differing opinions about what reading comprehension means (Pearson & Hamm, 2005). According to some researchers, reading comprehension skills are considered a cognitive and individual process to create meaning (e.g. Riddle Bury & Valencia, 2002), and by others, a social process that is developed in collaboration with others (for example, Chinn, Anderson

& Waggoner, 2001). In contrast, some reading researchers (Sweet & Snow, 2002; Westlund, 2013) try to see connections between social activities, context and cognitive processes as important aspects in the development of reading skills. The last perspective is supported in this thesis. *Reading comprehension* in this study refers to Sweet and Snow's (2002) definition:

“As the process of extracting and constructing meaning through interaction and involvement with written language. The reading comprehension process includes three dimensions: the reader, the text and the activity.” (p. 23–34).

These three dimensions cannot be considered in isolation because the capabilities of the reader always relate to a particular text and activities, which in this study, refers to reading mathematics texts, which differs from reading texts in other subjects (e.g. Barton & Heidema 2002; Fuentes 1998; Shanahan & Shanahan, 2008; Shepherd et al., 2012). For example, in school mathematics, texts are generally written in a short, unimaginative style, where few contextual clues are given to help the readers decode the meaning of specialized words (Reehm & Long, 1996). It is also common for these texts to have more than one concept per sentence and per paragraph than other kinds of texts, such as in history or social science (Barton & Heidema, 2002; Shuard & Rothery, 1988). Further, several semiotic resources are often involved, such as illustrations, symbols and words, that must be interpreted not only individually but also together. However, as previous research stress, reading the mathematics textbook puts specific demands on the students' reading skills, and in this study, these skills refers to disciplinary literacy.

The roots of the disciplinary literacy concept are threefold: They can be found in the historical development of content area reading, cognitive analyses of expert readers, and functional linguistics (Shanahan & Shanahan, 2012). Content area reading prescribes reading approaches and study techniques that can help with comprehension or to remember text better, independent of the type of text, whereas disciplinary literacy emphasizes the description of unique uses and the implications of literacy use within the various disciplines. Thus, learning a subject involves learning the specific language of the subject (Schleppegrell, 2004; Unsworth, 1997). Concerning the cognitive

requirements of interpreting or learning any text is much the same independent of the subject according to content area reading. In some cases, research in this area has evaluated student learning using texts drawn from particular disciplines (e.g. Herber, 1970), but despite this, nothing has been particularly specialized or discipline-specific about the reading guidance provided to the students, such as paraphrasing reading strategies to suit the specific discipline, which concerns the second part of disciplinary literacy. In previous research functional linguistics, which is the third part of the root of disciplinary literacy, has been proven as a tool to identify differences in the language used in the various disciplines (Fang & Schleppegrell, 2008; Bergwall, 2016), which also is used as a part of the conceptual framework in the thesis, Systemic Functional Linguistics (see chapter 3). Consequently, disciplinary literacy involves content specific challenges and the specific meaning the language has in the different subjects, which in this study, refers to reasoning in mathematics. This is supported in Möllehed's (2001) study of students in Grades 4–9 in Sweden and in Vilenius-Touhima, Aunala and Nurmi's (2008) study of Grade 4 students in Finland, where a correlation was found between their level of reading skills connected to the school mathematics text (relating to word problems) and their mathematical skills. According to Korhonen, Linnanmäki and Aunio's (2012) five-year longitudinal study, this correlation seems to remain throughout a student's school life.

Furthermore, a correlation between reading skills in language tests and mathematics skills has been found in Jordan, Hanich and Kaplan's (2003) study of 180 Grades 2 and 3 students with low reading skills and students with both low reading skills and mathematical skills. Difficulties in basic mathematical areas such problem solving were found to correlate with the students' general reading comprehension competences. Similar findings were found in Thurber, Shinn and Smolkowski's (2002) study of 207 Grade 4 students at four different schools where general reading comprehension was highly correlated with both math computation (such as solving multiplication and division tasks), and math applications (being able to apply basic math facts and principles to addition, subtraction, multiplication and division as well as mathematical concepts). Thus, for being able to reason in school mathematics, both general and specific reading

skills are necessary to take into consideration and should not be overlooked when drawing conclusions about the students' ways of reasoning as well as their reasoning competence in mathematics. Reading skills comprises two important aspects – decoding and linguistic comprehension (Hoover & Gough, 1990) – which is known as the simple view of reading. In next section, decoding, linguistic comprehension and prior knowledge will be further discussed in light of how they are closely related to mathematics reasoning.

2.2.1 Decoding

Usually, in the western language cultures, the reader reads from the top of the page to the bottom and from left to right when decoding a text. In mathematics, the students must also be able to read the text not only from left to right but also from right to left (consider an integer number line), from bottom to top and vice versa (with tables), and even diagonally (with some graphs) (Barton & Heidema, 2002). The same applies to Arabic cultures and Chinese cultures in mathematics, but vice versa. When decoding a text, the reader needs to understand that printed words are symbols for words in spoken language and are composed of letters (graphemes) that represent individual speech sound (phonemes). Another issue when decoding texts in mathematics is that they often involve symbols, which is more complex than decoding words, as it involves adapting a translation from symbols to words and vice versa (Carter & Dean, 2006). Furthermore, students need to understand how to decode the textbook when, for example, graphs, diagrams and illustrations are related to the text (Noonan, 1990), meaning, it is multimodal. The second part of reading comprehension is linguistic comprehension.

2.2.2 Linguistic comprehension

In school mathematics texts, phrases and words are often integrated with numerals, symbols, and illustrations (Barton et al., 2002), which means they are multimodal (Jewitt, 2005). The individual meanings of the vocabulary, symbols and illustrations need to be separated and sometimes combined to understand the text (Love, 2008). Several studies have shown the interrelation between vocabulary knowledge and reading comprehension in general (Nagy, Anderson & Herman, 1987, Nation, 2001), and Snow and Juel (2007) specifically claim that vocabulary is the most important aspect in reading comprehension.

Another issue that affects students' language comprehension of mathematics textbooks is the vocabulary (Adams, Thangata & King, 2005; Carter & Dean, 2006; Lee, 2006). This is also shown in Abedi and Lord's study (2001), where the performance of children in arithmetic word problems is 10–30% less than in problems presented in a numerical format, suggesting that the vocabulary used is a core issue. Furthermore, the vocabulary can also be difficult to comprehend because some of the terms are only found in mathematics, such as 'rectangle' (Lee, 2006), but some of the terms can have different meanings in conversational language, for example, 'odd' and 'volume' (Adams, 2003; Adams et al., 2005; Lee, 2006). Nation (2001) stresses that the students need to know 95% of the words to generally comprehend a text. This is not consistent in mathematics because the students must know all the words to comprehend the content provided, especially as the text is often compact and complex (Shanahan & Shanahan, 2014).

Nevertheless, the students also need to recognize the symbols used in mathematics and be able to translate from symbols to words and from words to symbols as they search for solution and meaning (Reehm & Long, 1996). Symbols often indicate that a specific operation needs to be performed in school mathematics texts (Adams, 2003). When the students learn a symbol, they also need to link the symbols to the mathematical ideas that are represented and with the words that correspond to that idea (Noonan, 1990). This can be difficult for some students because the symbols are not phonetic. Instead, they are representations of vocabulary words and ideas to which meaning must be attached (Carter & Dean, 2006).

Symbols in mathematics can be numeric such as 1, 8 and 22, and non-numeric, involving symbols for mathematical operations, such as addition, multiplication, subtraction and division. These can efficiently tell the learner what to do (Adams, 2003; Hammill, 2010), such as '+' or algebraic symbols such as ' $5x = 15$ '. Symbols are often considered an important part of mathematics texts (Morgan, 1998); however, a literature review of academic mathematics text comprehension (Österholm & Bergqvist, 2013) shows that it is often not clearly described how mathematical symbols are included in the descriptions of the mathematical language. Similar issues could arise with illustrations, which are also a common feature in mathematics texts (ibid, 2013).

According to Pettersson (2008), illustrations in general have a direct emotional appeal that texts often lack. Some researchers (see Arizpe & Styles, 2002; Levie & Lentz, 1982) consider illustrations to support students' learning in a positive way when reading texts, while other researchers (Eklund, 1990; Watkins, Miller & Brubaker, 2004) consider illustrations to contribute to difficulties and thereby hinder the students' learning. In school mathematics texts, the illustrations are often categorised into decorative material, related but non-essential material, and essential material (Noonan, 1990). The purpose of decorative material is to make the page more attractive, but it serves no instructional purpose. Related but non-essential material repeats ideas given in words and can give support to written material, and essential material is often in the form of graphs and tables and is referred to but not repeated in the text; It must be "read" along with the rest of the text (Noonan, 1990) to be able to comprehend the content.

Jellis' (2008) focus group study of 128 students in Grade 3 shows that the students paid great attention to the provided illustrations in the mathematics textbook, but they found it difficult to determine if the information expressed in the illustrations was important or not when it came to solving the tasks. Sometimes the pictures were decorative, and sometimes they were essential for comprehending the task, and thus, the students had to decide this for themselves. Further, in Åberg-Bengtsson's (1992) study of 30 students in preschool, as well as in Grades 4 or 6, were studied to determine how they made meaning of diagrams and graphs. The results showed that the diagrams were sometimes confusing for the students to interpret correctly because they did not have the prior knowledge about how to read the graphs, which also led to wrong assumptions about the content being expressed. The importance of prior knowledge for comprehending a text is further discussed in next section.

2.2.3 Prior knowledge

According to Reichenberg and Lundberg (2013), prior knowledge means that the reader is familiar with the content of the text, which helps the reader to integrate new information to their prior knowledge structures to create meaning. It is particularly important for students to identify the main idea of particular sections or paragraphs in a

text. Identifying the main ideas helps the students to activate their prior knowledge (Palinscar & Brown, 1984) which can contribute to understand the relationship between what they already know and the new information being learnt (Carter & Dean, 2006). In Löwing's (2004) study, of students in Grades 4-9 Grades 4 to 9, and in Riesbeck's (2008) study, of Grade 5 students, the results showed that students have difficulties with connecting their everyday language to the mathematics language in the textbook. This is likely to interfere with students being able to activate their prior knowledge, which often is connected to the main ideas being expressed in texts in the mathematics textbook. However, to be able to grasp the content in a text the students except for decoding, language comprehension and prior knowledge need to use successful reading comprehension strategies for being able to reason, which will be further discussed in next section.

2.3 Reading comprehension strategies

Reading comprehension strategies are defined as a consciously controlled mechanism that a proficient reader may use to improve her or his skills to understand a text (Vellutino, 2003). These strategies can be viewed as building conceptual structures or structures to support the comprehension of a text (Westlund, 2013). They also include accepting responsibility for and analysing one's own learning and monitoring comprehension to ensure understanding (Vellutino, 2003), such as re-reading the text and performing the mathematical operation again. Strategies of this type are known as superficial-level reading strategies (Murphy & Alexander, 2002). Another form is deep-level reading strategies, which are related to identifying the main ideas and clarifying them (Murphy & Alexander, 2002). According to Pressley and Allington (2015), these deep-level strategies usually involve processes such as predicting, questioning and summarizing the content, which can be found in both the Reciprocal Teaching model and in Transactional Strategy Instructions.

To date, research has been limited in examining which comprehension strategies students use for reasoning (Shepherd et al., 2012; Österholm, 2007). However, in both Weinberg, Wiesner, Benesh and Boester's (2012) study with 1,156 university students and Shepherd et al.'s (2012) study with 11 university students showed

that several of the students focused only on the information in the exercises. They did not pay attention to the details in the expository text in the textbook to predict about algebra, discrete mathematics, calculus or in introductory statistics, respectively functions and graphs, which led to wrong assumptions about the mathematical content being expressed. Similar findings were found in Shepherd et al.'s (2012) study, where identification of the current main ideas concerning functions and graphs were an issue for several of the university students. The reason for this was that they identified that students were mainly concentrated on the information in the exercises and did not pay attention to the details in the passages (information text) presented on the pages, which led to wrong assumptions about the mathematical content (functions and graphs). Students focusing on the exercises were also found in Lithner's (2003) study when analysing college students' strategies, as they worked through a set of textbook calculus exercises and found that the students tried to identify similarities between the exercises to determine the main ideas. These results indicate that exercises in a textbook play a central role in supporting the understanding of the mathematical content being presented. Thus, the way information is presented and placed in the exercises in the context of the textbook seems, in this case, to have had a big influence on several students' reasoning competence. Further, Österholm's comparative study (2006) of secondary school students and university students showed that the students had developed a special type of reading strategy when reading a mathematical text which contained symbolisation. The study also highlights that when the focus turned into the operative meaning (of symbols) and not on their semantic role when the text involved symbols of numbers. Pimm (1987) also stresses this as an issue because, for many students, the language of mathematics is regarded as mainly the language of numbers. However, to reason in mathematics education in an appropriate way, symbols and numbers are not always enough; other sources, such as words and/or illustrations, also need to be taken into account both separately and together.

To summarize, the texts provided in mathematics are usually short and dense with few clues to decipher the mathematical content being presented. These kinds of texts put special demands on the students' reading skills, and both general reading skills and specific mathematics

reading skills are needed, where prior knowledge is essential for being able to reason. Further, to grasp the content in a text, successful comprehension strategies are needed, which previous research has shown not all students use when reading the mathematics textbook. The focus is on the exercises (what to do) and not on the explanatory texts telling them what to learn. However, these studies focus on older students' reading comprehension strategies. Nevertheless, in Grade 4, a correlation between general reading skills and reading skills connected to mathematical tasks has been found, and it further correlates with the students' mathematics skills, which appears to follow the students throughout their entire school lives. Consequently, explicit teaching about how to read to be able to reason seems essential and necessary to help students comprehend the mathematical content being expressed. Moreover, several researchers have stressed the need for explicit teaching about reading comprehension strategies to support students' comprehension skills, which is also supported in this study in regard to developing students' reasoning competence and is further discussed in next section.

2.3.1 Teaching comprehension strategies to support students' reasoning competence

In research in other areas, evidence has been provided that persons who comprehend well tend to use successful strategies more often and more effectively than those who comprehend poorly (e.g. Palinscar & Brown, 1984; Pressley 2000). Thus, it seems relevant to teach reading comprehension strategies to help the students embrace the mathematical content being presented in a text. When considering the implementation of a new strategy, the strategy needs to be explicitly explained to connect the strategy to what the students are meant to learn and also to explain why the strategy is important and when it can be used (Gaskins, 2003).

It is also argued that comprehension strategies are worth teaching because they can effectively support students' metacognitive strategies for reasoning (Gaskins, 2003; Lederer, 2000; Palinscar & Brown, 1984; Pressley, 2000; Westlund, 2013). When specifically applied to reading, the concept of metacognition describes the reader's skills to evaluate one's own comprehension level (Pressley, 2002). This can lead to a deeper kind of reading involving the analysis and critical thinking

of the text itself (Keene & Zimmermann, 2003), which is essential in the process of reasoning (Palinscar & Brown, 1984). Palinscar and Schutz (2011) point out that the goal of direct instruction in reading comprehension is mainly to develop the students' skills to become more effective problem solvers when reading texts. Another important aspect concerning teaching about reading comprehension strategies involves connecting the strategies to the content provided in the text (Palinscar & Schultz, 2011). This is also confirmed in McKeown, Beck and Blake's (2009) study of Grade 5 students where a comparison was made between teaching reading comprehension strategies that focused on the content in texts with teaching that focused on teaching various reading strategies to use when reading a text. The result showed that students who participated in content-focused teaching intervention showed a greater skill in comprehending the content, and these students also gave longer and more detailed answers than the students who participated in strategy-focused teaching. Thereby, according to McKeown et al. (2009) and Shanahan (2014), it is not a question of whether or not to teach reading comprehension strategies. Instead, depending on the text and the reader's needs, the reading strategies should be carefully chosen if the text is complex for the students to grasp (Palinscar & Schutz, 2011). However, according to Siebert and Draper (2008), previous content-area literacy instructions tend to mostly focus on activities, text and genres that deemphasize and misrepresent mathematics, such as historical and popular texts involving little mathematical content and few involve mathematical texts which differs from these texts.

However, in previous research in school mathematics, the reasoning strategies in the Reciprocal Teaching model (Huber, 2010; Quirk, 2010) and Transactional Strategy instructions (Borasi et al., 1998) have been used to support students' problem solving competence in school mathematics. In the next section, these studies are further discussed, as they can provide more insight into the present research study.

2.3.2 Reciprocal Teaching model (RT model) and Transactional Strategy instructions (TSI)

This model, involves the comprehension strategies of prediction, clarification, questioning and summarization (Palinscar & Brown,

1984), which were developed in the mid-1980s to support students in Grade 7 with reading comprehension problems.

Predicting involves students predicting about the content of a text, clarification involves clarifying elements of the text, such as concepts and words, questioning involves asking questions about the content of the text, and finally, summarization involves summarizing the content of the text (Palinscar & Brown, 1984). All of the strategies are recommended to be used and have been extensively used with successful results in other disciplines than mathematics with students in middle school (Lederer, 2000; Takala, 2006). Lederer (2000) conducted research on the use of RT in social studies with 128 students from rural New Mexico in Grades 4, 5 and 6, with a specific focus on students with learning disabilities. As the teachers taught and modelled the four comprehension strategies of RT, this significantly improved the students' skill to comprehend the content after the use of RT in the social studies classroom. Through scaffolding, the teachers provided support in using the strategies as students began to lead RT groups to comprehend a social studies text. For example, in Finland, Takala (2006) researched RT in Grades 4 and 6. All the students in the mainstream classes were given explicit instructions in the four RT strategies during a five weeks period. The result indicated that RT was beneficial for improving students' comprehension in social studies.

In mathematics, the RT model has been used to support students' reasoning competence to problem solving (Collen, 2011; Huber, 2010; Quirk, 2010) and proofs (Yang & Lin, 2012).

Huber's (2010) study of students in Grade 4 showed that the students who used the RT approach during eight weeks for mathematical problems received significantly better results in the pre-test and post-test compared to a control group that did not use the RT approach. A similar result was found in Quirk's (2010) Educational Design Research study of Grade 5 students who also used the RT approach to solve mathematical word problems where a five-stage process was applied and aligned with Polya's four stages (see, plan, do and check) during fifteen lessons.

In contrast, Collen's (2011) study of four Grade 5 classes involving 35 students showed that the application of the RT model to mathematical word problems did not contribute to a significant improvement concerning students' problem solving competence after comparing

the results with a control group where the RT model was not used. One reason might be that the strategies were only explicitly discussed during one lesson starting with the strategy of summarization. In the results of Yang and Lin's (2012) study in Taiwan with twenty-two Grade 9 classes, the RT approach showed that reading-oriented tasks facilitated the students' comprehension of geometry proofs more than doing writing-oriented exercises. However, the result was lower than expected. One reason might be that all four reading strategies were introduced at the same time, but the recommendation is to introduce the strategies one at a time (Palinscar, w.y.).

Transactional Strategies Instruction (TSI) involves the four strategies in the RT model, but they are also built on "thinking load" from the strategies where the teacher first tells how he/she understands the text and then the students and the teacher tell each other how they understand the text (Westlund, 2009). A graphical model is also included where the students are to paint a picture about the content presented. However, transactional strategies have not been extensively used in mathematics education research, and only one study (Borasi et al., 1998) has previously reported about these strategies in mathematics education. This study involved students in Grades 8–11 where the strategies of 'say something', 'cloning an author' and 'sketch-to-stretch' were introduced to help them learn mathematics. In 'say something', the students shared what they felt was confusing in addition to questions and feelings after reading a text. In the strategy 'cloning an author', the students wrote down important words on cards. In 'sketch-to-stretch', the students read a text and then drew what they gleaned from reading the text. The result of the study showed that the students understood the texts better when they used different reading strategies, and this also encouraged further investigations and understanding of the mathematics involved. However, the students thought that instead of the teacher collecting their cards, they should keep them as an aid when discussing the text.

In this study, the reading comprehension strategies in the RT model are in focus for several reasons. One reason is that the work with transactional reading strategies in the mathematics classroom focuses on much longer mathematics texts, which are in contrast to the mathematics text found in Swedish mathematics textbooks. Also, it focuses on older students (Borasi et al., 1998) and the reciprocal

teaching model has been used more extensively and with similar-aged students in mathematics and other topics with successful results. The RT model is also considered the baseline for developing the students' reasoning competence (Palinscar & Brown, 1984) and is a part of the conceptual framework of this thesis. To deepen the students' comprehension strategies and thereby their reasoning competence, writing activities connected to the reading comprehension strategies are suggested (Westlund, 2009), which is also supported in this study.

2.3.3 Writing to support students' reasoning competence when reading texts

When reasoning in written text, the information must be presented and structured in a way that the writer can understand it, which is a useful process for making sense of mathematics (Burns, 2004). The students' thoughts then become visible and provide opportunities for reflection (Keene & Zimmermann, 2003). The information also must be presented and structured in a way that the reader, who might not be present or involved in the text creation, can follow and understand without the context and interaction between text producer and reader (Meaney & Flett, 2006). Further, writing down thoughts makes it practical and possible to find them again and follow their evaluation, which is a prerequisite for a more effective thought process (Liberg, 2008).

Research by Pugalee (2004) of Grade 9 students in the U.S. showed that when the students worked with mathematical problems, the students who wrote descriptions of their thinking performed better than those who verbalized their thinking. Through writing, the students also produced more execution and orientation statements than the students who verbalized. Liljedahl's (2006) study in problem solving from two classes in elementary school and two classes in secondary school showed that when the students were asked to write a problem solving journal along with a reflective journal, the descriptions of the problem solving processes got richer and less visible aspects of problem solving such as insight, aesthetics and intuition were shown. The writing, according to the students, helped them keep track of their thinking and solutions. Further, Davidson and Pearce's (1992) research with Native American Students in Grades 7 and 8 who previously had been behind in mathematics showed that the students

became more involved with mathematics by writing explanatory texts and mathematical vocabulary words appeared more frequently. The most obvious benefit was the students' self-confidence. The students became motivated when they saw the purpose of mathematics and when it began to make sense to them. Another positive effect about writing was found in Baxter, Woodward and Olson's (2005) study of Grade 7 students where the mathematics teacher in the class found that by reading the students' responses, a more complete picture of what the students did and did not understand could develop. This also helped the teacher plan the lessons and integrate particular problems and strategies based on difficulties that students showed in their writing. She also found that the journals were a way for her to encourage students privately. However, Minson and Takeda's (2012) study of a Grade 5 class in Japan showed that the students had more difficulty to write down descriptions to the operations they employed in arriving at a correct solution to a geometry problem than to explain their reasoning. Another important aspect they noticed was that many of the students did not use mathematical terms that they had worked with when they described the operation.

Lundberg and Sterner (2006) and Gibbons (2002) stress the need to build up a dictionary in a subject to extend and clarify students' knowledge of words and concepts during the course of study. However, Barton et al. (2002) state that giving students vocabulary lists with definitions or asking them to look up the definitions is not enough for them to develop the conceptual meaning behind the words or to read and use the vocabulary accurately. Instead, the students need to have opportunities to explain the terms in their own words (Pimm, 1987) because when the students develop their own definitions, they have mental ownership of the terms and their understanding is not the result of memorizing the words in a book or from a teacher or classmate (Adams et al., 2005). Other positive effects of the students writing their own definitions were found in Davidson and Pearce's (1992) research with Native American Students in Grades 7 and 8 who had been underachieving in mathematics. The students became more involved with mathematics through writing their own definitions of mathematical words and concepts, which in turn, improved their performance in mathematics, with the mathematical terms being used more frequently than they would likely otherwise have. Thus, it seems

important for teachers to help students make connections between new vocabulary and prior knowledge and also provide multiple opportunities to engage students in meaningful ways to apply the vocabulary across situations (Bryant, 2005; Chinn & Ashcroft, 1998).

However, when implementing writing activities in school mathematics, it is important that the students understand the aim of the writing and to whom they write (Meaney, Trinick & Fairhall, 2009; Morgan, 1998) and that they realize that the main audience for their writing, in this case, is themselves, not the teacher in the role of an examiner (Morgan, 1998). In a study of two Grade 8 classes, Fried and Amit (2003) studied students' notebooks and examined their characteristics to determine the role they play in learning. It appeared that the notebooks in the classrooms were considered public objects that could be opened for inspection, and they only contained finished work, not private notes where students can record preliminary ideas and reflections. The students had a hard time understanding and appreciating the connection between doing mathematics in their heads and writing mathematics down. They believed that writing in mathematics was only meant to show the teacher that "you understand the material and that you did not just guess" (Fried & Amit, 2003, p. 106).

Meaney, Trinick and Fairhall's (2009) study with students from Grades 1–11 found that many of the students felt that calculations were their favourite type of writing in mathematics, and using words in mathematics was their least favourite type of writing. Further, in Teledahl's (2016) study, the result showed that the students' writing skills in school mathematics was limited, and they lacked opportunities to develop their writing skills to write in a way that better reflects their knowledge of mathematics.

It has been suggested that teachers need to train and develop the students to describe and explain (reasoning) the different steps in the solution to a task, and not simply write down the correct answer (Minsono & Takeda, 2012). Instructions, structures and guiding is then needed to visualize what reasoning in mathematics involves (Gibbons, 2009; Meaney et al., 2009), which in this study, refers to reading and writing in school mathematics. Scaffolding has then been suggested as an important tool to support students' language development, and scaffolding is also considered an essential part of the

Reciprocal Teaching model (Palinscar & Brown, 1984) and Systemic Functional Linguistics (Halliday & Matthiessen, 2004), which are the two parts of the provided theoretical framework in this thesis (see chapter 3). In the next section, scaffolding will be further discussed.

2.3.4 Scaffolding

Wood, Bruner and Ross et al. (1976) describe scaffolding as a form of adult assistance “that enables a child or novice to solve a problem, carry out a task or achieve a goal which would be beyond his [sic] unassisted efforts” (p. 90). This involves helping students move forward in what Vygotsky called the Zone of Proximal Development (ZPD), which is “*the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers*” (Vygotsky, 1978 p. 86).

To bridge this gap between the actual and the potential level depends on the resources or the kinds of support provided. The goal of scaffolding is to lead the learner to adopt the processes he/she is being helped to accomplish (Rogoff, 1990), which in this study refers to reasoning. However, the challenge of successful scaffolding is to find a balance between support and challenge so that support is provided through scaffolding and challenge is provided through the learner’s interest in completing the task (Roehler & Cantlon, 1997) when suitable instructions about implement the RT strategies are limited (Rosenshine & Meister, 1994). Further, few studies have been conducted that investigate the scaffolding of language in mathematics (Meaney, Fairhall & Trinick, 2012; Smit, 2013), which is essential for being able to reason. Researchers (e.g. Moschkovich, 2010; Schleppegrell, 2007), emphasize the need for more practical teaching studies in which suitable activities and tools are examined to support students in their learning of the language in a discipline. To achieve this, several researchers (see, for example, Gravemeijer & Cobb, 2006; Tjernberg, 2013) express the importance of researchers being present in the school environment, and in dialogue with the teacher, contribute to develop the teaching and activities. But few studies of this kind have been conducted to find suitable activities for developing students’ reasoning competence further. However, in Jitendra, Griffin, Deatline-

Buchman and Sczesniak's (2007) Educational Design Research study of 56 Grade 3 students in two heterogeneous classrooms, the teachers implemented scaffolding by utilizing the schema-based instructions of 'change', 'group', 'compare' to problem solving involving addition and subtraction tasks during 15 weeks. A pre-test and post-test were conducted where the results indicated that, for many students, especially those with learning disabilities, scaffolding instructions helped the students to apply the strategies to solve the different tasks. In contrast, the results also indicated that the high-performing students improved the least.

Further, Smit and Van Eerde's (2011) Educational design research (EDR) study of 22 multilingual students aged 10–12 found that the students had become significantly better of reasoning about line graphs. This result was after nine lessons and after the teacher had used scaffolding language connected to different tasks in a whole-class setting. Seven different strategies were enacted: reformulate students' utterances by using a more academic language, ask students to be more precise in their spoken language, repeatedly correct students' utterances, refer to features of the text type, use gestures or drawings to support verbal reasoning, remind students to use supporting materials (i.e. word lists) and the last strategy involved asking students how their written text can be produced or improved.

Eerde and Haijer (2009) conducted an EDR study in two multiethnic classes in secondary school, through which the aim was to make language more visible and audible in the classroom and thereby encourage interaction and language. The result indicated that additional materials focusing on the language of mathematics helped individual students to learn mathematics and the teachers' awareness about enriching their lessons through scaffolding language increased.

2.4 Concluding remarks

Reasoning in school mathematics is a competence that, according to several researchers, is deemed essential for learning school mathematics, which also can be found in several countries' curricula, including Sweden. However, the descriptions of everyday culture in Swedish mathematics classrooms does not fall in line with the research that suggests how reasoning and mathematics language can be supported. To reason puts certain demands on the students'

reading and writing skills, both concerning general and disciplinary literacy, and thereby, influences their reasoning competence. In several countries, and in Sweden in particular, reliance on the mathematics textbook is widespread, where a common practice is for students to work individually with their textbook. In Grade 4 in Sweden in particular this can be problematic because it is in this Grade that the texts become longer and more complex than before. Further, a significant correlation between reading skills and mathematics skills begins to appear in this Grade and seems to follow the students during their whole school lives. Therefore, teaching how to read these texts seems essential.

In previous research, the RT model has been shown as appropriate to implement to support students' reasoning competence in other subjects and in school mathematics to support students' reasoning competence to problem solving. However, suitable instructions, exercises and guidance for implementing these strategies are difficult to find. Several researchers express the need for more practical teaching studies in which tools and suitable activities are examined to support students in their learning of the language of a discipline. But what mathematic reasoning consist of are not clearly stated. In this thesis, an attempt to conceptualize mathematical reasoning by coordinating the RT Model with SFL into a conceptual framework is provided; which is further discussed in the next chapter.

3. THEORETICAL PERSPECTIVES

The aim of this chapter is to describe the theoretical framework used to answer the addressed research questions. Theoretical frameworks often involve applying a formal theory to explain phenomena, and they can be practical, theoretical or conceptual (Lester, 2005). The framework of this study is a conceptual framework. A conceptual framework is often inspired by various theoretical sources visited over time and in relation to the specific circumstances of the study (Eisenhart, 1991). This is a common approach in mathematics education not only for studying the improvement of teaching and learning in a certain setting, for example, but also for guiding the process of an educational design research (Prediger, Bikner, Ahsbahs & Arzarello, 2008).

In this study, the conceptual framework coordinates Palinscar and Brown's (1984) RT model with Halliday's (1993) SFL. The RT model is influenced by cognitivist, behavioural and sociocultural perspectives in the field of educational psychology (Palinscar, 2007), and within it, four comprehension strategies are addressed: prediction, clarification, questioning and summarization. The other model, SFL, is directed more towards socio-semiotic theory, where language is viewed as a system of meaning that can be analysed at the clause level to understand how contexts such as professional settings and classrooms are reflected in the participants' linguistic choices. SFL is a well-developed direction in socio-semiotics, where signs such as words, illustrations and symbols are considered to express meaning about the content of a text and how this relates to the social environment. In other words, the social context in which signs are used or where they are created and organised is essential for the meaning being

constructed. SFL is helpful in analyses of texts but cannot be used on its own to describe and suggest how teaching might support students' attempts to reflect on and develop their reasoning competence.

Therefore, in the present study, Palinscar and Brown's RT model has been coordinated with SFL to find ways of identifying and determining the process through which students develop an awareness of mathematical reasoning when reading and writing. In this study, the conceptual framework is not only used as a tool for analysis but also as a theoretical foundation when justifying the activities in the intervention, with Paper I being an exception (Segerby, 2016). Similar approaches have previously been used, for example, in Sjöblom's (2015) Educational Design Research study, where Alrø and Skovsmose's (2004) IC model is coordinated with Fuente's (2009) as a conceptual framework for designing and analysing activities for supporting secondary students' oral reasoning competence in mathematics problem solving.

In Paper I (Segerby, 2016), the students' perceptions about the kinds of notes they write in mathematics were examined. Britton et al.'s (1975) categories of 'transactional', 'expressive' and 'poetic' were used as tools in the analysis. In the second study SFL was used as analysis tool and in the other studies, students' reasoning were analysed by using the RT model coordinated with SFL.

In the next section, the theories in the conceptual framework will be clarified and discussed. Theories from the field of educational psychology are briefly introduced, as the RT model is derived from this field. In relation to this, the RT model is then introduced. Thereafter, socio-semiotics is briefly discussed in general before SFL is introduced. This is followed by how SFL and RT are coordinated; the aim is to explain this within a conceptual framework and show how socio-semiotics and educational psychology might serve as the backdrop where SFL and RT meet and connect for the purposes of the present study.

3.1 Educational psychology

The RT model was influenced by three theories in educational psychology: cognitivism, socioculturalism and behaviourism (Palinscar, 2007). In psychology, *Cognitivism* is the study of mental processes, such as how people perceive, think, remember, learn and

solve problems (Mandler, 2002). Knowledge is viewed as mental models or cognitive structures and learning means how humans take in, process and combine information (Barsalou, 1992). According to Bruner (1990), the initial aim of cognitivism was to focus on thinking and meaning making – how humans understand and develop knowledge. But later, the focus changed to how humans process information. For example, Collin and Smith (1982) focus on processing skills when comprehending a text. The two processes involve comprehension monitoring and the use of clues in a text to evaluate and revise hypotheses about current and future events. These are known as metacognitive strategies, from which the RT model is inspired. Metacognitive strategies are mechanisms that come from the reader's awareness of the cognitive demands, the purposes and the goals of reading the text (Vellutino, 2003).

The idea of metacognition applied to mathematical thinking in mathematics education was first introduced by Schoenfeld (1985), and certain studies emphasise metacognition. For example, Hannula (2001) connected the meta-levels of emotion and cognition, and Österholm (2006) examined students' metacognitive strategies when reading mathematical texts.

Collins and Smit (1982) were interested in finding the specific processes of visualising thinking skills but not the methods that could be used to teach students about mastering these strategies, which is an important aspect in the RT model. According to Palinscar (2007), the RT model is inspired by Vygotsky's (1978) sociocultural perspective, which built on the idea of teachers as experts helping the novice (student) to reach the Zone of Proximal Development (ZPD). Vygotsky (1962) argued that one's culture is the primary determining factor for knowledge construction and that learning occurs through this cultural lens by interacting with others and following the rules, skills and abilities shaped by our culture. A body of research uses this perspective in, for example, studies involving teacher-orchestrated discussions within whole-class settings or interactive student discussions within small-group problem solving (e.g., O'Connor, 2001; Riesbeck, 2008). The RT model also emphasises guided learning in social contexts as a key to developmental change, where scaffolding is viewed as an important aspect in the teaching (Palinscar, 2003). The RT model is also influenced by applied behaviour analysis (ABA), a direction in

behaviourism. In this direction, the process of systematically applying interventions is based on the principles of learning theory to improve socially significant behaviours to a meaningful degree. It is also used to demonstrate how the interventions employed are responsible for any improvement in behaviour and, if necessary, to make changes that improve their effectiveness (Baer, Wolf & Risley, 1968). In behaviourism, knowledge is viewed as constructing knowledge as a repertoire of behaviours, which focuses on the social significance of the behaviour studied (Skinner, 1976). Previous research in mathematics education which has used applied behaviour analysis has been limited apart from Neef, Nelles, Iwata and Page's (2003) study in which two students with learning disabilities were taught to facilitate the solution of arithmetic word (story) problems.

The RT model and ABA aim to make students aware of the factors that influence learning thus resulting in them appreciating their activity as a reader and the outcomes of this activity (Palinscar, 2007).

I make no claims to explain the background for RT in its entirety. The aim is to provide an understanding of how I have read and used the framework in order to construct a theoretically based teaching and to analyse student's strategies for reasoning in mathematics.

3.1.1 The Reciprocal Teaching model

The RT model involves the four comprehension strategies of prediction, clarification, questioning and summarization aimed at helping students understand the factors that interact with and influence their comprehension of a text and are taught as a mean for enhancing comprehension in a meaningful context, rather than an end in themselves (Palinscar, 2003). These four comprehension strategies provide a dual function which not only improve comprehension but also give an opportunity for the students to check whether it is occurring. By focusing on the processes requisite to successful comprehension, the students are provided with tools for learning independently. The teaching is then reciprocal, where the teacher and the students change roles and tell each other about the different strategies. Palinscar and Brown's (1984) recommendation is to implement the strategies one at a time.

Prediction concerns students predicting future content as well as testing and drawing inferences in a text. In advance, through a brief overview, the reader is to make assumptions or guesses about what

the text is about based on the title, headings and images (Palinscar, 2003). To do this successfully, students need to activate the relevant background knowledge that they already possess regarding the topic. The resources (e.g. words, illustrations, symbols) used for constructing this meaning do not have inherent meaning, but rather they need to be interpreted based on previous experience and in understanding the current situation which supports the students to link the new knowledge they meet in the text to their prior knowledge and thereby develop their knowledge (Carter & Dean, 2006).

Clarification is a strategy to explicitly explain words and/or phrases, visual representations and symbols in a text which are important to understand to grasp the content being presented (Palinscar & Brown, 1984). For example, measuring as well as figures such as squares and circles are essential concepts for understanding geometry.

Questioning involves students asking questions about the content to determine if they have understood it. Questioning increases students' awareness of a text's important ideas (Palinscar & Brown, 1984; Doolittle et al., 2006). When generating questions, the reader needs to identify what is important enough in the text to provide the substance for a question (Palinscar, 2003). Thus, through creating questions, the reader's own understanding can be tested (Westlund, 2009). Furthermore, when students formulate their own questions, they need to create logical connections. Different kinds of questions can be asked: They can be open, which generates many possible alternatives, or they can be closed, which only generates one answer (Sullivan & Lilburn, 2002). Closed questions require an answer or a response, such as those which involve a procedure like addition or subtraction. Open questions demand deeper thinking and involve more than simply reproducing a skill or recalling a fact (Sullivan & Lilburn, 2002).

Summarization relates to the major content which distinguishes important and less important information in the text. This involves the students' skill to control their own understanding of the text (Westlund, 2009). This strategy requires students to recall and organise the important ideas in a text (Palinscar, 2003). Further, "Summarizing provides the input to create a context for understanding the specifics of a text" (Doolittle, Hicks, Young, Triplett & Nichols, 2006, p.107).

The RT model is used to visualise the students' comprehension strategies when reasoning. But the RT model cannot provide

information about the students' linguistic choices and how the textbook context, classroom context and social context can all have an impact on students' reasoning competence, which SFL can contribute with which is a direction in socio-semiotics.

3.2 Socio semiotics

According to Saussure (1996), semiotics is “a science that studies the life of signs within society” (p.16), where sign is the fundamental concept. A sign is fundamental in the construction of meaning, and thus, a sign can be anything, including words, images, sounds, objects and gestures (Eco, 1979). Given that mathematics is regarded as the “quintessential study of abstract sign systems” (Ernest, 1997, p.1) and mathematics education is “the study of how persons come to master and use these systems” (ibid), semiotics seems relevant for this study because previous research raises issues about students mastering these sign systems (see chapter 2) and also because of my own personal experience as mathematics teacher.

However, Halliday (Halliday & Hasan, 1985) problematized how signs are often seen as isolated entities, and this resulted in the modification and expansion of semiotics from a study of sign systems to a study of semantic meaning where social processes influence how meaning is expressed. Thus, Halliday modified and expanded semiotics from the study of sign systems to the study of meaning where social processes influence how meaning is expressed; this type of study is known as socio-semiotic.

In this perspective, “meaning” refers to an active process generated through social interaction (Chapman, 1993). Within social semiotics, one will find the high regard for all aspects of language use where context and relationships are important within the functional setting (Halliday, 1978). An important aspect of social semiotics for understanding the role of language in school mathematics is that it allows for the understanding of both language and mathematics as resource systems – systems of possible ways of meaning (Chapman, 1993). Furthermore, language and mathematics can both be understood as semiotic systems for expressing meaning and systems for the construction of meaning, given that they are both resource systems for the creation of meaning (Lemke, 2003). Halliday further explains:

Language is being regarded as the encoding of a 'behaviour potential' into a 'meaning potential'; that is, as a means of expressing what the human organism 'can do', in interaction with other human organisms, by turning it into what he 'can mean'. What he can mean (the semantic system) is, in turn, encoded into what he 'can say' (the lexicogrammatical system, or grammar and vocabulary). (Halliday, 1978, p.21)

This study draws upon Halliday's SFL, a direction in socio-semiotics. Through this approach, I can study the students' linguistic choices when reasoning about the mathematical content as embedded in the context of culture, such as the use of the mathematics textbook, the teacher's and students' role. The aim of this is twofold. First, it allows for analysing the impact of the cultural and situational aspects on the mathematical meaning making, and second, it allows theory to be used as a tool for developing strategies to improve students' reasoning and reading competence. In previous mathematics education research, this approach has been a helpful tool for studying spoken and written texts in mathematics classrooms (Atweh, Bleicher & Cooper 1998; Bergwall, 2016; Herbel-Eisenmann, 2007; Morgan, 1998, 2006; Temple & Doerr, 2012; Wagner, 2012). In line with this, two directions in mathematics education apply SFL (Schleppegrell, 2010). One direction involves communication that appears in the classroom – between students and the teacher or between students – to negotiate a content (for example Atweh et al., 1998; Herbel-Eisenmann, 2007; Morgan, 2006). The other direction studies language use in mathematics (Bergwall, 2016; Meaney et al., 2012, Morgan, 1998; Schleppegrell, 2007). Both directions are used in this thesis: The first is used in the intervention (Paper V) and the second in Papers II–IV.

3.2.1 Systemic Functional Linguistics

SFL is a linguistic perspective, which views language as a resource that people use to accomplish a purpose through expressing meanings in context (Halliday, 1978; Halliday & Hasan, 1985; Halliday & Matthiessen, 2004). The goal is to explain, within any given linguistic and cultural community, what people can mean and how they use their linguistic resources to do this (Wells, 1994). The language is viewed as a system of meaning which can be analysed at the clause

level to understand how contexts, such as professional settings and classrooms, are reflected in participants' linguistic choices.

In Halliday's terminology, a text is any instance of a language that plays some part in a context of situation which means that every text is about something, is addressed to someone, and is based on the text structure. Furthermore, Halliday argues that texts are developed through the context of situation which is inherently surrounded by the context of culture (Halliday & Hasan, 1985).

Context of culture and context of situation

A situation is an instance of culture, and a culture lies behind all the types of situations that can occur (Halliday, 2007). Therefore, the context of culture and context of situation are not two different things, but rather the same thing seen from different points of view.

Context of culture is what goes on outside language – with the happenings and conditions of the world and with the social processes that are involved (Halliday & Matthiessen, 2004). It refers to the lifestyles, beliefs and value systems of a language community (Halliday, 2007) and involves certain assumptions and expectations. For example, context of culture can refer to how mathematics education is designed in Sweden with the use of textbooks and how that affects the teacher's and the students' roles. The context of situation is the “environment in which meanings are being exchanged” (p.12) in any given occasion. The context of situation is, according to Halliday, comprised of three elements: field, tenor and mode. Field is defined as “the total event, in which the text is functioning, together with the purposive activity of the speaker or writer; it thus includes the subject-matter as one element in it” (Halliday & Hasan, 1994, p. 22). Just as field is related to human experience and representations of the world (Halliday & Matthiessen, 2004), tenor refers to who is taking part; the roles and relationships among the participants (Halliday & Hasan, 1985). In addition, mode refers to what role the language has in developing the functions of text within the context (Halliday & Hasan, 1985) and involves which semiotic resources appear, such as illustrations, symbols and words, and how they jointly contribute to create cohesiveness.

The three elements of field, tenor and mode constitute the context of a text and “enable us to give a characterization of the nature of this

kind of text, one which will do for similar texts in any language”. They are materialised through lexico-grammatical choices which, in turn, are realized through sound and/or writing systems by including the three metafunctions: ideational, interpersonal and textual (see Table 1). Halliday (1989) argues that the concept of metafunction is one of a small set of principles necessary for explaining how language works, and in this case, to explain how meanings are expressed through language. Together, these three metafunctions create the register – the structure of the language.

Table 1. Relation of the text to the context of situation, inspired by Halliday’s table (Halliday & Hasan, 1985, p. 26) but modified to the present study.

SITUATION:		TEXT:
Feature of the context	(realised by)	Functional component of semantic system
Field of discourse (what is going on)		Ideational meanings Naming and processes (material, relational and mental)
Tenor of discourse (who is taking part)		Interpersonal meanings (personal pronouns, modality, imperatives, interrogatives)
Mode of discourse (role assigned to language)		Textual meanings (mode, such as illustrations, words and symbols, cohesive relations)

Halliday (1978) defines the mathematical register as:

A set of meanings that is appropriate to a particular function of language, together with the words and structures that express these meanings. We can refer to ‘mathematics register’. In the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes. (p.195)

This means that to learn mathematics, students need to grasp and use language in new ways to serve new functions. This does not only

involve learning new words but also new “styles of meaning and modes of argument and of combining existing elements into new combinations” (Halliday, 1978, pp. 195–196). Furthermore, Halliday (1993) asserts that “Language is the essential condition of knowing, the process by which experience becomes knowledge” (p. 94).

Thus, studying linguistics choices contributes to visualising how meaning is expressed through the three mentioned metafunctions: ideational, interpersonal and textual. These metafunctions can be used differently and focus on different aspects of the metafunctions due to SFL’s praxis which allows one to use parts of the SFL (Herbel-Eisenmann & Otten, 2011). In the following, I will describe how they are used in this thesis.

The ideational metafunction is related to human experience and is constituted by the field. Field is defined as “the total event, in which the text is functioning, together with the purposive activity of the speaker or writer; it thus includes the subject-matter as one element in it” (Halliday & Hasan, 1994, p.22). As noted in chapter 2, textbooks are often considered to contain dense language with specific terms, symbols or illustrations carrying much of the meaning. Therefore, in considering what kinds of linguistic choices are made, it is important to consider vocabulary as carrying meaning. In this study, the ideational metafunction involves examining what kind of everyday language, such as “three corner” (*trekant*) or mathematics-specific vocabulary such as “triangle” (*triangel*) that the students use. This involves examining the **naming** of objects that are relevant for the context (Halliday & Matthiessen, 2004).

The other part of the ideational-meta function in this study is **transitivity** – categories of processes that address experiences and are described as *material*, *existential*, *relational*, *verbal* or *behavioural*. The most common processes involve the material, relational and mental processes (Halliday, 1973). These three processes are also in focus in this study in relation to the analysis of what the students write they do when solving different tasks. *Material processes* refer to an activity that is happening or something that has happened. For example, “counting” or “adding” (I am adding the numbers 12 + 26.) *Relational processes* involve identifying or describing someone or something, and the verbs “be” and “have” are common when expressing relations. In mathematics, this can refer to the identification of mathematical objects, such as in “a triangle has three corners”. And

the third most common type, *mental processes*, refers to sensing; for example, what a person thinks, feels and believes, such as “I *think* that geometry is easy to understand”, which refers to human experiences constituted by the field.

Interpersonal metafunction involves participants and how semiotic resources enact “personal and social relationships with the other people around us” (Halliday, 2004, p. 29). The interpersonal metafunction is constituted by the tenor which refers to who is taking part, their role and status, and what role relationships obtain among the participants (Halliday & Hasan, 1985). The relationships among the participants can be visualised by examining the “voice” through identifying the use of **personal pronouns**, **imperative**, **interrogative** and **modal verbs** in texts (Herbel-Eisenmann, 2007; Morgan, 1996). Personal pronouns, such as ‘you’ and ‘we’, identify those considered to be the main and secondary actors in the text. Imperative verbs command listeners or readers, for example, to do something such as “draw”, whereas modality indicates the level of certainty associated with particular actions, such as “shall”. This approach which examines “voice” is used in this study to examine the interpersonal meta-function.

The **textual metafunction** is related to the construction of a text (in whatever form), and refers to the formation of whole entities that are communicatively meaningful (Halliday & Matthiessen, 2004) and constituted by the mode. *Mode* refers to what role the language has in developing the functions of text within the context (Halliday & Hasan, 1985). For instance, illustrations, text, and symbols are all examples of **different semiotic resources** with important functions in a mathematical text. The textual meaning also involves **cohesive relations** in the texts (Halliday & Hasan, 1985). In this study, cohesive relations in the text are examined by analysing the text in the mathematical textbook, the choice of the students’ reading strategies connected to the textbook, and how the texts in the students’ notes contribute to cohesiveness about the content being expressed. Kress and van Leeuwen’s (2001) framework is adopted to this study to be able to examining illustrations.

3.3 Coordinating the RT model with SFL

A conceptual framework can be connected in different ways, and a common approach is to combine or coordinate the frameworks

(Prediger et al., 2008). When theories are combined, different theoretical approaches are used to study a subject area but are not necessarily complementary or compatible (Wedge, 2009). Coordinating can be used when the elements from different theories are well-fitting, and they must be complementary or compatible (Wedge, 2009). The theories in the conceptual framework of this study are coordinated, and how they are coordinated is expressed in Table 2.

Table 2. A summary of how the parts of the conceptual framework are coordinated in this study.

Conceptual	Framework
Reciprocal Teaching Model	- Systemic Functional Linguistics metafunctions
Clarification: Understand and explain mathematics concepts and be able to use the concepts when explaining their thinking.	- Ideational (naming and material process (doing)) - Interpersonal (personal pronouns) - Textual (which semiotic resources appears and how they contribute to cohesiveness in the text)
Prediction: Predict about the content in a text.	- Ideational (naming relevant for the content, i.e. main ideas) - Textual (which semiotic resources appears and how they contribute to cohesiveness in the text)
Questioning: Construct questions (tasks) to the content being learnt.	- Ideational (naming and material process) - Interpersonal (personal pronouns, imperative, interrogative) - Textual (which semiotic resources appears and how they contribute to cohesiveness in the text)
Summarization: Summary the main ideas in a topic.	- Ideational (naming, material, mental (sensing) and relational processes (relations between objects) - Textual (which semiotic resources appears and how they contribute to cohesiveness in the text)

To connect SFL with other frameworks in order to understand social practices such as participants learning has been done in previous educational research (e.g. Holmberg, 2012; Morgan, 2006). Holmberg (2012) combines SFL with Activity theory and Lave and Wenger's notion of situated learning. Furthermore, Morgan (2006) suggests Bernstein's pedagogical discourse as a complimentary theory. In this study, SFL is coordinated with the RT model as a tool to make well-judged choices in the design/constructing phase and also a way to develop an analytical tool. To analyse the students' written texts, SFL and the RT model are used as an analytical tool in all the papers except for Paper I (Segerby, 2016) where the students' writing was categorised by Britton et al.'s definitions, transactional, expressive and poetic writing for examining what kind of writing that the students perceived that they wrote during their mathematics lessons. The reading writing examined in Papers I, IV–V, instead focused on the students' use and development of mathematical reasoning competence.

3.3.1 Similarities between SFL and the RT model

Sfard (2007) offers two metaphors; the acquisition and the participation perspective, for how knowledge is developed in society. The acquisition metaphor refers to human development, as a result of acquiring knowledge, which is related to biological capacities of an individual. The other metaphor, participating, refers to human development as a result from engaging in social activities with others. Here, both frameworks fall within the second perspective. Similarly, the consideration that language learning is integral and not separate from or prior to cognitive development concerns both of parts of the conceptual framework. Halliday (1975; 1978) stresses that:

In learning a language, the child's task is to construct the system of meanings that represents his own model of social reality. The process takes place inside his own [sic!] head; it is a cognitive process. But it takes place in contexts of social interaction, and there is no way it can take place except in these contexts. (Halliday, 1975, p.140)

This is similar to assumptions that reading comprehension involves cognitive as well as socio cultural aspects; both are essential for students learning language (Palinscar & Brown, 1984; Sweet & Snow, 2002; Westlund, 2013).

Further, the RT model also builds on an idea of teachers as experts, helping the students as novices by scaffolding (Palinscar, 2003). This is in line with Halliday's idea of the optimal learning environment: "A milieu that is child-centred but in which the teacher functions as a guide, creating structure with the help of the students themselves" (Halliday, 1978, p.210). Within the RT model, this is referred to scaffolding. In this study, scaffolding not only involves developing the students' mathematics register when reasoning by examining and implementing suitable activities connected to the Reciprocal Teaching comprehension strategies and SFL but also seeing that the teacher is informed and shown how reasoning can be expressed (see Table 2).

Another similarity is that both the RT model and SFL take behavioural aspects into consideration. The RT model aims to make students aware of how they can influence their learning and thereby appreciate their activity as reader. Through a SFL approach, this can be further examined through analysing the behavioural processes in the ideational metafunction.

3.4 Summary

In this thesis, a conceptual framework for reasoning in school mathematics is provided, where the RT model is coordinated with SFL. The RT model consists of the four reasoning strategies of prediction, clarification, questioning and summarization, which are considered the baseline for reasoning. *Prediction* involves predicting about the content being presented, which is essential for activating prior knowledge. In addition, *Clarification* involves clarifying words, *questioning* involves asking questions about the content being provided, and finally, *summarization* involves summarizing what is most important in a text.

The other part of the framework is Halliday's Systemic Functional Linguistics, where texts are considered as developed through the context of situation which, in itself, is surrounded by the context of culture. The *context of culture* refers to what occurs outside language, such as the events and conditions of the world and the social processes involved (Halliday & Matthiessen 2004), which in this study, refers to the textbook's role as well as the students' and teachers' roles. The *context of situation* is the "environment in which meanings are being exchanged" (Halliday & Hasan 1985, p. 12) and includes

three metafunctions: ideational, interpersonal, and textual, which are examined in the students' notes. The *ideational* metafunction is constituted by the field and refers to what is happening in the text. The *interpersonal* metafunction refers to the participants in the texts and is constituted by the tenor, and finally, the *textual* metafunction is constituted by the mode and refers to how the text is presented.

By coordinating the RT model with SFL, the students' strategies for reasoning could be visualized by the RT model, and SFL contributed to the analysis of the students' linguistic choices in their texts. However, according to previous research, suitable activities when enacting these comprehension strategies in the RT model are difficult to find (Rosenshine & Meister, 1994). Therefore, there is a need for more research that details effective instructions to support students' in their learning of the language of mathematics (Schleppegrell, 2007), which in this study, involves the students' reasoning competence. In this thesis, Educational Design Research (EDR) is conducted to examine suitable activities to support students' reasoning competence when reading and writing texts connected to the text provided in a Grade 4 mathematics textbook. To frame the activities in the EDR study, the provided conceptual framework is also used. In the next chapter, the methodology will be presented where EDR is further discussed.

4. METHODOLOGY

Earlier studies have identified the lack of, and need for, approaches in addressing practical problems, developing solutions and accomplishing valuable goals in educational research (e.g. Carlgren, 2011; McKenney & Reeves, 2012). There are a variety of methodological approaches where theory and practice interact, for example, learning studies, action research and educational design research. When comparing these approaches, it is apparent that at their core is a process of predicting and reflecting on developing solutions to practical problems. Further, these approaches do not only aim to produce effective lessons but also involve teachers in understanding why and how learning processes are developed in the context of the classroom (Carlgren, 2011).

There are also differences between the approaches. Within action research, the teachers identify a problem area. A plan is settled where the change process is the result of not only consultation with a supervisor and colleagues but also inspiration from relevant literature and lectures. The teacher tries new approaches and thus alters their practice, which is systematically documented and discussed with a researcher; however, the researcher does not participate in designing the activities and analysis (see, for example, Rönnerman, 2008). In learning studies, the teachers (suggested in a team teaching) and researchers in collaboration agree on a learning object related to the curriculum to focus on, and theory of variation is common to use. Initially, one lesson is conducted, which is revised in other settings (see, for example, Holmqvist, Gustavsson & Wernberg, 2007; Runesson, 2010).

Another approach is Educational Design Research (EDR), which requires the researchers and teachers to collaborate in planning, implementation and evaluation in a specific setting (see de Beer, Gravemeijer & van Eijck, 2015; McKenney & Reeves, 2012; Prediger, Gravemeijer & Confrey, 2015; Van Eerde & Hajer, 2009). Within EDR, activities can be tested and revised (Nieveen, McKenney & Van den Akker, 2006). The research process of the present research project is framed within the structure of Educational Design Research. The basic motive for using this approach is that the type of language-oriented mathematics education involving students' reasoning could not be found in already existing classroom settings, as discussed in chapter 2. Further, professional learning is strongly shaped by the context in which the teacher and students interact. This is usually the classroom, which in turn, is influenced by the wider school culture, meaning the curriculum, the educational context, and the community and society in which the school is situated (Timperley, 2008). When trying to change some aspects of the teaching practice, teachers' daily experiences in their practice context shape their understandings, and their understandings shape their experiences. It is not until the teacher sees evidence that students have gained new knowledge and/or skills that they can be convinced about the new way of teaching (Timperley & Alton-Lee, 2008). This evidence can take a long time to produce and I had the possibility to be in a class for one school year. Therefore, in the present study, an EDR study seemed appropriate to conduct. This is because it would not only facilitate engagement in a collaborative design process with teachers to produce suitable activities that potentially could support students' reasoning competence but also the methodological setting to implement, evaluate and redesign the activities used in the classroom settings until a satisfactory result was achieved. A satisfactory result in the present study can be understood as the students' achieving the competence to reason in an appropriate way (e.g. using their mathematical register in an appropriate way when reading and writing texts), which is further discussed in Paper V. The timescale of the EDR in this study was one semester (15 weeks) and involved collaborating with a Grade 4 mathematics teacher. In the next sections, the Educational Design Research methodology is further discussed along with the stages of its actual use in the present study.

4.1 Educational Design Research (EDR)

EDR addresses educational problems in real-world settings, and it has two primary goals: to develop knowledge and to develop solutions (McKenney & Reeves, 2012). According to Plomp (2007) Educational Design Research is:

“the systematic study of designing, developing and evaluating educational interventions (such as programs, teaching-learning strategies and materials, products and systems) as solutions for complex problems in educational practice, which also aims at advancing our knowledge about the characteristics of these interventions and the processes of designing and developing them” (p.13)

EDR is especially useful for understanding the underlying reasons why something is happening, or the conditions under which a particular occurrence or interaction could happen. It focuses on understanding the messiness of real-world practice, with context being a core part of the story and not an extraneous variable to be trivialized (Barab & Square, 2004). This means letting theory and practice interact in a specific setting when identifying and improving practical problems. The activities are implemented through a number of so-called cycles in which an understanding of how to design for implementation is developed (McKenney, Nieveen & van der Akker, 2006; McKenney & Reeves, 2012).

Collaboration between researchers and practitioners is not only essential, but even required, in the identification of real teaching and learning problems and to understand the practice and how the context influence the learning (Barab & Squire, 2004). The initial contextual steps undertaken in the present study are presented in sections 1.2 and 4.2.

EDR builds on theories of both the process of learning and the means designed to support that learning (Cobb et al., 2003). In this study, this is fulfilled through applying a theoretical framework coordinating SFL and RT to conceptualise mathematical reasoning.

There are three levels of educational theories that can be produced by EDR: local theory, middle-range theory and high-level theory (McKenney & Reeves, 2012). Local theory is produced when limited manifestations of a certain phenomenon are studied and the purpose is to describe, explain, predict or prescribe a certain phenomenon.

The theory consists of conjectures about a possible learning process together with conjectures about possible means of supporting that learning process (Gravemeijer & Cobb, 2006). Middle-range theory is developed when multiple manifestations of an intervention are studied in several settings, and it can be used to explain, describe or predict phenomena. High-level is based on sets of assumptions, on paradigms and epistemologies, and the theory can be used to explain, describe or predict certain phenomena (McKenney & Reeves, 2012). In the present study, a local theory can be developed because the research is limited to one class and related to a specific context, which is further discussed in 1.2 and 4.2.1. Nevertheless, based on descriptions from the study, teachers can make decisions about what is transformable to their practice. The descriptions need to involve both design features of the intervention and the impact of these features on learning and participation, but the descriptions also need to visualise the context and emerging theory (Barab & Squire, 2004; Ruthven, Laborde, Leach & Tiberghien, 2009). In this study, it is done through describing how activities that complement each other (connected to the RT model) were used to support students' reasoning competence and how these practices can be understood with help from SFL's notions of context of culture and context of situation. Specifically, four field-based studies (see Paper I-IV) were also conducted to examine the context of situation and context of culture before the intervention took place.

Another purpose of EDR is to develop new forms of learning, which consist of a theory about the students' learning process for a given topic and theories about the means of supporting that learning process; this is also described as the rationale that underpins an instructional sequence on a given topic (Gravemeijer, 1998). The intent is to examine the possibilities for education improvement by bringing new forms of learning in order to study them.

Furthermore, EDR is iterative, which means that conjectures are adapted or possibly even refused and new conjectures are developed and tested. This involves scrutinising and improving the initial design by testing and revising conjectures as informed by ongoing analysis of both the students' learning and the learning environment (Cobb et al., 2003).

Three parts are identified in EDR: analysis and exploration, design and construction, and evaluation and reflection (McKenney & Reeves, 2012), see figure 2.

Figure 2. Model of EDR and its phases (McKenney & Reeves, 2012, p.77).

In next section the different phases within an Educational Design Research study is presented and connected to the present study.

4.2 The present study's approach

I have chosen to name the first phase to only “Exploration” because the analysis to the design is provided in the third part “Evaluation and Reflection”. There is analysis in the first phase too but this analysis helps to constitute the starting point since exploration involves problem identification and diagnosis. The second phase “Design and construction” provide solutions to the problem. Theories are used to justify the choices made in the design/construction phases for the intervention and finally evaluation and reflection involves evaluation the intervention and reflections about the results. Hence, recurring planning, implementation and reflection have occurred.

4.2.1 Exploration – starting point

Exploration phase is often the most extensive, because there is a need to have a starting point. The goal of this first phase is to characterise the instructional context, establishing baseline performance or conditions prior to introducing the intervention and to identify factors that enhance and inhibit movement towards a specified pedagogical goal (Reinking & Bradley, 2008). Three main activities structured the starting point: *a literature review, initial orientation and field-based investigation* (McKenney & Reeves, 2012). Drawing on and synthesising the prior research literature is essential when identifying ideas central to the design of the study (Gravemeijer &

Cobb, 2006). How exploration was conducted in this thesis initially will be presented in the following section in a chronological order.

Initially, I started to explore research concerning reading and writing in general but also connected to mathematics, educational design research, mathematics education (both national and international), mathematical literacy, mathematical reasoning, primary school, Reciprocal Teaching, Systemic functional linguistics. The keywords were used individually and in combination. Different databases, such as Eric via EBSCO, ERC and Mathematics education and Google scholar were used to retrieve peer-reviewed articles, but other relevant literature were also taken into consideration, such as handbooks and conference papers.

The results from the literature review are presented in section 1.2 and in chapter 2. From this literature search, it became clear that students' opportunities to develop their reasoning competence is scarcely when reading and writing texts in school mathematics. Further, to teach about language along with mathematics is, according to mathematics teacher, an issue since they consider that they lack knowledge in this area and suitable activities are difficult to find. In Sweden, this is especially problematic in Grade 4 for several reasons concerning reasoning in mathematics. One reason is that the texts are considered to become longer and more complex than before (Myndigheten för skolutveckling, 2008) and a correlation between students' mathematical skills and reading skills begins to appear from this Grade in Sweden (Möllehed, 2001). Another issue to address is that the students' general reading skills, which is also essential for reasoning, has decreased in Grade 4 since 2006, according to the result of the Progress in International Reading Literacy Study (PIRLS) in year 2011 (Skolverket, 2012a). Four field-based studies were conducted to frame the intervention. The role of field-based studies is to portray the setting and other relevant factors (Gravemeijer & Cobb, 2006; McKenney & Reeves, 2012). The first field based study is discussed in next section.

Field-based study 1 (Segerby, 2016) - Questionnaire to examining students' perceptions about their writing in school mathematics

This study aimed to study how the new curriculum, Lgr 11 (Skolverket, 2011), might have influenced the context of the culture in mathematics, since this curriculum stressed the importance of language and reasoning. A second aim was to identify in what students' age and Grade it would be appropriate to conduct the intervention study. Given the question set, a survey study was conducted involving a postal questionnaire with 300 students in Grade 3, 4 and 5 and it was the Swedish government personal address register that randomly had selected the students. 100 from Grade 3, 100 from Grade 4 and 100 from Grade 5 students were invited to participate from all over Sweden and 136 students responded. When randomly selected samples are used, every unit in the target population has then the same opportunity to participate (Bryman, 2008).

The questionnaire survey contained an information letter (see Appendix 1) and 13 statements or questions (see Appendix 2). The questionnaire had three different parts; the first examined students' attitude to mathematics, the second concerned the design of the students' mathematics education and the third part involved students' perceptions about what they write during their mathematics lessons and the purpose of keeping those notes. All questions in the questionnaire except for one had closed-response answers.

In February 2012, permission from the Swedish Tax Agency (Svenska skattemyndigheten) was gained to collect randomly selected addresses of the female guardians of 100 students born in 2000, 100 students born in 2001 and 100 students born in 2002, a total of 300 students. To use a randomly selected sample provided me with information about experiences from many classrooms in Sweden.

Pre-testing the questions in the questionnaire is crucial to its success (Bryman, 2008). Therefore, before sending the questionnaire, a pilot study involving 15 students in Grade 3 was conducted to examine how the youngest students were likely to reply to the questions and statements. First, the students answered the questionnaire individually and then a discussion in groups of four was conducted. From this pre-test, some of the questions/statements were reformulated and for some of the questions the number of response alternatives were expanded.

At the beginning of April 2012, the postal questionnaire with individual information letters to the parents and the children was sent out, see Appendix 1. After one reminder letter, 136 students replied; 50 answers from Grade 3 students, 40 answers from Grade 4 students and 46 from Grade 5 students. Statistical Package for the Social Sciences (SPSS), a software for compiling and analysing quantitative data, was used as analysis tool (see Segerby, 2016). However, the focus of my research has been refined as a result of the questionnaire's findings, and the first and second parts were left out of this study since they did not focus on examining students' opportunities to reason. Thereby, this paper only reports on the analysis of the third part, which concerned the students' perceptions about their writing and the aim for keeping these notes in mathematics. Departing from literature review and the results from this field-based study, I decided to conduct the study in a Grade 4 class.

Getting in contact with the teacher

In autumn 2012, during a seminar in mathematics about students' writing, I got in contact with a teacher that expressed that he wanted to learn more about how to work with the language in mathematics, because he considered that to be an issue for many students and he addressed that he wanted to participate in my research. The goals for the intervention were then set up after several meetings with the teacher during the school year 2012/2013. In this study, the goal involves developing students' reasoning competence when reading and writing in mathematical activities. Furthermore, the different roles were also discussed, settled, and clarified before the intervention took place. Together we, I and the teacher, should design and evaluate the activities, the teacher teach and integrate the activities and I analyse the results and the mathematics textbook should initially be the base material. The teacher and I, also agreed that when I was in the classroom I would not take an active part in teaching, but if the students initiated interacted with me I could respond to the students and help them.

A second field-based study was conducted to examining potential issues for reasoning when reading the mathematics textbook, which is presented in next section.

Field based study II – Analysis of a textbook page
(Ebbelind & Segerby, 2015)

This study uses empirical data from two empirical studies with different aims but interrelated aims. The first study, which is directly connected to the intervention, concerns the analysis of a page in a widely used, Swedish mathematics textbook *Matte Direkt Borgen* (Andersson Picetti & Sundin, 2003, p. 13) for Grade 4 students (age 10) in the area number sense. The focus was to identify potential difficulties students may face when reading the page for reasoning but also examine which opportunities there are for the students to reasoning when working with the page. As analysis tool Systemic Functional Linguistics metafunctions; ideational, interpersonal and textual metafunctions were used (see section 2.4.3 for a deeper descriptions of the metafunctions).

This study was expanded to analyse the whole number sense chapter, which contains totally 374 exercises (Segerby, manuscript in process)

Towards mapping of the hindrances for reasoning in school mathematics in Grade 4, I needed to find out how the students read mathematics texts in their textbook and thereby closer examine the potential problems addressed in this study. Field-based study III and IV were conducted in the class where the intervention should take place.

Field based study III- Interview study concerning six Grade 3 students' reading strategies when approaching a textbook page in mathematics (Segerby, 2014)

In this study, different reading comprehension strategies were studied from letting six students, with different achievement levels in mathematics, were interviewed while reading one page in the mathematics textbook for Grade 4. Five questions were provided, connected to the three reading comprehension strategies prediction, clarification and summarization and as analysis tool the RT model was used which was coordinated with SFL to visualize students' mathematical reasoning and identify similarities and differences between the students when reading the text.

A video recorder and dictaphone were used to document the interviews, which were conducted individually, in approximately fifteen minutes. All of the interviews were transcribed by Transana, which is a transcript tool for quantitative data.

The analysis tool was the conceptual framework where the strategies; prediction, clarification and summarization, in the RT model was coordinated with SFL (see Table 2).

Field based study IV- Questionnaire study to examine Grade 4 students' reading strategies and writing skills when approaching two textbook pages in mathematics (Segerby, submitted)

As a further and larger research of the two studies; Ebbelind & Segerby (2015) and Segerby (2014), a questionnaire study with six questions was conducted to examining eighteen Grade 4 students' reading and writing skills (reasoning skills) when approaching two pages in the mathematical textbook to examining both their reading and writing skills when reasoning about number value.

The analysis tool, the RT and SFL, is the same as in paper III (see Segerby, 2014).

Summary of the four field-based studies.

The four field-based studies include both quantitative and qualitative data, which is common in Design research (Plomp, 2007; Reinking & Bradley, 2008). The first two field based studies (Segerby, 2016; Ebbelind & Segerby, 2015) examined reasoning in a broader view and the third (Segerby, 2014) and fourth (Segerby, submitted) explored reasoning in the specific setting where the intervention should take place.

The two first field-based studies are concerned with the exploration of the context of culture and involve reading and writing in school mathematics activity from a broad perspective when reasoning. The two later studies focused on exploring in more detail the comprehension strategies of the students in the class when reading and writing texts when reasoning in school mathematics before the intervention took place. The results showed that there were limited opportunities for the students to develop their reasoning competence, that specific reading and writing strategies were needed for reasoning and that all students needed to develop their reasoning competence, but how varied. A short summary of the four field based studies is followed in Table 3 and in the text below. To read more about each field-based study, see section 5.1 - 5.4 and the respective papers.

Table 3. Summary of the four field-based studies.

Study	Aim of the study	Participants and data collection	Analysis tool
I	Examine Grades 3-5 students' perceptions about what they write in mathematics and what they consider the purpose for keeping those notes to be.	A total of 300 students in Grades 3–5 were invited to participate by answering a postal questionnaire study (spring 2012).	Expressive, transactional and poetic writing.
II	Identify potential issues for reasoning when reading the mathematic textbook.	Analysis of a page in a Grade 4 mathematical textbook (spring 2013).	SFL (ideational, interpersonal and textual metafunction).
III	Examine six students, considered high, middle and low achievers, reasoning (comprehension) strategies when approaching a page in a mathematical textbook.	Interview study with six Grade 3 students involving prediction, clarification and summarization in the RT model and analysis of the textbook page (spring 2013).	SFL (ideational, interpersonal and textual metafunction) and Reciprocal teaching model (prediction, clarification and summarization).
IV	Examine students' mathematical reasoning which involves both their reading and writing skills when approaching two pages in a mathematical textbook.	Questionnaire study to investigate 18 students' reading and writing strategies when approaching two pages in the mathematics textbook (autumn 2013).	SFL (ideational, interpersonal and textual metafunctions) & Reciprocal teaching model (prediction, clarification and summarization).

In EDR studies, the so-called initial orientation also involves identifying the context and the participants, which is presented in next section. Considering the context is particularly significant when considering attempts to create changes in educational settings (Wells, 1994).

Participants in the study

The intervention study involved one teacher and a Grade 4 class. The teacher had worked as a mathematics teacher for fifteen years in Grades 4–9 at three different schools in lower socioeconomic areas of a large city in southern Sweden. When the intervention began, the Grade 4 class was new to the teacher. During the intervention, the teacher was the class mentor and taught the class in the subjects of science and mathematics. He also taught two other classes at the school in the same subjects (one Grade 5 class and one Grade 6 class). The school where the teacher was located is in a small town in the south of Sweden. At the time when the study was conducted, five classes were located at the school: two in Grade 4, two in Grade 5 and one in Grade 6.

In the Grade 4 class, all of the students were between nine and ten years old, and according to the teacher, several of the students have a low socioeconomic background. Eighteen of the twenty-two students in the class had, from the beginning, agreed to participate in the intervention. All the students except for one have Swedish as their first language. During the intervention, three of the students handed in their notes sporadically, which made it difficult to follow their development. As a result, the number of participating students was reduced to fifteen.

The following section details the context of culture. In the class of the present study, the context of the culture needs to be taken into consideration when designing the intervention because it plays a significant role in framing the intervention (Figueiredo, van Gale & Gravemeijer, 2009).

Context of the culture

According to the students in the class, before Grade 4, the mathematics lessons had mainly involved the students' working individually with their textbooks. The exercises aimed to prompt the students to perform procedures. Whole-class and group discussions rarely took place, and tasks in mathematics were supposed to take, at most, ten seconds to complete. Preferably, the answer should be short and should usually

include a number. Furthermore, their knowledge in mathematics was evaluated by tests and diagnoses, thus reinforcing the idea that how far along the students were in the mathematics textbook indicated how good they are at mathematics.

In this study, the first part worked as a baseline for constructing the design of the implementation of the comprehension strategies to support students' reasoning competence when reading and writing texts, but also a base line for two other interventions where the aims were to develop students' evaluation and problem solving competence. However, these two last interventions involving students' evaluation and problem solving are not a part of this thesis because they focused on other aspects than the comprehension strategies in the RT model and SFL. From this phase a local instruction theory, which in this study involved the RT model which is coordinated with SFL, is conducted where conjectures about a possible leaning process, together with conjectures about possible means of supporting that learning process is anticipated (Gravemeijer & Cobb, 2013). In this study, the design involves the implementation of the reading comprehension strategies; prediction, clarification, questioning and summarization, in the RT model aiming to support students' reasoning competence, which involved three areas; number sense, 'addition and subtraction' and geometry that the class worked with during a period of 15 weeks.

4.2.2 Design and construction

The design experiment consists of cyclic processes of thought experiments and instruction experiments, see figure 3.

Figure 3. Cyclic processes of thought experiments and instruction experiments (Gravemeijer & Cobb, 2013, p. 81).

This part of the study aims to create prototype solutions, based on existing design principles regarding planning, testing and refinement. The prototype should not be seen as a static model, but as in the making until satisfactory outcomes have been reached (Reeves, 2006).

Initially, we implemented the comprehension strategy clarification to support students' reasoning competence when writing because to clarify concepts was an issue for most of the students' according to the results from the field-based studies; where mathematical concepts relevant for the context barely appeared, which refers to SFL' ideational metafunction, and their explanations were usually short and not cohesive, which from SFL's perspective refers to the textual metafunction (Segerby, submitted). This involved students being able to understand and describe mathematical words and use them to explain different solutions to mathematical tasks. A dictionary and whole-class discussions were introduced into the students' mathematical teaching in order to facilitate their reasoning. In collaboration with the teacher, I identified difficult common mathematical concepts, which were digit, number, unit digits, tens digits and hundreds digits. The teacher and the students jointly defined the chosen concepts on the whiteboard and the dictionary was handed out to the students as homework. The students were then asked to describe some of the concepts and also perform tasks where they were asked to explain how they constructed different numbers, for example the highest number of using four digits. The students could describe the concepts by imitating part of the dictionary but they did not use the concepts from the dictionary when they explained their solutions to constructing different numbers, which refers to SFL's ideational metafunction. Other complementing activities were implemented, for example different games with dials for supporting students' understanding of place value. The strategy clarification became revised to except for continuing building up the dictionary with new concepts, for example odd and even number, number line and size arrange, exercises were provided were students were asked to clarify the provided concepts with own explanations to try to make the students' notes involve several semiotic resources, which refers to SFL's textual metafunction. The teacher now by scaffolding showed how they could clarify mathematical concepts by clarifying size arrange by providing an example and explanatory text to the example. He also highlighted the importance of using several semiotic resources to visualize their understanding, which refers to

SFL's textual meta-function. The first chapter about number sense ended with the students using the reading comprehension strategy summarization, which was new for them. The teacher asked the students to summarize number sense but no specific guidance was provided by the teacher. This involved students to identify main ideas in the topic (the concepts from the dictionary), and refers to SFL's ideational meta-function, but almost none of the students were able to identify some of the mathematics concepts when summarizing and in collaboration with the teacher a mind-map was constructed as a tool to summarize number sense.

In the area, 'addition and subtraction', the strategy prediction was introduced for the students which involved identifying main ideas on different pages in the mathematical textbook, which refers to SFL's ideational metafunction. The teacher highlighted the importance of identifying the main ideas and the teacher gave the students explicit oral and written instructions about the importance of the heading and information in the information box. Initially, in collaboration between the teacher and the students the main ideas were identified and later the students performed this by themselves. The students were also initially asked to write down something about addition and subtraction as a preparation for clarifying addition and subtraction, new concepts for the dictionary. At this point, clarification was modified to except for clarifying mathematics concepts also involve students clarifying their solutions to addition and subtraction tasks. This was difficult for the students and here the teacher's scaffolding again become important where he clarified and discussed with the students the use of naming relevant for the context (SFL's ideational metafunction) and the use of several semiotic resources (SFL's textual metafunction) and again provided addition examples with explanatory text. At this point, we changed the recipient of the students' notes – from the teacher to a fictive Grade 3 student – in order to support the students' reasoning competence involve naming relevant for the context and how they performed the tasks (ideational metafunction), involve several semiotic resources (textual meta-function) but also to use personal pronouns (interpersonal meta-function).

For the second time the students used the strategy *summarization*. Through this step, I would be able to study if there was any progression in the students' reasoning concerning identify main ideas

(SFL's ideational metafunction) and the use of semiotic resources (SFL's textual metafunction) when summarizing. During this chapter, complementing exercises where the students were asked to perform other addition and subtraction tasks (procedures) were included and specific exercises for reasoning needed to be conducted during two of the five weeks, but these exercises also facilitated the students' reasoning competence.

In next topic, geometry, the students worked with all of the four comprehension strategies for the first time. Initially, the students were asked to write down (summarize) what they knew about geometry before starting to work with the area to find out about the students' prior knowledge in this topic. The students then continued with the strategy *prediction* during the whole geometry chapter and complemented the dictionary with the concepts: distance, measurements; m, dm, cm and mm, figures such as triangle, square, rectangle and circle, and perimeter. During the geometry chapter, the students were also asked to formulate a question after reading and working with the week's pages. This strategy – conceptualized as questioning in the RT model – was new for the students. As well, the comprehension strategy *clarification* was expanded – added to the dictionary was to answer their own questions. Practically, this meant that the students started with writing down their questions and answers individually, and after that communicating the same in groups. The students' questions and answers were then discussed during a whole-class discussion. Initially, the teacher talked about what a question is but no specific guidance was provided and in the beginning several of the students' questions did not relate to the mathematics content (SFL's ideational meta-function) being expressed on the pages. The teacher then highlighted the importance of the question should involve the mathematical content (ideational meta-function) next time the students should provide a question and to use the information they got when predicting about the content on the pages. To extend the students' questioning skills the teacher talked to the students about their construction of the questions and instead of asking questions the teacher asked the students to construct a task on their computer. The use of imperatives and interrogative (SFL's interpersonal metafunction) were then discussed. Further, in this topic the students for the first time worked in pair to clarify mathematics

concepts; such as square, triangle and rectangle, before the whole-class discussion where the new concepts were clarified. Further, group discussions were also conducted where the students discussed their questions and prediction before whole-class discussions. During the whole-class discussion the importance of using several semiotic resources (SFL's textual metafunction) was discussed. The geometry chapter ended with the students having to identify and summarize the main ideas in the geometry by constructing a mind map.

To read more about the activities and result of the intervention, see section 5.5 and paper V (Segerby & Chronaki, submitted).

4.2.3 Evaluation and reflection

Evaluation and reflection according to McKenney and Reeves (2012) include thoughtful consideration of what has emerged in both research and development (including theoretical inputs, empirical findings and subjective reactions) in the intervention.

4.2.3.1 *Evaluation*

Evaluation consists of data collection and analysing the findings (McKenney & Reeves, 2012).

Data collection

During the intervention, the empirical data mainly consisted of the students' notes and observation notes from the lessons, when we mainly worked with reasoning. The observation notes were kept in a Word document during the fifteen weeks. The classroom observations gave valuable information about how the new approach during mathematics lessons was received by the students and involved examining SFL's context of culture. An interview with the students was also conducted during the intervention, but since the students were not interviewed continuously, this data is not included in the thesis, but was used to inform the study.

The students' notes were collected, usually once or twice each week during the fifteen weeks, except for one week in number sense and two weeks in addition and subtraction where no specific notes for reasoning were collected. Other notes involved other complementary exercises, such as addition and subtraction tasks, problem solving tasks and

homeworks were collected. These notes were not copied and analysed by the conceptual framework, as they did not involve the specific tasks for reasoning, but they were important as a complement to determine how the students had interpreted the provided mathematical content. The notes that were analysed concerned the specific tasks involving reasoning in the topics number sense, addition and subtraction and geometry. In table 4, the collection of transcripts in each topic are presented.

Table 4. Collection of the students’ notes concerning reasoning in the different topics.

Topic	Collection of transcripts concerning reasoning
Number sense	Clarify digit (clarification) Construct the highest and lowest number of four digits and explain how (clarification). Clarify size arrange (clarification) Clarify number line (clarification)
Addition and subtraction	Predicting about addition and subtraction (predict) Explain different solutions to addition and subtraction I (clarification) Equal sign (clarification) Explain different solutions to addition and subtraction tasks II (clarification) Summarize addition and subtraction (summarization)
Geometry	Prior knowledge about geometry (summarization) Distance (prediction, questioning and clarification) Measurements, such as m, cm (prediction, questioning and clarification) Clarify the concept distance (clarification) Geometrical figures and perimeter (prediction, questioning and clarification) Summarization of geometry (summarization)

Data analysis

The analysis during the intervention was conducted continuously during the design process and the approach was abductive, meaning going from micro to macro levels and allowing the empirical findings and theory to interact, which is a typical approach in EDR (de Beer et al, 2015). However, during the intervention, there was no time for a deeper analysis of the data, but nevertheless, it facilitated the

intervention study and a second analysis is necessary (Gravemeijer & Cobb, 2013). As a result of this first round of data analysis, a sequence of conjectures and refutations that are tied to specific episodes are shown. A second analysis was conducted after the intervention, where the data set was analysed and reread several times. By the second part of the analysis a distinction of the material was needed and a decision was made to focus on major shifts in the students' reasoning competence when reporting on the results. This approach is inspired by Cobb's (2002) approach.

In next section, the analysis tools for each of the four reading strategies are presented, where the analysis is divided into 'during intervention' and 'after intervention', except for prediction, where the analysis was the same during and after the intervention. As analysis tool, the conceptual framework of the Reciprocal Teaching model and Systemic Functional Linguistic was used. Depending on the method, the theoretical underpinnings of the focus of the metafunctions in Systemic Functional Linguistics can be different in different studies (Herbel-Eisenmann & Otten, 2011). This was also the case with this study, which will be discussed in regard to each of the strategies in the RT model and is also visualized in Table 2.

Prediction was implemented in the addition and geometry chapters – but also as an analytical concept referring to SFL's ideational metafunction, where naming (i.e. use of mathematical concept/words) relevant for the context is in focus. In this study naming referred to the main ideas presented in the textbook pages in the two topics, for example distance and perimeter.

Clarification was the strategy the students worked with in all of the topics. During the intervention, the following questions were raised for analysing the results: Can the students describe the mathematical concepts by using own examples and explanatory text? This question relates to SFL's textual meta-function by looking at use of semiotic resources and how they contribute to cohesiveness in the text. Another question that was asked was "Can the students use the concepts in another relevant context?" This aspect refers to SFL's ideational metafunction concerning naming. After the intervention, the textual metafunction and ideational meta-function were analysed again and the ideational metafunction was expanded to analyse *material process*, which refers to what the students are 'doing' when they perform the tasks, such as 'adding' and 'starting' and refers to SFL's ideational

metafunction. Further, the *interpersonal* metafunction was also analysed which involves examining relationships between participants by examining the use of personal pronouns, such as 'you' and 'I' in the students' explanatory texts to different solutions.

Questioning was implemented as a strategy in the geometry chapter and used as an analytical concept referring to SFL's ideational, interpersonal and textual metafunctions, just as with the strategy of clarification. During the intervention, the following questions were asked to analyse the students' notes: Are the students able to ask a question about the mathematical content provided on the pages? (SFL's ideational metafunction concerning naming). Can the students construct a task that involves reasoning, such as "Explain how you perform $36 + 12$ " or does the questions relate to procedures such as "Add $36 + 12$ "? (SFL's interpersonal meta-function). After the intervention, SFL's ideational meta-function and textual metafunction were analysed again by the semiotic resources the students used in their questions/tasks, such as illustrations. The analysis also involved examining the interpersonal meta-function again, which was expanded to the uses of personal pronouns.

Summarization is a strategy being implemented during all of the topics, but also an analytic concept referring to SFL's **ideational and textual metafunction**. Ideational metafunction refers to naming and material, mental and relational processes. The following questions were addressed during the intervention: Can the students identify the main ideas after working with different areas (ideational metafunction)? Are the main ideas described in appropriate ways (textual metafunction)? The analysis after the intervention summarization was expanded to the ideational metafunction to except for naming also involve the material processes and concerns what to do, such as 'to measure', and relational processes, such as "A distance has a beginning and an end" and mental process involving sensing, such as feelings. The textual metafunction was examined more in detail and analysed which semiotic resources appear and if and how they together contribute to cohesiveness. In table 5, a summary of the analysis tools is provided.

Table 5. A summary of the analysis tool

	Ideational metafunction
Prediction, Clarification, Questioning and Summarization	Naming relevant for the context, such as ten digits and hundred digits.
Summarization & Clarification	Material processes (what to do) and Relational processed (relations between objects).
Summarization	Mental processes (sensing), such as feelings.
	Interpersonal metafunction
Clarification and Questioning	Personal pronouns (such as you and I)
Questioning	Imperatives/Interrogatives (describe, what)
	Textual metafunction
Clarification, Questioning & Summarization	Semiotic resources, such as words, symbols and illustrations, and how they contribute to cohesiveness.

4.2.3.2 Reflections

Reflection involves considerations about the process and the findings (McKenney & Reeves, 2012). In this study, it concerned to what extent the students had developed their reasoning competence, context of situation) when reading and writing about the mathematical content and also how the activities worked. The teacher’s and my reflections were discussed almost every day but in a specific meeting each Friday and on the forthcoming Monday the analysis were discussed and planned during the whole intervention. The students’ current understanding and the difficulties that they faced in successfully completing the learning activity were then taken into consideration into the planning. From this third phase the main outputs are answer to the research question, implications for design proposition and recommendations/issues for redesign, which in this study prompt new cycles of design and constructions as well as evaluation and reflection.

4.3 Ethical considerations

In this section, ethical considerations are presented which I have made in relation to my research. The four aspects of ethical considerations: information requirement, consent, confidentiality and utilization (Bryman, 2008; Vetenskapsrådet (the Swedish Research Council), 2007) are carefully taken into account in relation to different persons; the principal, students, their parents and the teacher, involved in the study.

Information about parts that will be included in the survey have been clearly stated in written text concerning the questionnaire study (see Appendix 1) and both orally and in written text concerning the intervention (see Appendix 3). Further, all participants in the study have been informed about the investigation's purpose, that it is voluntary to participate and that they have the right to decline participation at any time. Concerning the interview study (Segerby, 2014), the participating students had previously filled the consent letter (see Appendix 3).

The interviews in Segerby (2014) were carried out in a room at the students' school. To conduct the interview in a familiar environment is making the child more relaxed and will promote the child's motivation and reduce the anxiety (Grieg & Taylor, 2007). Just before the interview was about to start each of the students were informed that it was voluntary to participate and that they could at any time decline their participation by saying that they did not want to continue the interview. This is stressed as essential so the students know what to do if they do not want to continue with the interview (Morgan, Gibbs, Maxwell & Britten, 2002). Initially in the interview, the students' were asked to tell about their mathematics education as a way to find out about the context in which the students are situated. Thereby awareness of language, communicative norms and patterns could be visualized (Eder & Fingerson, 2003). The questions in the interview study had previously been tested with two children in similar age as the students in the interview study to find out if the questions were suitable for this Grade.

In the information letters, information was provided concerning confidentiality, that findings will be reported in a way that does not allow individuals to be identified and that all research material will stored on a safe place where unauthorized people cannot share

it. As well, it was stated that the findings would only be used for research purposes which concerns utilization (Bryman, 2008; Kvale & Brinkmann, 2009). Before the intervention study, I verbally informed the students and they had the opportunity to ask questions concerning their participation and the research. Furthermore, the project was presented during a parents' meeting at the beginning of the school year, and the parents had the opportunity to ask questions as well.

The information letter attached to the questionnaire explained that by letting the children fill in the questionnaire, the child and the guardian had agreed to participate in the study. In the intervention study, the guardians and their child signed a consent letter attached to the information of the study (see Appendix 3). The construction of the information letter where the consent was included was inspired by Björklund Boistrup's (2010) letter that involved students in similar age.

4.3.1 Trustworthiness

In qualitative research, the four aspects for assessing trustworthiness are credibility, dependability, confirmability and transferability (Bryman, 2008).

During the intervention, I have studied the implementation of the RT model during a semester and keeping observation notes during the mathematics lessons and collected the students' notes usually one's or twice a week. To strengthen the credibility, I have triangulated this data by combining the lesson observation notes with the students' notes. I discussed the findings continuously with the teacher during the intervention but also at two occasions after the intervention took place where we discussed the findings that what was going to be included in the thesis. The teacher found that the information harmonized well with his point of view of what took place in the classroom but some information about the teacher and the school needed to be revised after these meetings. To make justice to the students' transcripts, an English native English speaker has been involved in the translations of the students' notes see Paper V (Segerby & Chronaki, submitted).

Regarding dependability, I have described the different phases of the research process, such as the research questions, choices of participants, data collection and decisions concerning analyses in as much details as possible to make the research transparent. Further, I

have also discussed the research findings with fellow researchers on seminars and conferences at several occasions which has been helpful since this kind of research, EDR, demands spending a lot of time out in the field with little time to evaluate and analyse the activities.

Concerning the aspect transferability, I have explained, in so much detail as possible, the design of my research and the choices, however; the results are connected to one specific context. Within EDR, it is critical that the aim is to develop design or innovations that can be used to support learning productively in all other settings. Instead, the descriptions of the research can function as reference or frame for teachers who want to adapt the instructions to their own classroom settings (Gravemeijer & Cobb, 2013).

Regarding confirmability, I only have the possibility to describe, explain or understand limited part of the processes during the implementation of the RT model in this study. These interpretations are based on personal experiences where my background as a teacher has influenced my research interest and my methodological choices. I am biased in the sense that I think that what are happening in the classroom are essential for understanding learning processes. To distance me from my teaching role the provided conceptual framework has contributed to helping me to focusing on certain aspects and thereby other aspects were not taken into consideration. Further, the choice of the data collection, mainly classroom's observations and students' written notes, were chosen because the focus was on the students' development in reasoning in mathematics not the teacher's development. This was clearly stated for the teacher before the intervention took place and it was under this condition the teacher agreed to participate in the study. As well, the constant discussing and reflecting on the research results with fellow researchers and with the teacher have helped me to simultaneously use my knowledge and distance myself from preconceptions, which helped me to step out of the role as a teacher and become a researcher.

4.4 Methodological discussion

When examining solutions to problems in authentic setting in education, several researchers require that theory and practice interact during several cycles. This involves identifying the problems and finding solutions involving developing theoretical and practical

considerations (e.g. Carlgren, 2011; McKenney & Reeves, 2012). EDR, as being used in this study, is suggested to be one way to build a bridge between theory and practice where collaboration between teachers and researchers is required (Cobb, 2002). When conducting research together with a teacher an ethical aspect considers the compromises, the collaboration builds on. The teacher and I had our roles stated before the intervention took place, the teacher should teach and I observe and analyse the data, and together we planned the activities. The collaboration between the teacher and me involved co-learning. Still, we had different roles and possibilities to influence what was happening. The collaboration between the teacher and me was however based on both parties recognising and acknowledging the particular skills and expertise of the other and together we provided the design of the intervention. The teacher had knowledge about the students' different prerequisites and practical aspects and my responsibility was to provide theories (SFL and RT) into the intervention. During the collaboration, we had an open climate when talking and discussing the design, evaluation and analysis. However, much of the provided new activities were new or partly new to teach for the teacher and during his teaching, I was an observer. These aspects made the teacher very exposed but the teacher and I discussed this initially and during the intervention where I clearly stated that I observed the activities that we together had planned, not him as a teacher. An open climate where we respected our areas of expertise was strived during the whole intervention where we almost daily discussed the findings and approach in the intervention.

Since complementing exercises were needed, the students got a lot of single paper that they should collect in a map, which was problematic for the students in the beginning. This approach was new for the students and before the intervention took place they were used to keep all their notes in their mathematics textbook but now they had except for two notebooks in mathematics, linear papers, square papers and computer papers. Initially, I and the teacher thought that the students should keep most of their notes on their computer but since the students were not familiar with using the computer and there continuously were problems with the computers, it was only used one time during this intervention.

The results of this study shows that it is difficult to determine how long time an EDR study will take – it is difficult to predict the

outcome and the possibilities for making changes. As Gravemeijer and Cobb (2006) stress “If you want to change something, you have to understand it, and if you want to understand something, you have to change it” (p. 17). It is also difficult to determine when the result is satisfactory. Further, in the beginning it was difficult to determine how much change that would be appropriate to suggest when discussing the planning with the teacher concerning both the teacher but also the students. Here the open climate between the teacher and me was necessary where the teacher could say no to suggested changes. The teacher did not get any extra time for participate in the research and planning and evaluation was done at his planning time.

During the intervention, sometimes other things needed to be prioritised and the activities could not be conducted at that time that we had planned for different reasons. For example, things that had happened on the break or on another lesson needed to be discussed. Practical things could be another reason where problems with the students’ computers needed to be solved. An important part in the intervention was to clarify the different roles the teacher and I had and this was done before the intervention took place, which facilitated the collaboration. However, in this study few participants made the research vulnerable since the study needed to be conducted during one specific school year. The teacher and/or students could at any time decide that they did not want to participate in the research any more for different reasons.

A comparison with another classroom, other teachers and other students could have made me see things that I could not from my own horizon. On the other hand, the long time in the same classroom opened up for seeing – and making – change. However, this study is a small-scale study in a specific context and if the study would have been conducted in another context the choices in the intervention would probably been different.

The conceptual framework was used to both design and develop the activities but also to analyse the students’ development of their reasoning competence in written texts. Thereby, specific parts of reasoning could both be analysed and developed which limited what reasoning involves and other tools could contribute to develop students’ reasoning in different ways.

5. RESULTS

In this part a summary of each of the five papers are provided. First the four field-based studies are presented (Paper I-IV), which is followed by the result from the implementation of the RT model during one semester (Paper V). In figure 4, an overview of the empirical field-based studies is presented and then discussed.

Figure 4. Overview of the four field-based studies in this thesis which framed the intervention.

Initially, the perceptions of students in Grades 3–5 in Sweden about their writing in school mathematics were explored (Paper I). To examine this further, an analysis of a page in a well-known mathematics textbook was conducted to examine possibilities for the students to develop their reasoning competence and potential issues when reading the textbook were also identified (Paper II). These

potential issues were further investigated in the next study, Paper III, by interviewing six students from the class where the intervention should take place and a larger study involving the whole class, was conducted to investigate this further and is presented in Paper IV where also the students' writing skills when reasoning were examined. The results from these four field-based studies (Papers I–IV) worked as a baseline for designing the intervention, where the RT model was implemented into the students' mathematics education during one semester, which is presented in Paper V.

However, close to the submission of the thesis, it became clear to me that the two papers that I had previously written about the intervention, which not is included in this thesis, mainly referred to the strategies of *clarification* and *questioning*, but all four strategies in the RT model were implemented during the semester. This made me decide to merge these two papers and also expand them to involve the processes in the implementation of *prediction* and *summarization*. According to Palinscar and Brown (1984), it is essential to implement all the strategies otherwise the intended effect might get lost. Thus, it became important to visualise the whole process of the implementation of the RT model, which is presented in Paper V – (Segerby & Chronaki, submitted), but with important analysis for the understanding of conceptualizing reasoning and to design and analyse strategies for mathematical reasoning with help from the RT model and SFL. In the following section, a summary of each of the papers included in this thesis is provided. For further information, see each respective paper.

5.1 Paper I

Title: Writing in mathematics lessons in Sweden (Segerby, 2016)

In this study, the aim was to examine students' perceptions of their writing in school mathematics and their assumptions about the purpose of keeping such notes in Grades 3–5 in Sweden. In this paper, the following research questions were addressed: “What kinds of writing do Year 3, 4 and 5 students do during their mathematics lessons?” and “What do these students consider to be the functions of this writing?”

Previous research has shown benefits for both students and teachers when students are encouraged to write in different ways in mathematics (Johanning, 2000; Meaney et al., 2012), such as when clarifying mathematical concepts and solutions. A consequence of this is that a greater emphasis is placed on the students' understanding of the subject – an aim which is also expressed in the current Swedish curriculum (Skolverket, 2011).

To explore students' views of writing in mathematics, a postal questionnaire was sent to 300 randomly selected students throughout Sweden, 100 in Grade 3, 100 in Grade 4 and 100 in Grade 5. The questionnaire comprised of four questions that concerned different aspects about the students' writing in school mathematics. The data was analysed by Britton, Burgess, Martin, McLeod and Rosen's (1975) categories: transactional, expressive and poetic writing. *Transactional* writing focuses on the final product, as its purpose is to describe, explain, persuade or instruct. *Expressive* writing is more exploratory and personal and can be thought of as 'thinking aloud on paper', as it expresses elements such as feelings and is intended for the writer's own use. It serves as a kind of diary where students can record and explore their feelings, opinions, moods and preoccupations of the moment. *Poetic* writing encourages the use of one's imagination and may come in the form of poetry, drama, plays and stories. Another term for poetic writing is 'creative writing'.

In total, 136 of the students responded, 50 students in Grade 3, 40 students in Grade 4, and 46 students in Grade 5. The result showed that writing activities in mathematics were primarily concerned with transactional writing connected directly to calculations involving performing procedures, with few involved students describing, explaining or justifying their thinking (i.e. reasoning). Additionally, the results seemed to show that few classes in Sweden use writing in mathematics extensively. However, in Grade 3, a different kind of writing appeared involving reasoning more frequently but then decreased in Grade 4 and 5. Another important finding was that many of the students considered their notes in mathematics to be worthless, both for their learning and for keeping track of their development in mathematics.

5.2 Paper II

Title: Systemic Functional Linguistics as a Methodological Tool in Mathematics Education (Ebbelind & Segerby, 2015)

The aim of this paper was to illustrate how SFL can be used as a methodological tool to unpack two different kinds of texts: a page in a mathematics textbook (Segerby) and a transcript from an interview with a student teacher (Ebbelind). In this paper, SFL in particular is discussed to unfold different texts to reveal traces of context that contribute to the formation of these texts and how this unpacking can lead to relevant interpretations of how conceptions of mathematical literacy operate in these texts. The framework of SFL makes interpretations in mathematical literacy possible, such as revealing different actors' roles in the texts. Also, there are similarities between the texts, (i.e. the textbook and the interview); for instance, both are developed through the linguistic choices of the contributors to the texts. However, there are differences as well. In the first study, the focus was on examining what a participant (the reader) might be engaged in, while the second study involved what a participant (a student teacher) is engaged in. This aspect affects which components of the three metafunctions are more interesting in the two studies (Herbel-Eisenmann & Otten, 2011). The result of the study showed that the main focus of the metafunctions can have both similarities and differences, depending on the text, which demonstrates the usefulness of using SFL as a methodological tool to illustrate the relationship between contexts and texts in mathematics education research.

In my study, the aim was also to explore the relationship between the reader and the textbook and how that relationship affects the reader in a participatory way and potential issues when reading the mathematics textbook by analysing a textbook page in a commonly used mathematics textbook. By using the framework from SFL, potential issues were pointed out concerning the mathematical vocabulary. For instance, the text in the information box which aimed to clarify might actually be confusing for students. Also, the concepts of *odd* and *even numbers* can also have alternative meanings in other contexts which may also lead to confusion. Furthermore, the questions being asked on the page position the students as doers, performing procedures individually. None of the tasks asked the students to reflect, describe or explain their thinking or to collaborate with other

students. In other words, the possibilities to train for reasoning were small. Also, the analysis illuminated problems with multimodal texts. In two exercises on a single page, the students needed to connect information in written text with illustrations. This kind of task places demands on the students' reading skills and competence to deconstruct the multimodality. Furthermore, some of the exercises require prior knowledge beyond traditional mathematics (involving the distribution of street numbers). The lack of this prior knowledge might hinder the students from completing the exercises correctly.

Co-authorship in the article: All of the text in this paper has been created in collaboration between the authors, except for the results and analysis of the textbook page which Segerby was responsible for and Ebbelind was responsible for the transcript from the interview.

5.3 Paper III

Title: Reading strategies in mathematics: A Swedish example (Segerby, 2014)

The aim of this study was to identify the reading strategies students use when reading the mathematics textbook. The following research questions were raised: "What kind of reading strategies do Year 3 students use when they approach a page in a Year 4 mathematics textbook?" and "What kinds of comprehension problems do students indicate they have when reading this page?"

In this study, six 10-year-old students with different levels of mathematical achievement were individually interviewed about which reading strategies they used when reading a page about number sense in a well-known mathematical textbook in Sweden. Five questions were provided in the study, which involved the three reading strategies of *prediction*, *clarification* and *summarization* from Palinscar and Brown's reciprocal model and also Halliday's System Functional Linguistics (SFL).

From this small study, it can be concluded that several of the students did not know how to predict about the mathematical content, which is essential to develop students' knowledge in an area. Some of the mathematical concepts were confusing for several of the students because some of the words have another meaning in another context. The result showed that the high-achieving students had, to a larger

extent, found successful reading strategies in this mathematical context than the other students. For example, the high achieving students read all information in the information box when summarizing the content. This is in comparison to the other students who only read parts of the information box, which led to wrong assumptions about the content. Some of these students considered the information box to be instructions on what to do on the page and not as a learning resource. The two high-achieving students also used appropriate strategies when predicting about the content, while the others either looked in the exercises or did not know how to determine the content. The illustrations on the page were also essential for understanding the content but did not always support the students in learning the content. None of the tasks on the page asked the students to describe or explain their thinking (i.e. reasoning). Instead, they were only required to perform procedures, and thus, the students' understanding and/or misunderstanding was not visualized. Additionally, what this study showed was that, irrespective of the students' mathematical achievement, all the students needed guidance, but the degree of guidance varied.

5.4 Paper IV

Title: Four Grade students' comprehensions strategies when reading their mathematics textbook. (Segerby, submitted)

In this study, the aim was to investigate 18 students' strategies in comprehending two adjacent pages in the beginning of their mathematics textbook. Here, the following research questions are posed: What kind of comprehension strategies do the students in Grade 4 indicate they use when approaching two pages of a mathematics textbook? and How does the context of culture influence the students' comprehension strategies?

In this study, the students' reading strategies as well as their writing skills when reasoning were examined in a class of 18 ten-year-old students at the beginning of an intervention study. Drawing on the pilot study (Segerby, 2014), a questionnaire study (see Appendix 4) was conducted to investigate how the students read the first two pages in a commonly used mathematics textbook that the students were supposed to work in individually. Similar to the pilot study, the questions are connected to Palinscar and Brown's reciprocal activities

of *prediction*, *clarification* and *summarization* and are also related to Halliday's System Functional Linguistics (SFL). The result shows that, as with the pilot study (Segerby, 2014), the high-achieving students used several appropriate resources when predicting the content in cases where the heading and the information in the information boxes are central; this is in contrast to many other students who only used some of the information on the page.

Mathematical concepts also caused confusion among the students. The illustrations in the information boxes helped few of the students to comprehend the content, just as in the pilot study (Segerby, 2014). In particular the illustrations of manipulatives, which were presented in one of the information boxes, did not seem to be familiar to many of the students, which led to further misunderstanding about the content being presented by several students. This is an issue because the content on the following pages in the mathematics textbook builds on that knowledge.

Additionally, many of the students had poorly developed reading and writing skills when *predicting*, *summarizing* and *clarifying* mathematical concepts on the textbook pages (i.e. reasoning).

5.5 Paper V

Title: Primary students' participation in school based mathematical reasoning practices: Coordinating Reciprocal Teaching and Systemic Functional Linguistics to support reasoning in the Swedish context. (Segerby & Chronaki, submitted).

From the results of the field-based studies and the literature review, an intervention study was conducted where the aim was to explore how a specific collaborative intervention design that coordinates RT and SFL may support Grade 4 students' reasoning process and what might be the engendered potentialities and boundaries in such an endeavour

An Educational Design Research study in a Grade 4 class in a small town in Sweden was conducted. Activities connected to the reading comprehension strategies of *prediction*, *questioning*, *clarification* and *summarization* were implemented into the mathematical teaching during a semester to the students' reasoning competence. *Prediction* involved predicting the content on the pages by using suitable resources, *questioning* involved asking questions about the content being presented, *clarification* involves clarifying mathematical

concepts relevant for the content being presented but also to solutions to different tasks, and *summarization* involves summarizing students' knowledge in a topic.

To inform the study, a conceptual framework framed within the ideas of Reciprocal Teaching (RT) and Systematic Functional Linguistics (SFL) are coordinated and used in both the analysis and development of the activities. The results from the four field-based studies (Ebbelind & Segerby, 2015; Segerby, 2014; Segerby, 2016; Segerby, submitted) and literature review contributed to the starting point for the intervention.

Critical moments, as in, changes in the students' reasoning competence, were identified and analysed to visualize the processes in the intervention. Three phases could be identified that were connected to all the reading comprehension strategies except for *prediction*. First, the students imitated the dictionary, teacher or textbook. In the second phase, scaffolding became essential; this is where the teacher highlights important aspects to be able to reason. Implicit issues related to reading and writing when reasoning were then made explicit which contributed to developing the students' reasoning skills. Nevertheless, the students' reasoning competence were still limited, but they began to use their own words and examples. In phase three, the strategies were modified to mathematics to support richer reasoning of the students. Furthermore, in connection with *clarification*, the receiver (a fictive Grade 3 student) of the students' notes became essential: *questioning* was modified to construct a task, and finally, *summarization* worked as a revision and to visualize their development in the topic in a way that tasks, diagnoses and tests usually do not do.

What the result shows is that all the strategies in the RT model were essential in the development of the students' reasoning competence and that *summarization* was the most difficult strategy for the students to apply in an appropriate way. This strategy was quite complex because it aimed to identify main ideas and situate them in a larger context, which in this study, concerned summarizing a topic. For this to happen, the students needed to predict about the mathematics content on pages, clarify mathematics concepts by using their own examples and explanatory text, and finally, to construct tasks in relation to the mathematical content being presented.

During the intervention, having changed the teaching method in mathematics was problematic at the beginning. The students initially expressed frustration over the new approach because they were not accustomed to having whole-class discussions and needing to explain their thinking (i.e. reasoning) when solving different tasks. Instead, working individually in different mathematics textbooks was common and mostly procedural tasks where the solution to the tasks usually involved a number were provided. Thereby, this was the context of culture the students were familiar with before the intervention took place. The students' resistance decreased throughout the course of the intervention, which was reflected in the students' notes when appropriate ways to reason began to happen, and their participation in the whole-class discussion increased. An essential aspect was the teacher's scaffolding during the whole-class discussions where implicit things were made explicit about reasoning in mathematics, but also in group and pair discussions, where the students discussed their findings about reasoning. Further, the students' notes containing their reasoning visualized their understanding and misunderstanding, which became valuable in and of themselves, as well as for the teacher and with the revision of the activities. All 15 students' notes were taken into account during the 15 weeks' intervention, but in the paper four students were in focus to show different paths of the intervention. One of the students was a high achiever in both mathematics and reading, one student was a high achiever in mathematics and a low achiever in reading, one student was a middle achiever in both reading and mathematics, and finally, one student was a low achiever in mathematics and in reading.

The result showed that all of the students had developed their reasoning competence and that the activities contributed to showing different levels of understanding by the students. Moreover, it seemed that the low achieve student facilitated the intervention more than the other students when implicit aspects when reasoning become explicit. What became clear in this study was that learning how to read and write texts in school mathematics would be a step in the right direction for supporting students' reasoning competence, but it is not enough. Instead, the need to change the perceptions of the context of culture, which operates around the context of situation, is a more optimal way to support the development of the students' reasoning

in school mathematics. The results of this study also highlight that it takes time to make changes; therefore, long-term studies would be the most beneficial way to conduct further research about how changes in the mathematics classroom can make a difference for students' reasoning competence in mathematics.

Co-authorship in the article: Segerby has in collaboration with a teacher framed the intervention, collected and initially analysed the data. A deeper analysis and the discussion part have been conducted in collaboration between Segerby and Chronaki. Chronaki has been responsible for the construction of the text in the paper.

6. CONCLUSION & DISCUSSION

In this chapter, I will describe the conclusions of the study and discuss these reflexively in the light of further research. The discussion is divided into these three parts which all concern resources for reasoning: a conceptual framework for mathematical reasoning, supporting activities and the context of culture. The chapter ends with implications for practice and research.

6.1 Conclusion

Reasoning in mathematics is stressed as an important competence in both national (Lithner, 2003; 2008) and international research (Cobb, 2002) as well as in the curricula of other countries (Skolverket, 2011; NCTM, 2003). Yet, the lack of an explicit and theoretically based distinction about what mathematical reasoning is and could be in school is still the case (Lithner, 2008). In this thesis, a conceptual framework for how reasoning in school mathematics can be conceptualized is suggested. The conceptual framework builds on Palinscar and Brown's (1984) Reciprocal Teaching (RT) model which is coordinated with Halliday's (1973) Systemic Functional Linguistics (SFL). This conceptual framework offers tools for making meaning from a text when reasoning by using the strategies of prediction, clarification, questioning and summarization, but also to expand the students' awareness of the mathematical register. In addition, I highlight the importance of understanding that linguistic choices are embedded in a context of culture and a context of situation which influence each other.

However, this study shows that change is not easy; interventions in practice may not lead to actual change. Halliday (1993) helps us to understand how change is needed on a cultural level. For example,

I could see how the practice of working individually with the textbook limited what the students considered learning mathematics. Depending on the textbook genre, the opportunities for reasoning can be limited, according to certain field-based studies (Ebbelind & Segerby, 2016; Segerby, 2014; Segerby, submitted) which also correlates with previous research (see, for example, Sidenvall, 2014). Complementary exercises are therefore needed which, in this study, refer to writing activities connected to the RT model and SFL.

The complementary activities also contributed to visualizing the students' understandings and misunderstandings and are thus essential for both the students and the teacher by employing SFL's metafunctions and the RT model as analysis tools. However, previous research shows that the opportunities for students to develop their reasoning competence in mathematics education is limited due to "traditional" teaching. This implies a change of teaching strategies is needed through working more in collaboration and beyond the textbook. In the present study, the teachers' guidance and providing of structures for reasoning (i.e. scaffolding) becomes essential and is necessary to support the students' reasoning competence. The teacher made implicit processes about reasoning in mathematics explicit for the students; thus, they did not have to "crack the code" by themselves. However, these activities are not necessarily suitable for other classroom settings, as classroom cultures may differ. It must be up to the teacher to find ways to support reasoning through supporting the use of language but what is obvious is that there is a need for making mathematics reasoning explicit so all students can become a participant and not something for the students to figure out by themselves.

6.2 Discussion

In this section, resources for reasoning are discussed. Initially, I elaborate my thoughts on making the language for reasoning in school mathematics explicit by using the conceptual framework where the Reciprocal Teaching model is coordinated with Systemic Functional Linguistics. I then share my thoughts and elaborate on the context of situation involving activities to support students' mathematical reasoning and the context of culture that must be taken into consideration when exploring the development of students' mathematical reasoning.

6.2.1 A conceptual framework for mathematical reasoning - Exploring the mathematical language for reasoning into a local theory

Referring back to the issues with students working individually in their mathematics textbook (Boesen, 2014; Skolinspektionen, 2009) – which also was the case in the class before the intervention took place – this approach left it up to each student to find out about the mathematical content. Previous research show that specific reading comprehension skills are needed that concerns both general and specific decoding and language comprehension skills for being able to reasoning in school mathematics (Barton & Heidema, 2002; Shanahan & Shanahan, 2008; Shepherd et al., 2012) and refers to disciplinary literacy (Shanahan & Shanahan, 2008).

In two of the field-based studies (Segerby, 2014; Segerby, submitted), a correlation between the Grade 4 students' mathematical skills and reading skills appeared just as previous research shows with students of a similar age (Möllehed, 2001). According to Korhonen et al.'s (2012) study, this correlation seems to follow the students during their whole school life. Another important finding in the field-based studies Segerby (2014) and Segerby (submitted) was that all of the students had developed comprehension (reasoning) strategies when reading the mathematics textbook but the strategies were more or less successful. The reasoning strategies in this study involves the RT model's four comprehension strategies; prediction, clarification, questioning and summarization, which according to Palinscar and Brown (1984) are considered as a baseline for reasoning. The results showed for example, that several of the low achiever students only focused on the information being expressed in the exercises on the pages when predicting about the mathematical content, which was not enough and led to wrong assumptions about the mathematical content. This unsuccessful strategy, focusing on the exercises when identifying main ideas, has also been found in previous research among secondary and university students (Shepherd et al.'s, 2012; Weinberg et al., 2012). This indicates that the strategies the students develop in younger years may follow the students during their whole school life and thus influence the students' learning in mathematics in a positive or negative way for a long time. However, the number of students in the field-based studies was small, and no generalization can be made. It is also unclear whether it is the students' knowledge of

mathematics that supports their reading or if it is the opposite – good reading strategies result in higher achievement in mathematics. Thus, further research is suggested to investigate this in larger settings.

What also could be found in the field-based studies (Segerby, 2014; Segerby, submitted), was that, independent of the students' achievement level in mathematics, all of the students needed opportunities to develop their reading and writing skills for reasoning in school mathematics. But according to the field based studies results (Ebbelind & Segerby, 2015; Segerby, 2014, Segerby, 2016; Segerby, submitted) and previous research (Bieda, Drwencke & Picard; 2014; Brehemer et al., 2015; Thompson, 2014) the students' opportunities for mathematical reasoning can be very limited when working in the mathematics textbook. As Hiebert and Grown (2007) point out, the seemingly commonsensical, but at the same time important, fact that students do not learn content and approaches that they have never been exposed to. This is problematic because according to Nunes et al.'s (2012) study, students' reasoning competence is an overall strong predictor of their future mathematical performance, and all other basic skills also need to be taken into account. In this study, the local theory concerns how to read and structure written texts in school mathematics when reasoning and the comprehension strategies in the RT model – prediction, clarification, questioning and summarization – are coordinated with the SFL metafunctions, see Paper III-V (Segerby, 2014; Segerby, submitted; Segerby & Chronaki, submitted). The RT model has been limitedly used in school mathematics, and even then, it tends to be connected to problem solving tasks in mathematics for developing students' mathematical reasoning (see, for example, Huber, 2010; Quirk, 2010). In this study, the focus is instead to develop students' ways of explaining and describing mathematical phenomena such as the meaning of ideas, concepts, operations and processes as they evolve in specific mathematical tasks. However, the mathematical language differs from other languages and in this study it refers to disciplinary literacy (Shanahan & Shanahan, 2008), where Systemic Functional Linguistic is an essential part that contribute to show the structure for a language within a specific discipline. SFL is the second part of the provided conceptual framework in this study, the local theory. Three metafunctions are involved; the ideational, interpersonal and textual metafunctions. These three metafunctions

are considered as a small set of principles necessary for explaining how language works (Halliday, 1989).

By using this conceptual framework, the students were given tools to engage in mathematical reasoning, which also gave the students a common language for reasoning. This is stressed by Schleppegrell (2007) as essential because “Learning the language of a new discipline is a part of learning the new discipline; in fact, the language and learning cannot be separated” (p.140). During the implementation, the students were provided with tools for not only comprehending the content but also learning independently to reason in mathematics regardless of the topic. Thereby, the students developed their metacognitive strategies, meaning their awareness of the cognitive demands, goals and purposes of reading the text (Vellutino, 2003) in school mathematics when reasoning.

In this study the provided conceptual framework also contributed to designing the activities after analysing what was missing or needed to be developed in the students’ mathematical reasoning. By using the approach, Educational Design Research (EDR), the activities could be revised until satisfactory results were achieved. However, there are limitations with the conceptual framework because no other comprehension strategies are taken into consideration except for the four in the RT model, which limits the findings. In addition, other comprehension skills are not taken into consideration, such as to reread, think aloud and depict the content with an illustration. The same concerns the use of the mathematical register, which focuses on three aspects: ideational, interpersonal and textual metafunctions. Thus, specific parts of the written and oral text are taken into account while others are not, which also restricts the findings to a couple of rules when analysing the texts both in the textbook and in the students’ notes.

In next section, the complementing activities to support the students’ reasoning competence connected to the conceptual framework in this study are further discussed.

6.2.2 Activities to support students’ mathematical reasoning

I and a teacher designed and implemented complementary activities connected to the students’ mathematics textbook during one semester (fifteen weeks) to support students’ reasoning competence. The complementing activities were connected to the reasoning strategies

of prediction, clarification, questioning and summarization in the RT model which were coordinated with SFL. These activities more or less encouraged the students to reason by developing their reading and writing skills in school mathematics.

In this study, the four strategies were implemented one at a time, which is recommended (Palinscar, w.y) and not as in Yang and Lin's (2012) study, where all of the strategies were implemented at the same time where their result become less successful to support students' reasoning in geometry. This study thereby support Palinscar's recommendations. Initially, the clarification strategy was implemented since according to the results from the field-based studies (Segerby, 2014; Segerby submitted) all of the students needed to develop this reasoning strategy. Connected to this strategy a dictionary, complementing exercises and whole-class discussions and peer discussions were implemented. Next strategy to implement was summarization and mind maps were used as an essential tool for summarizing, which initially was constructed in collaboration between the students and the teacher. Prediction was the third strategy to implement where initially specific instructions during whole-class discussions were provided by the teacher and finally the strategy questioning was implemented where the students should construct a question/task after working with several pages in the textbook where the students discussed their finding first in groups and before the whole-class discussions were conducted.

In this study the strategy prediction was the easiest strategy to implement however, specific instructions were then provided and the students' reasoning only involved single words, i.e. the main ideas, for example perimeter. The most difficult strategy for the students to comprehend was summarizing and for using this strategy in an appropriate way the students in this study needed to be able to predict about the main ideas on different pages, be able to clarify these main ideas and be able to transform them to another context. This might be why the result of Collen's (2011) study was not so successful because in her study they started to implement the strategy summarization. But research is suggested to investigate which strategy that might be best to taught first.

During the intervention the whole-class discussions became essential; this was when the teacher explicitly emphasized how the strategies in the RT model could contribute to developing the

students' mathematical reasoning. By this, the language was given a function in relation to the content pointed out as important by several researchers (Gibbons, 2009; Halliday, 1973; Halliday & Hasan, 1985; Palinscar & Brown, 1984). Implicit aspects when reasoning could then be made explicit to the students. With these activities, the main ideas were discussed and elaborated and involved SFL's ideational metafunction and textual metafunction. Further, during the whole-class discussions, the students received opportunities to discuss with the teacher what reasoning is and can potentially be, and the teacher explicitly showed the core of reasoning. For example, when predicting the content, reading the heading and the whole explanatory text (information box) were highlighted as essential, which reflects SFL's textual metafunction. In previous research, this kind of scaffolding has been emphasized as an important tool to support the students' comprehension in different subjects (Halliday, 1993; Palinscar & Brown, 1984; Shanahan & Shanahan, 2012), specifically, school mathematics (Eerde & Hajer, 2009; Jitendra et al., 2007; Smit & Eerde, 2011). Moreover, the students were given opportunities to discuss their reasoning with their peers.

During the intervention, the teacher did the teaching and I analysed the students' notes, where the conceptual framework (SFL and RT) was used as analysis tools. Through this analysis, I recognized that the activities needed to be revised because something was missing or needed to be developed in the students' use of the mathematical register in regard to the ideational, interpersonal and textual metafunctions in the different strategies. The teacher and I reflected on the analysis and approach, which usually led to a revision of the activities. During the intervention, critical moments were identified which can be viewed as when the students' mathematical reasoning was developed. From analysing this development, I also could point to three different phases (c.f. Cobb, 2002). These are named: imitative, limited and richer reasoning.

In the first phase, the *imitative* phase, the students' reasoning was obviously influenced by the textbook, dictionary or the teacher and appeared connected to all four comprehension strategies but in different ways. Concerning prediction and summarization, the students imitated the teacher; in clarification, the students imitated the dictionary, and finally; with questioning, the students imitated the

textbook. This phase was suitable for prediction where the teacher's strategies by reading the heading and information in the information box were imitated when identifying the main ideas on the pages. But connected to the other strategies the students' reasoning did not contribute to show how they had made meaning of the content.

In phase two, a limited amount of reasoning appeared, which can be identified in the students work in the strategies clarification, questioning and summarization. Thus, the students started to use their own words or examples (SFL's textual metafunction), for which the teacher's scaffolding became essential through showing the importance of using several semiotic resources (textual metafunction). Still, the students' reasoning was very limited. Nevertheless, at this point, the students could identify the main ideas (ideational metafunction) and use their own words or give examples (textual metafunction); for example, when summarizing addition, but still their reasoning was limited and explanatory texts barely existed.

Phase three, refers to richer mathematical reason, which was supported through strategies for clarification, questioning and summarization and involved students to reasoning about different mathematics concepts and solutions. For the students to reach phase 3, the activities needed to be revised and involved changing the receiver of the students' notes to become a fictive Grade 3 students to develop students clarification skills, change the strategy questioning to instead construct a task, and to summarization highlight the purpose of keeping the notes as a resource for the students to show their development in a topic, which also could be used as a repetition paper next time they worked with the topic but also a way to visualize their own development. In this third phase, the reasoning became richer and more cohesive and involved more explanatory texts where several semiotic resources appeared (analysed through SFL's textual metafunction) with mathematics-specific words and processes (ideational metafunction) connected to both summarization and clarification. Further, in the strategy clarification also involved personal pronouns (interpersonal metafunction) were included.

However, what the results of this study indicate is that perhaps the students initially need to imitate the teacher or copy the textbook to be able to move forward to develop their reasoning competence connected to all of the strategies. Research is needed to examine

students' approach when developing their mathematical reasoning further. It would then be interesting to examining further what Lithner's (2008) category Creative Reasoning can involve with students in the lower Grades.

According to the result of the intervention (see Segerby & Chronaki, submitted), all of the students had developed their mathematical reasoning competence positively. However, the implementation was much more time consuming than the teacher and I imagined before starting the intervention and sometimes the focus on mathematical reasoning was limited or taken away during the EDR because other activities needed to be prioritized. Examples of these involved students' basic skills in mathematics, such as algorithms for addition and subtraction. That these exercises were put in the first room was not surprising because this kind of procedural exercise is the most common task in school mathematics (Bergqvist et al., 2010; Boesen et al. 2014; Stylianides, 2009).

In this study, learning to reason in school mathematics is also considered a matter of being socialized into the culture of school mathematics, which is necessary to take into account when examining students' reasoning competence. This is further discussed in next section.

6.2.3 Context of culture for mathematical reasoning

In this study, according to the students in the class, the tasks in school mathematics should be performed fast and individually, which was the right way to do school mathematics. This showed that answers to the exercises usually involved procedures where the answers contained a number. This was also found in Meaney, et al.'s (2009) study as the students' favorite kind of writing. This is consistent with many of the exercises provided in the mathematics textbook (Boesen et al., 2014; Ebbelind & Segerby, 2015; Herbel-Eisenmann, 2007; Segerby, 2014; Segerby, submitted) and tests (Nolan, 2012). The exercises are a strong part of the context of culture; what mathematics is and should be according to students as well as the teachers. In the intervention, this was reflected in the students' written texts; initially, their notes just contained a few words or digits. But to be able to use several semiotic resources in school mathematic is stressed as essential, and by reasoning students' understanding and misunderstanding can be shown (Baxter et al., 2005), which not a single symbol or word

can do. To develop the students' mathematical reasoning the student need to have structures to what mathematical reasoning involves. Again, in the present study the teacher's scaffolding was essential, where implicit aspects about reasoning were made explicit for the students, which contributed to decrease the students' resistance and instead increase the students' participation because they got a common language (structures) to use for mathematical reasoning, but mostly the low achievers. Similar findings have been found in Jitendra et al's. (2007) study.

In previous research (Lampert & Cobb, 2003; Jaworski & Potari, 2009), approaches involving whole-class discussions and peer discussion have been suggested to support students' mathematics skills and connected to reasoning several studies have shown positive effects on the students' mathematical reasoning when working in collaboration with their peers (Nunes et al., 2015; Riesbeck, 2008; Rojas-Drummond & Zapata, 2005) and in whole class (Larsson, 2015). But initially in the intervention, the students found it difficult to understand and appreciate whole-class discussions. Instead, they wanted to concentrate on the tasks being provided in their mathematics textbook, which was the context the students previously was familiar to. It seemed that the students did not know how to discuss for example mathematical concepts and their solutions to different tasks so again the teachers scaffolding become essential.

Several of the students were frustrated that the "new" exercises took much longer to perform than they were used to and that they needed to write (and think) in a different way. Thus, the new approach in the intervention was not in line with the context of culture the students were familiar with. However, during the intervention, it seemed that the students' notes started to become valuable not only for them but also to the teacher and me, as these notes show how the students had made meaning about the mathematical content. Thus, their notes became an important resource for students to become aware of their development, and these notes also showed another understanding than what tasks and tests usually do not provide. This seemed to lead to decreased resistance from almost all of the students. The teacher also highlighted that the main audience for their writing was the students themselves and not the teacher, in order to check that they had not guessed (which several students thought was the purpose of their notes in Fried and Amit's (2003) study). This is essential to

stress (Morgan, 1998) that the aim of the writing is explicitly expressed so that the student understands that their notes are valuable for their learning of school mathematics where they can write down their experience of mathematics and thereby follow their development and get help with their misunderstandings and thereby not become worthless, which approximately half of the students believed them to be in Segerby (2016).

In other classrooms and countries, the context of culture is likely to be different but is essential to take into consideration when exploring suitable activities for developing students' mathematical reasoning competence and further research is suggested to investigate how the context of culture influences students' mathematical reasoning in other settings and countries. But within SFL the tools for examining the context of culture are limited. An approach that can be used to further examine the context of culture is Skott's (2013) framework, Patterns of Participation (PoP). PoP draws on two main theoretical perspectives, symbolic interactionism and social practice theory. Symbolic interactionism views humans as actors and reactors in situations and positions meaning as something that one engages in a specific situation. In social practice theory, identity formation and learning results from shifted participation in relevant social practices (Skott, 2013). The combining of SFL and PoP has previously been used by Ebbelind (2015) when analysing the experience of pre-service teachers and could be useful when examining elementary school students' shifted participation in reasoning during the intervention.

6.3 Implications for practice and research

Research to identify and support the development of students' reading and writing skills when reasoning in mathematics is an under-developed field in mathematics education research, especially concerning the lower Grades. Focusing on lower Grades seems essential given that, in this study, just as in previous research, there appears to be a correlation between students' reading skills and their mathematics skills (Möllehed, 2001; Vilenius-Touhimaa et al., 2008). This correlation seems to follow the students during their whole school life (Korhonen et al., 2012). From the results of this study, teaching and implementing activities for the comprehension strategies in the RT model (analysed and planned through a framework coordinated with SFL) is suggested as one relevant way to support students'

reasoning competence in mathematics. However, the strategies should then be adapted to school mathematics texts because these texts differ from texts in other subjects, (Barton & Heidema, 2002; Shanahan & Shanahan, 2008; Shepherd et al., 2012), which shows the importance of discussing and developing disciplinary literacy. Much of the previous research concerning the RT model tends to focus on low achieving readers and their positive development in other areas (e.g. Lederer, 2000). The result of this study is similar, but it also shows that the rest of the class was also helped by working with the RT model, which also contributed to show different levels of understanding and/or misunderstanding, which is raised as an important aspect about mathematical reasoning (Cobb, 2002). However, as stated in the introduction, according to the current curriculum, (Skolverket, 2011) teaching about reading comprehension strategies for reasoning is not a task for mathematics teachers. Instead, it is a task for teachers in the subjects of Swedish, mother tongue language classes, and Swedish as a second language. As a result, modifying the activities to mathematics can be difficult if the teacher is not familiar with teaching mathematics. Because it is not clear which teachers are ultimately responsible for developing students' reading comprehension strategies in mathematics, the strategies may end up falling through the cracks. This means that the specific comprehension strategies, which refers to the disciplinary literacy needed for mathematical reasoning, might not be supported by teachers or developed by students. This indicates that teacher education needs to prepare the teacher training students to support students' language development in the subject(s) they teach (Hattie, 2009; Liberg, 2008). Further research is suggested where collaboration between language teachers and mathematics teachers along with language and mathematics researchers, investigate different reasoning strategies and activities that can be used to support both first- and second-language learners' mathematics reasoning in different Grades. This is crucial because learning mathematics (or any school subject) is very much a matter of learning to communicate properly inside the discipline by using a specific register where the context of culture influence how the students choose to engage in these activities.

7. SAMMANFATTNING PÅ SVENSKA

Denna avhandling är resultatet av en studie inom området skolmatematik med fokus på att utveckla elevers resonemangsförmåga. Att resonera anses av många forskare vara en viktig kompetens för att lära sig matematik (Kilpatrick, 2001; Lithner, 2006; Niss 2003), vilken även har en central roll i flera länders läroplaner, exempelvis Sverige (Skolverket, 2011). Dock visar tidigare forskning på att elevernas möjligheter att utveckla sin resonemangsförmåga är begränsad (Brehemer et al., 2015; Thompson, 2014) och fokus är framförallt på problemlösning. Nunes et al. (2012) poängterar att möjligheterna att utveckla resonemang i förhållande till alla grundläggande matematiska kompetenser och inte endast problemlösning är viktigt, såsom att resonera kring aritmetiska operationer och matematiska begrepp, eftersom resonemang har visat sig vara viktigt för elevernas framtida matematiska lärande. I denna studie utgår resonemangsförmågan ifrån att kunna förutsäga, klargöra, ställa frågor och summera matematiska begrepp och procedurer där läsning och skrivning är i fokus.

I tidigare forskning har det visat sig att läsning av texter i matematiken är en utmaning för elevernas läsförmåga och skiljer sig ifrån att läsa texter i andra ämnen (Barton & Heidema, 2002; Shanahan & Shanahan, 2008) och relaterar till disciplinär literacitet (Shanahan & Shanahan, 2008). Till exempel är texterna i matematiken oftast multimodala och kompakt skrivna med få ledtrådar att ta till sig det matematiska innehållet som presenteras (Barton & Heidema, 2002). Emellertid anser många matematiklärare att de saknar kunskaper om hur de kan arbeta med språket i matematiken (Liberg, 2008). Detta är problematiskt eftersom en korrelation mellan elevers läsförmåga och matematiska förmåga redan uppträder från årskurs 4 och detta

verkar fortsätta genom deras resterande skoltid (Korhonen et al., 2012). Dessutom är det i Sverige vanligt att elever arbetar självständigt i matematikboken under matematiklektionerna, vilket innebär att var och en av eleverna förväntas skapa sin egen förståelse för matematik genom att läsa i matematikboken (Boesen et al., 2014; Skolverket, 2009).

Syftet med denna avhandling är att utforma och analysera strategier och aktiviteter för att utveckla elevers matematiska resonemang i en årskurs 4 klass. I studien koordineras Palinscars och Browns Reciprokala Undervisningsmodell (Palinscar & Brown, 1984) med Hallidays Systemisk Funktionell Lingvistik (Halliday & Hasan, 1984) till ett konceptuellt ramverk för resonera i matematik. Genom att använda SFL kan klassrumskontexten, elevrollen, lärarrollen, d v s kontexten av kulturen, samt elevernas lingvistiska val kopplat till ideationella, interpersonella och textuella metafunktioner analyseras. Dessa metafunktioner kan användas olika i olika studier (Herbel Eisenmann & Otten, 2008) och i denna studie utgår den ideationella metafunktionen ifrån naming, d v s relevanta begrepp (ord) för kontexten, samt processerna- materiell (vad som görs) och relationella processer (relationer mellan objekt) är i fokus. Den interpersonella meta-funktionen utgår ifrån personliga pronomen, modala hjälpverb, imperativ samt frågeord och slutligen den textuella meta-funktionen. Dessutom studeras vilka semiotiska resurser som används och hur de bidrar till att skapa en sammanhängande och förståelig text, dvs strukturer för att resonera, vilket är en viktig komponent inom disciplinär literacitet, som förespråkas i denna studie. Däremot kan SFL inte analysera resonemangsstrategier, vilket RU modellen bistår med. RU modell består av förståelsestrategierna förutsäga, klargöra, fråga och summera som i denna avhandling anses vara grundläggande strategier för att kunna resonera, vilka även kan återfinnas i Skolverket (u.å) som beskrivning för resonemangsförmågan.

Följande tre forskningsfrågor ställs:

- Hur kan matematiskt resonemang bli konceptualiserad till en lokal teori genom att operationalisera den Reciprokala Undervisningsmodellen med Systemisk Funktionell Lingvistik?

- Hur kan specifika läs- och skrivaktiviteter kopplade till den Reciprokala Undervisningsmodellen och Systemisk Funktionell Lingvistik stödja årskurs 4 elevers resonemangsförmåga i matematik?
- På vilka sätt påverkar klassrumskulturen elevernas möjligheter att utveckla sitt matematiska resonemang?

Denna studie är en Education Design Research (EDR) där aktiviteter kopplade till RU modellen implementerats under 15 veckor i en klass från årskurs 4. För att forma interventionsstudien har fyra förstudier utförts. I den första förstudien (Segerby, 2016) undersöktes vad elever i årskurs 3, 4 och 5 anser att de antecknar i matematiken samt vilken funktion dessa anteckningar anses ha för deras lärande. Resultatet visade att elevernas möjligheter att resonera var mycket begränsade och att eleverna hellre förespråkade att lösa proceduruppgifter. Dock var tillfällena att träna elevernas resonemangsförmåga mer frekventa i årskurs 3 jämfört med årskurs 4 och 5. Ett annat resultat var att ungefär 50% av eleverna ansåg att deras anteckningar i matematiken var oanvändbara för deras lärande oavsett ålder.

Den andra förstudien undersökte potentiella svårigheter med att läsa en sida i en matematikbok (Ebbelind & Segerby, 2015). Resultatet visade att texten på sidan i matematikboken ställde krav på elevernas läsförmåga gällande både avkodning och förståelse av multimodala texter. Resultatet visade även att specifika förkunskaper krävdes gällande den information som uttrycktes på sidan, t.ex. hur gatunummer är organiserade, liksom förståelsen för specifika matematiska begrepp såsom begreppet udda. Dessa aspekter undersöktes sedan vidare i den tredje förstudien (Segerby, 2014), genom en intervjustudie där sex elever, två högpresterande, två medelpresterande och två lågpresterande elever i matematik intervjuades. Elevernas resonemangsförmåga undersöktes när de läste en sida i matematikboken. Det visade sig att de högpresterande eleverna hade utvecklat mer framgångsrika strategier när de resonerade jämfört med de andra eleverna. Däremot så behövdes samtliga elevers strategier utvecklas. För att undersöka detta vidare expanderades studien till att omfatta hela klassen där interventionen skulle utföras, vilket därmed utgjorde den fjärde förstudien (Segerby, inskickad). I denna studie undersöktes elevers strategier när de läste två sidor i en

matematikbok, men även deras skriftliga förmåga när de resonerade studerades. Liknande resultat framkom i denna studie som i den tidigare studien av Segerby (2014), men i denna studie framkom det även att elevernas skrivförmåga när de resonerade var begränsad. Utifrån resultatet ifrån förstudierna designades interventionen i samarbete med en matematiklärare gällande innehållet samt upplägget med lämpliga aktiviteter. Läraren utförde sedan undervisningen där jag observerade och förde fältanteckningar. Utvärdering och reflektion gällande aktiviteterna skedde sedan kontinuerligt av mig och läraren, där jag analyserade elevers anteckningar vilket resulterade i revidering av aktiviteterna i interventionen.

Inledningsvis implementerades strategin *klargöra* där en ordlista med aktuella matematiska begrepp i området 'taluppfattning' byggdes upp. Denna ordlista kompletterades med uppgifter där eleverna skulle förklara begreppen med egna ord. Inledningsvis var elevernas beskrivningar av olika matematiska begrepp begränsat trots att läraren diskuterat och väglett (scaffolding) hur de kunde gå tillväga för att beskriva ett begrepp tydligare genom att ge exempel med förklarande text, men de flesta eleverna gav antingen ett exempel eller skrev en text, vilket inte var tillräckligt. Området taluppfattning avslutades med att införa strategin *summera*, där eleverna nu skulle summera området taluppfattning. Endast två elever kunde identifiera något viktigt begrepp ifrån taluppfattning trots att de arbetat med en ordlista under hela aktiviteten. Läraren introducerade då en tankekarta och tillsammans med eleverna byggde de upp denna tankekarta gällande taluppfattning.

I nästa aktivitet 'addition och subtraktion' infördes strategin *förutsäga* där läraren gav tydliga instruktioner om hur denna strategi skulle användas samt förklarade vikten av att kunna använda denna strategi. Eleverna fick även skriva något som de kände till om addition respektive subtraktion innan de började arbeta med aktiviteten som hjälp vid den fortsatta uppbyggnaden av en ordlista. För att utveckla elevernas skriftliga resonemangsförmåga gällande strategin *klargöra* ändrades mottagaren till en fiktiv åk 3 elev istället för läraren. Detta bidrog till ett rikare resonemang när matematiska begrepp och uträkningar skulle förklaras. Eleverna summerade nu för andra gången en aktivitet och denna gång kunde samtliga elever identifiera viktiga begrepp, dock var elevernas resonemang begränsat.

I den tredje aktiviteten 'geometri' användes alla fyra resonemangsstrategierna för första gången i klassen. Geometrikapitlet inleddes med att eleverna skulle *summera* vad de kände till om geometri innan de började arbeta med kapitlet. Under kapitlets gång arbetade eleverna med de tre andra strategierna, *förutsäga, fråga och klargöra*, kontinuerligt. Här visade det sig att strategin "fråga" var svår för eleverna att använda för att utveckla deras matematiska resonemang och istället fick eleverna konstruera egna uppgifter. När eleverna arbetat klart med kapitlet fick de *summera* vad de nu kände till om geometri och en jämförelse mellan vad de ansåg att de kände till innan de började arbeta med det de nu kände till om geometri genomfördes. Därmed kunde deras utveckling i geometri synliggöras. Tidigare hade elevernas kunskaper endast utgått ifrån resultat på prov och diagnoser, men genom detta tillvägagångssätt kunde elevernas kunskaper synliggöras på ett annat sätt.

Resultatet av interventionen visade på att elevernas läs- och skrivförmåga kopplat till resonemangsförmågan hade utvecklats positivt. Eleverna använde mer framgångsrika lässtrategier och deras resonemang blev rikare genom att de innehöll fler matematiska begrepp och representationsformer. Under interventionen kunde tre faser identifieras; imitativ, begränsat resonemang och rikare resonemang. I den första fasen imiterade eleverna textboken, ordlistan eller läraren och använde inte egna ord eller exempel. I den andra fasen började eleverna använda egna ord eller exempel. Helklassdiskussionerna blev mycket väsentliga inslag där elevernas förståelsestrategier och anteckningar lyftes och diskuterades. Lärarens vägledning (scaffolding) var likaså väsentliga för utvecklingen av elevernas resonemangsförmåga. Emellertid krävdes det en anpassning av förståelsestrategierna till skolmatematiken vilket relaterar till disciplinär literacitet (Shanahan & Shanahan, 2012), för att eleverna skulle nå den tredje fasen, rikare resonemang. En annan oerhört väsentlig aspekt under interventionen var den rådande kulturen som gällde under matematiklektionerna eftersom den påverkade elevernas utveckling av deras resonemangsförmåga och den behövde förändras parallellt för att elevernas resonemangsförmåga skulle utvecklas optimalt.

Inledningsvis mötte detta nya tillvägagångssätt motstånd bland eleverna där de uttryckte en frustration över det nya tillvägagångssättet.

Eleverna uttryckte bland annat att de inte var vana vid att ha helklassdiskussioner där de behövde förklara sitt tänkande, det vill säga resonera, när man löser olika uppgifter samt att kunna förklara olika matematiska begrepp. Den kontext som eleverna tidigare var vana vid utgick ifrån att de arbetade individuellt i olika matematikböcker vars uppgifter ofta utgick ifrån procedurräkning där svaret involverade ett tal eller ett ord. Motståndet minskade dock under interventionen. Ett väsentligt inslag under interventionen var lärarens vägledning, där implicita aspekter gällande resonemang i matematik uttrycktes explicit, men också gruppdiskussioner och pardiskussioner där eleverna fick möjlighet att diskutera sina resonemang med sina klasskamrater.

Det som blev tydligt i denna studie var att det inte endast handlade om att utveckla elevernas läs- och skrivförmåga i skolmatematiken, utan att även kulturen i klassrummet påverkade elevernas utveckling av deras resonemangsförmåga. I denna studie krävdes en förändring av kulturen, vilken tog lång tid att genomföra. Fortsatt forskning där lämpliga aktiviteter för att utveckla elevers resonemangsförmåga förespråkas där även den kontextuella kulturen behöver beaktas för den har en betydelsefull roll i elevernas lärande.

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8. REFERENCES

- Abedi, J., & Lord, C (2001). The language factor in mathematics tests. *Applied Measurement in Education*, 14(3), 219-234.
- Adams, T. L. (2003). Reading mathematics: more than words can say. *The reading Teacher*, 56(8), 786-795.
- Adams, T.H., Thangata, F., & King, C. (2005). "Weigh" to Go! Exploring Mathematical language. *Mathematics Teaching in the Middle School*, 10(9), 444-448.
- Alatalo, T. (2011). *Skicklig läs- och skrivundervisning i åk 1–3: Om lärares möjligheter och hinder* (Doktorsavhandling, Gothenburg studies in educational sciences, 311). Göteborg: Centrum för utbildningsvetenskap, Göteborgs universitet.
- Alatalo, T. (2015). Professional Content Knowledge of Grades One – Three Teachers in Sweden for Reading and Writing Instruction: Language Structures, Code Concepts, and Spelling Rules. *Scandinavian Journal of Educational Research*, 60 (5), 1-23.
- Alrø, H., & Skovsmose, O. (2004). *Dialogue and learning in mathematics education. Intention, reflection, critique*. Dordrecht: Kluwer Academic Publishers.
- Andersson, A., Picetti, M., & Sundin, K. (2003). *Matte direkt borgen*. Stockholm: Bonniers utbildning AB.
- Arizpe, E., & Styles, M. (2002). *Children reading pictures: interpreting visual texts*. London: RoutledgeFalmer
- Atweh, B., Bleicher, R. E., & Cooper, T. J. (1998). The construction of the social context of mathematics classrooms: A sociolinguistic analysis. *Journal for Research in Mathematics Education*, 63-82.
- Baer, D.M., Wolf, M.M., & Risley, T.R. (1968). "Some current dimensions of applied behavior analysis". *Journal of Applied Behavior Analysis*, 1(1): 91-97.

- Ball, D., & Bass, H. (2003). Making mathematics reasonable in school. I J. Kilpatrick, W. G. Martin & D. Schifter (Eds), *A research companion to principles and standards for school mathematics* (p. 27-44). Reston, VA: National council of teachers of mathematics.
- Barab, S., & Squire, K. (2004). Design-based research: Putting a stake in the ground. *The Journal of the Learning Sciences*, 13(1), 1-14.
- Barsalou, L.W. (1992). *Cognitive psychology: an overview for cognitive scientists*. Hillsdale, N.J.: Erlbaum
- Barton, M. L., & Heidema, C. (2002). *Teaching reading in mathematics* (2nd ed.). Aurora, CO: Mid-continent Research for Education and Learning.
- Barton, M. L. Heidema, C., & Jordan, D. (2002). Teaching reading in mathematics and science. *Educational Leadership* 60, 24-28.
- Baxter, J. A., Woodward, J., & Olson, D. (2005). Writing in mathematics: An alternative form of communication for academically low-achieving students. *Learning Disabilities Research & Practice*, 20(2), 119-135.
- Bergqvist, E., Bergqvist, T., Boesen, J., Helenius, O., Lithner, J., Palm, T., & Palmberg, B. (2010). *Matematikutbildningens mål och undervisningens ändamålsenlighet* [elektronisk resurs]: Grundskolan våren 2009. Göteborg: Nationellt centrum för matematikutbildning, Göteborgs universitet.
- Bergvall, I. (2016). *Bokstavligt, bildligt och symboliskt i skolans matematik: en studie om ämnesspråk i TIMSS*. Diss. (sammanfattning) Uppsala: Uppsala universitet, 2016. Uppsala.
- Bieda, K. N., Ji, X., Drwencke, J., & Picard, A. (2014). Reasoning-and-proving opportunities in elementary mathematics textbooks. *International Journal of Educational Research*, 64, 71-80.
- Bishop, J., Lamb, L., Philipp, R., Withacre, I., Schappelle, B., & Lewis, M. (2014). Obstacles and Affordances for Integer Reasoning: An Analysis of Children's Thinking and the History of Mathematics. *Journal for Research in Mathematics Education*, 45(1), 19-61.
- Bjuland, R., Luiza Cestari, M., Bjuland, R., Luiza Cestari, M., & Borgersen, H. E. (2008). The Interplay Between Gesture and Discourse as Mediating Devices in Collaborative Mathematical Reasoning: A Multimodal Approach. *Mathematical Thinking and Learning*, 10(3), 271-292.
- Björklund Boistrup, L. (2010). *Assessment discourses in mathematics classrooms: a multimodal social semiotic study*. Diss. Stockholm: Stockholms universitet
- Boaler, J., & Brodie, K. (2004). The importance of depth and breadth in the analysis of teaching: A framework for analyzing teacher questions. In *the Proceedings of the 26th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 773-80).

- Boaler, J., & Staples, M. (2008). Creating mathematical futures through an equitable teaching approach: The case of Railside School. *The Teachers College Record*, 110(3), 608-645.
- Boesen, J., Lithner, J., & Palm, T. (2010). The relation between types of assessment tasks and the mathematical reasoning students use. *Educational studies in mathematics*, 75(1), 89-105.
- Boesen, J., Helenius, O., Bergqvist, E., Bergqvist, T., Lithner, J., Palm, T., & Palmberg, B. (2014). Developing mathematical competence: From the intended to the enacted curriculum. *The Journal of Mathematical Behavior*, 33(0), 72-87.
- Borasi, R., Siegel, M., Fonzi, J., & Smith, C. (1998). Using transactional reading strategies to support sense-making and discussion in mathematics classrooms: An exploratory study. *Journal for research in mathematics education*, 29(3), 275-305.
- Britton, J., Burgess, T., Martin, N., McLeod, A., & Rosen, H. (1975). *The development of writing abilities*. London: Macmillan Education.
- Bruner, J. S. (1990). *Acts of meaning*. Cambridge, Mass.: Harvard Univ. Press.
- Bryant, D. P. (2005). Commentary on early identification and intervention for students with mathematics difficulties. *Journal of Learning Disabilities*, 38(4), 340-345.
- Bryant, B. R., Bryant, D. P., Kethley, C., Kim, S. A., Pool, C., & Seo, Y. J. (2008). Preventing mathematics difficulties in the primary grades: The critical features of instruction in textbooks as part of the equation. *Learning Disability Quarterly*, 21-35.
- Bryman, A. (2008). *Social Research Methods*. Oxford: Oxford University Press.
- Burns, M. (2004). Writing in math. *Educational Leadership*, 62 (2), 30-33.
- Carlgren, I., (2011). Forskning ja, men i vilket syfte och om vad? Om avsaknaden och behovet av en klinisk mellanrumsforskning. *Forskning om lärande och undervisning*, 5, 64-79.
- Carlgren, I. & Marton, F. (2000). *Lärare av i morgon* [Teacher of tomorrow]. Lärarförbundets förlag.
- Carter, A.T., & Dean, O.E. (2006). Mathematics intervention for grades 5-11: Teaching mathematics, reading, or both?, *Reading Psychology* 27(2-3), 127-146.
- Chapman, A. (1993). Language and learning in school mathematics: a social semiotic perspective. *Issues in educational research*, 3(1), 35-46.
- Chen, C.-L., & Herbst, P. (2012). The interplay among gestures, discourse, and diagrams in students' geometrical reasoning. *Educational Studies in Mathematics*, 83(2), 285-307. doi: 10.1007/s10649-012-9454-2

- Chinn, C. A., Anderson, R. C., & Waggoner, M. A. (2001). Patterns of discourse in two kinds of literature discussion. *Reading Research Quarterly*, 36(4), 378-411.
- Chinn, S., & Ashcroft, R. (1998). *Mathematics for dyslexics*. London: Whurr publishers.
- Chronaki, A., & Christiansen, I.M. (red.) (2005). *Challenging perspectives on mathematics classroom communication*. Greenwich, Conn.: IAP-Information Age Pub.
- Cobb, P. (2002). Reasoning With Tools and Inscriptions, *Journal of the Learning Sciences*, 11(2-3), 187-215.
- Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher*, 32(1), 9-13.
- Cobb, P., & Gravemeijer, K. (2008). Experimenting to support and understand learning processes. In A. Kelly, T. Lesch and J. Baek (Eds.), *Handbook of Design research methods in education*, 9-13.
- Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational psychologist*, 31(3-4), 175-190.
- Collen, M. H. (2011). *Fifth Grade Children's Use of Reciprocal Teaching to Solve Word Problems in Mathematics*. Doctoral thesis, state university of New York at Albany.
- Collins, A., & Smith, E. (1982). Teaching the process of reading comprehension, pp, 173-185). In D. K. Detterman and R. J. Sternberg, (Eds.). *How and how much can intelligence be increased*. Norwood, N.J.: ABLEX.
- Darling-Hammond, L. (2000). Teacher quality and student achievement. *Education policy analysis archives*, 8, 1.
- Davidson, D., & Pearce, D. (1992). The influence of writing activities on the mathematics learning of Native American students. *The Journal of Educational Issues of Language Minority Students*, 10, 147-157.
- de Beer, H., Gravemeijer, K., & van Eijck, M. (2015). Discrete and continuous reasoning about change in primary school classrooms. *ZDM*, 47(6), 981-996.
- Doolittle, P. E., Hicks, D., Young, C. A., Triplett, C. F., & Nichols, W. D. (2006). Reciprocal Teaching for Reading Comprehension in Higher Education: A Strategy for Fostering the Deeper Understanding of Texts. *International Journal of Teaching and Learning in Higher Education* 2006, 17(2), 106-118.
- Ebbelind, A. (2015). Systemic Functional Linguistics as a methodological tool when researching Patterns of Participation. In *CERME 9-Ninth Congress of the European Society for Research in Mathematics Education* (pp. 3185-3191).

- Ebbelind, A., & Segerby, C. (2015). System Functional Linguistics as a Methodological Tool in Mathematics Education. *Nordic Studies in Mathematics Education*, 20(1), 33-54.
- Eco, U. (1979). *A theory of semiotics*. (1st Midland book ed.) Bloomington: Indiana University Press.
- Eder, D. & Fingerson, L. (2003) *Interviewing children and adolescents in Inside interviewing*. Eds J.A, Holstein, J.F, Gubrium. London: Sage publications ltd.
- Eerde, H. A. A. van, Hajer, M., & Prenger, J. (2008). Promoting mathematics and language learning in interaction. in J. Deen, M. Hajer och T. Koole. *Interaction in two multicultural mathematics classrooms. Processes of inclusion and exklusion*, p 31-68.
- Eisenhart, M. A. (1991). Conceptual frameworks for research circa 1991: Ideas from a cultural anthropologist; implications for mathematics education researchers. In *Proceedings of the 13th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 202-219).
- Eklund, S. (1990). *Bedömning av kunskaper och färdigheter i bild, årskurs 2: del av en nationell utvärdering av grundskolan*. Umeå: Umeå universitet.
- Ernest, P. (1997). Introduction: Semiotics, Mathematics and Mathematics Education, *Philosophy of Mathematics Education, Journal* 10.
- Evens, H., & J. Houssart: 2004, 'Categorizing pupils' written answers to a mathematics test question: 'I know but I can't explain". *Educational Research* 46, 269–282.
- Fang, Z., & Schleppegrell, M. J. (2008). *Reading in secondary content areas*. Ann Arbor: University of Michigan Press.
- Figueiredo, N., van Galen, F., & Gravemeijer, K. (2009). *The actor's and observer's point of view*. *Educational Designer*, 1(3). Retrieved from: <http://www.educationaldesigner.org/ed/volume1/issue3/article10>
- Fried, M. N., & Amit, M. (2003). Some reflections on mathematics classroom notebooks and their relationship to the public and private nature of students practices. *Educational Studies in Mathematics* 53: 91-112.
- Fuentes, P. (1998). Reading comprehension in mathematics. *The Clearing House*, 72(2), 81-88.
- Gaskins, I. W. (2003). Taking charge of reader, text, activity, and context variables. *Rethinking reading comprehension*, 141-165. In A. Sweet and C. E., Snow (Eds.), *Rethinking reading comprehension*. New York: The Guilford Press.
- Gibbons, P. (2002). *Scaffolding language, scaffolding learning: teaching second language learners in the mainstream classroom*. Portsmouth, NH: Heinemann

- Gibbons, P. (2009). *English learners' academic literacy and thinking*, Portsmouth, NH: Heinemann.
- Gravemeijer, K. (1998). Developmental Research as a Research Method, In: J. Kilpatrick and A. Sierpiska (Eds.) *Mathematics Education as a Research Domain: A Search for Identity (An ICMI Study)*. Dordrecht: Kluwer Academic Publishers, Book 2, 277-295.
- Gravemeijer, K., & Cobb, P. (2006). Design research from a learning design perspective. In J. Van den Akker and K. Gravemeijer, S. McKenney and N. Nieveen (Eds.), *Educational design research*, pp. 17-51. London: Routledge.
- Gravemeijer, K. & Cobb, P. (2013). Design research from the learning design perspective. In: T. Plomp & N. Nieveen (Eds.). (2013). *Educational design research Part A: An introduction* (pp. 72-113), Enschede: SLO.
- Haggarty, L., & Pepin, B. (2002). An investigation of mathematics textbooks and their use in English, French and German classrooms: Who gets an opportunity to learn what? *British Educational Research Journal*, 28(4), 567-690.
- Halliday, M.A.K. (1973). *Explorations in the functions of language*. London: Edward Arnold.
- Halliday, M.A.K. (1975). Some aspects of sociolinguistics. In *Interactions between linguistics and mathematical education*. Copenhagen: UNESCO.
- Halliday, M. A. K. (1978). *Language as social semiotic: the social interpretation of language and meaning*. London: Edward Arnold.
- Halliday, M.A.K. (1989). *Spoken and written language*. Oxford: Oxford University Press.
- Halliday, M. A.K. (1993). Towards a language-based theory of learning. *Linguistics and education*, 5(2), 93-116.
- Halliday, M.A.K., & Matthiessen, C. (2004). *An Introduction to Functional Grammar*. London: Arnold.
- Halliday, M.A.K. (2007). The notion of "context". In J. J. Webster (Eds.), *The Language and Education. Collected Works of M.A.K. Halliday, Vol.9*. London and New York: Continuum.
- Halliday, M. A. K. & Hasan, R. (1985). *Language, context, and text: aspects of language in a social-semiotic perspective*. (2. ed.) Oxford: Oxford Univ. Press.
- Halliday, M.A.K. & Hasan, R. (1994). *Cohesion in English*. London: Longman.
- Hammill, L. (2010). The interplay of text, symbols and graphics in mathematics education. *Transformative dialogues: teaching & learning Journal* 3(3)1-8.

- Hanna, G. (2000). Proof, explanation and exploration: An overview. *Educational Studies in Mathematics*, 44 (1), 5-23
- Hannula, M. S. (2001). The metalevel of cognition-emotion interaction. In M. Ahte, O. Björqvist, E. Pehkonen & V. Vatanen (Eds.) *Research on Mathematics and Science Education- From Beliefs to Cognition, from Problem Solving to Understanding* (pp.55-66).
- Hattie, J. (2009). *Visible learning: a synthesis of over 800 meta-analyses relating to achievement*. London: Routledge.
- Herbel-Eisenmann, B. A. (2007). From intended curriculum to written curriculum: Examining the” voice” of a mathematics textbook. *Journal for Research in Mathematics Education*, 344-369.
- Herbel-Eisenmann, B. A., & Otten, S. (2011). Mapping mathematics in classroom discourse. *Journal for Research in Mathematics Education*, 42, 451–485.
- Herber, H. L. (1970). Teaching reading in content areas. Englewood Cliffs, NJ: Prentice-Hall.
- Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students’ learning. F: K Lester (Ed.). *Second handbook of research on mathematics teaching and learning*, 1, 371-404.
- Hirst, P. (1974). *Knowledge and the curriculum*. A collection of philosophical papers. Routledge and Kegan Paul: London and Boston.
- Holmberg, A. P. (2012). Kontext som aktivitet, situationstyp och praktik. En kritisk analys av kontextbegreppet i systemisk-funktionell teori. *Språk och stil*, 22(1), 67-86.
- Holmqvist, M., Gustavsson, L., & Wernberg, A. (2007). Generative learning: Learning beyond the learning situation. *Educational action research*, 15(2), 181-208.
- Hoover, W. A., & Gough, P. B. (1990). The simple view of reading. Reading and Writing. *An interdisciplinary Journal* (2), 127-160.
- Huber, C. W. (2010). *The Impact of Reciprocal Teaching on Mathematics Problems Solving for Grade 4 Students*. Doctoral thesis, Central Connecticut State University, New Britain.
- Inglis, M., Mejia-Ramos, J. P., & Simpson, A. (2007). Modelling mathematical argumentation: The importance of qualification. *Educational Studies in Mathematics*, 66(1), 3-21.
- Jablonka, E., & Johansson, M. (2010). Using texts and tasks: Swedish studies on mathematics textbooks. In B. Sriraman, C. Bergsten, S. Goodchild, G. Palsdottir, B. Dahl, B. D. Söndergaard, & L. Haapasalo (Eds.), *The first sourcebook on Nordic research in mathematics education: Norway, Sweden, Iceland, Denmark and Contributions from Finland* (pp. 363–372). Charlotte, NC: IAP-Information Age Publishing.

- Jaworski, B., & Potari, D. (2009). Bridging the macro- and micro-divide: Using an activity theory model to capture sociocultural complexity in mathematics teaching and its development. *Educational Studies in Mathematics*, 72(2), 219–236.
- Jellis, R. M. (2008). *Primary Children's Interpretation And Use Of Illustrations In School Mathematics Textbooks and Non Routine Problems: A School pBased Investigation*. Doctoral thesis, Durham University.
- Jewitt, C. (2005). *Technology, literacy and learning: a multimodal approach*. London: Routledge.
- Jitendra, A. K., Griffin, C. C., Deatline-Buchman, A., & Szczesniak, E. (2007). Mathematical word problem solving in third-grade classrooms. *The Journal of Educational Research*, 100(5), 283-302.
- Johanning, D. I. (2000). An analysis of writing and postwriting group collaboration in middle school pre-algebra. *School Science and Mathematics*, 100(3), 151-160.
- Johansson, M. (2006). *Teaching mathematics with textbooks- a classroom and curricular perspective*. (Doctoral dissertation, Luleå University, Sweden).
- Jordan, N.C., Hanich, L.B., & Kaplan, D. (2003). A longitudinal study of mathematical competencies in children with specific mathematics difficulties versus children with comorbid mathematics and reading difficulties. *Child Development*, 74, 834-850.
- Jurdak, M., & El Mouhayar, R. (2014). Trends in the development of student level of reasoning in pattern generalization tasks across grade level. *Educational Studies in Mathematics*, 85(1), 75-92.
- Keene, E.O. & Zimmermann, S. (2003). *Tankens mosaik: om mötet mellan text och läsare*. Göteborg: Daidalos.
- Kilpatrick, J. (2001). Understanding mathematical literacy: The contribution of research. *Educational Studies in Mathematics*, 47(1), 101-116.
- Knipping, C. (2003). 'Argumentation structures in classroom proving situations'. In: M. A. Mariotti (ed.): Proceedings of the Third Congress of the European Society for Research in Mathematics Education. Bellaria, Italy, ERME.
- Kress, G. R. & van Leeuwen, T. (2001). *Multimodality: a social semiotic approach to contemporary communication*. London: Routledge
- Korhonen, J., Linnanmäki, K., & Aunio, P. (2012). Language and Mathematical Performance: a Comparison of Lower Secondary School Students with Different Level of Mathematical Skills. *Scandinavian Journal of Educational Research*, 56(3), 333-344
- Krummheuer, G., (1995). The ethnography of argumentation, I P Cobb and H. Bauersfeld (Ed), *The emergence of mathematical meaning: Interaction in classroom cultures*, (p. 229-270). Hillsdale, NJ: Lawrence Erlbaum.

- Kvale, S. & Brinkmann, S. (2009). *Den kvalitativa forskningsintervjun*. Enskede: TPB.
- Lampert, M., & Cobb, P. (2003). Communication and language. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to the principles and standards for school mathematics* (pp. 237-249). Reston, VA: National Council of Teachers of Mathematics.
- Larsson, M. (2015). *Orchestrating mathematical whole-class discussions in the problem-solving classroom: theorizing challenges and support for teachers*. Diss. Västerås: Mälardalens högskola, 2015. Västerås.
- Lederer, J. M. (2000). Reciprocal teaching of social studies in inclusive elementary classrooms. *Journal of Learning Disabilities*, 33(1), 91-106.
- Lee, C. (2006). *Language for learning mathematics*. England: Open university press.
- Lemke, J. L (2003). Mathematics in the middle: Measure, picture, gesture, sign, and word. In M. Anderson, A. Saenz-Ludlov, S. Zellweger & V.V Citfarelli (Eds.). *Educational perspective on mathematics as semiosis: From the thinking to interpreting to knowing* (pp 215-234). Ottawa, Ontario, Canada: Legas
- Lester, F. K. (2005). On the theoretical, conceptual, and philosophical foundations of research in mathematics education. *ZDM - The International Journal on Mathematics Education*, 37(6), 457-467.
- Levie, W. H., & Lentz, R. (1982). Effects of text illustrations: A review of research. *ECTJ*, 30(4), 195-232.
- Liberg, C. (2008). Skrivandet i olika ämnen- lärares textkompetens. In R. T. Lorentzen and J. Smidt (Eds.), *Å skriv i alle fag* [To write in every subject]. Oslo: Universitetsforlaget.
- Liljedahl, P. (2006). Persona-based journaling: Alligning the product with the process. *International Group for the Psychology of Mathematics Education*, 4, 65.
- Lithner, J. (2003). Students' mathematical reasoning in university textbook exercises. *Educational Studies in Mathematics*, 52, 29-55.
- Lithner, J. (2006). *A framework for analysing creative and imitative mathematical reasoning*. Rapportserie, Matematik, Umeå: Umeå universitet.
- Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics*. 67(3), 255-276.
- Love, K. (2008). Literacy across the school subjects: a multimodal approach. In L. Unsworth (Ed.), *Multimodal semiotics: Functional Analysis in contexts of education*. London: Continuum International Publishing.
- Love, E., & Pimm, D. (1996). 'This is so': a text on texts. In A. J. Bishop (Ed.), *International Handbook of mathematics education*. Dordrecht, Boston, London: Kluwer Academic Publishing.

- Lundberg, I., & Sterner, G. (2006). *Räknesvårigheter och lässvårigheter under de första åren - hur hänger de ihop?* [Counting disabilities and reading disabilities during the first Years- how are they connected]. Stockholm: Natur och Kultur.
- Löwing, M. (2004). *Matematikens konkreta gestaltningar. En studie av kommunikationen lärare - elev och matematiklektionens didaktiska ramar.* [The concrete formation of mathematics teaching. A study of communication between teachers and pupils and the educational framework of mathematical classrooms] (Doctoral dissertation, Göteborg University, Sweden).
- Makar, K. (2014). Young children's explorations of average through informal inferential reasoning. *Educational Studies in Mathematics*, 86(1), 61-78.
- Mandler, G. (2002). Origins of the cognitive (r) evolution. *Journal of the History of the Behavioral Sciences*, 38, 339-353.
- McKenney, S., & Reeves, T. C. (2012). *Conducting educational design research*. London: Routledge.
- McKenney, S., Nieveen, N., & van der Akker, J. (2006). Design research from a curriculum perspective. In J. Van den Akker, K. Gravemeijer, S. McKenney, and N. Nieveen (Eds.) *Educational design research*, 67-90. London: Routledge.
- McKeown, M.G., Beck, I. L. & Blake, R.G.K. (2009). Rethinking reading comprehension. A comparison of instruction for strategies and content approaches. *Reading Research Quarterly*, 44, 218-253.
- Meaney, T. & Flett, K. (2006). Learning to read in mathematics classrooms. *The Australian Mathematics Teacher*, 62(2), 10-16.
- Meaney, T., Trinick, T., & Fairhall U. (2009). *Mathematics: She'll be wright!* Wellington: NZCER.
- Meaney, T., Trinick, T. & Fairhall U. (2012). *Collaborating to meet language challenges in indigenous mathematics classroom*. London: Springer.
- Minsono, T., & Takeda, K. (2012). *An investigation of the mathematical writing abilities of students in public elementary school in Japan*. Paper presented at the 12th international congress on mathematical education, July 8-15, Seoul, Korea.
- Morgan, C. (1996). The language of mathematics": towards a critical analysis of mathematics texts. *For the Learning of Mathematics*, 16(3), 2-10.
- Morgan, C. (1998). *Writing Mathematically: The discourse of investigation*. London: Falmer Press.
- Morgan, C. (2006). What does Social Semiotics have to Offer Mathematics Education Research? *Educational Studies in Mathematics*, 61(1), 219-245.
- Morgan, M., Gibbs, S., Maxwell, K., & Britten, N. (2002). Hearing children's voices: methodological issues in conducting focus groups with children aged 7-11 years. *Qualitative research*, 2(1), 5-20.

- Moschkovich J. (Ed.) (2010). *Language and Mathematics Education: Multiple Perspectives and Directions for Research*. Charlotte, NC: Information Age Publishing.
- Mullis, I. V. S., Martin, M. O., Foy, P., & Arora, A. (2012). *TIMSS 2011 international results in mathematics*. Chestnut Hill, MA: TIMSS & PIRLS International Study Center.
- Murphy, P. K., & Alexander, P. A. (2002). What counts? The predictive powers of subject-matter knowledge, strategic processing, and interest in domain-specific performance. *The Journal of Experimental Education*, 70(3), 197-214.
- Myndigheten för skolutveckling (the Swedish Agency for School Improvement) (2008). *Mer än matematik: om språkliga dimensioner i matematikuppgifter* [More than mathematics: the language dimensions in mathematics tasks]. Stockholm: Myndigheten för skolutveckling.
- Möllehed, E. (2001). *Problemlösning i matematik - en studie av påverkansfaktorer i årskurserna 4-9* [Problem solving in mathematics- a study of influencing factors in Year 4-9]. (Doctoral dissertation, Malmö University, Sweden). Malmö: Reprocentralen Lärarutbildningen.
- Nagy, W. E., Anderson, R. C., & Herman, P. A. (1987). Learning Word Meanings from Context during Normal Reading. *American Educational Research Journal*, 24(2), 237-270.
- Nation P. (2001). *Learning vocabulary in another language*. Cambridge: Cambridge University Press.
- National council of teachers of mathematics (NCTM) (2000). *The Principles and standards of school mathematics*. National Council of Teachers of Mathematics. Reston, VA: NCTM.
- National council of teachers of mathematics (NCTM) (2003). *Programs for Initial Preparation of Mathematics Teachers*. National Council of Teachers of Mathematics Reston, VA: NCTM .
- National council of teachers of mathematics (NCTM) (2008). *Navigating through reasoning and proof in grades 9-12*. National Council of Teachers of Mathematics Reston, VA: NCTM .
- National Curriculum (2008). National Curriculum. Value, aims and purposes. Retrieved from: <https://web.archive.org/web/20081221153026/http://curriculum.qca.org.uk:80/key-stages-1-and-2/subjects/mathematics/keystage1/index.aspx>
- NCF (2005). National curriculum framework. Retrieved from: <http://www.ncert.nic.in/rightside/links/pdf/framework/english/nf2005.pdf>
- Neef, N. A., Nelles, D. E., Iwata, B. A., & Page, T. J. (2003). Analysis of precurrent skills in solving mathematics story problems. *Journal of Applied Behavior Analysis*, 36(1), 21-33.

- Nieveen, N., McKenney, S., & Van den Akker, J.(2006). "Educational design research." *Educational design research*, 151-157.
- Niss, M. (2003). Mathematical competencies and the learning of mathematics: The Danish KOM project. In *3rd Mediterranean conference on mathematical education* (pp. 115-126).
- Niss, M., & Højgaard Jensen, T. (2002). In M. Niss & T. Højgaard Jensen(Eds.), *Kompetencer og matematiklæring: Ideer og inspiration til udvikling af matematikundervisning i Danmark*. København: Undervisningsministeriets forlag
- Nolan, K. (2012). Disposition in the field: viewing mathematics teacher education through the lens of Bourdieu's social field theory. *Educational Studies in mathematics*, 80(1-2), 201-215.
- Noonan, J. (1990). Readability problems presented by mathematics text. *Early child development and care*, 54, 57-81.
- Nordin, A. (2016). *Matematiska argument i helklassdiskussioner: En studie av elevers och lärares multimodala kommunikation i matematik i åk 3-5*. Lic.-avh. (sammanfattning) Stockholm : Stockholms universitet, 2016. Stockholm.
- Nunes, T., Bryant, P., Barros, R., & Sylva, K. (2012). The relative importance of two different mathematical abilities to mathematical achievement. *British Journal of Educational Psychology*, 82(1), 136-156.
- Nunes, T., Bryant, P., Evans, D., Gottardis, L., & Terlektsi, M. E. (2015). Teaching mathematical reasoning: Probability and problem solving in Primary School. *Nuffield Foundation*.
- O'Connor, M. C. (2001). "Can any fraction be turned into a decimal?" A case study of a mathematical group discussion. *Educational Studies in Mathematics*, 46, 143-185.
- Palinscar, A.S. (w.y). *Reciprocal Teaching*. Retrieved from: <http://www.psy.cmu.edu/~siegler/418-Palinscar.pdf>
- Palinscar, A. S. (2003). Collaborative Approaches to Comprehension instruction. In A. Sweet and C. E., Snow (Eds.), *Rethinking reading comprehension*. New York: The Guilford Press, 99-114.
- Palinscar, A. S. (2007). Reciprocal teaching 1982 to 2006: The role of research, theory, and representation in the transformation of instructional research. In *56th yearbook of the National Reading Conference*. Oak Creek, WI: *National Reading Conference*.
- Paliscar, A. S., & Brown, A. L. (1984). Reciprocal teaching of comprehension-fostering and comprehension-monitoring activities. *Cognition and Instruction*, 1 (2), 117-175.
- Palinscar, A. S., & Schutz, K. M. (2011). Reconnecting strategy instruction with it's theoretical roots. *Theory Into Practice*, 2(50), 85-92.

- Pearson, P.D., & Hamm, D.N. (2005). The assessment of reading comprehension: a review of practices-Past, Present and Future. In S.G. Paris & S.A. Stahl (Eds.), *Children's Reading Comprehension and Assessment* (p. 13-70). Mahway, New Jersey: L Erlbaum Associates.
- Pettersson, R. (2008). *Bilder i läromedel*. (2. uppl.) Tullinge: Institutet för info-logi. Retrieved from: <http://www.bilderlernen.at/Bilder%20i%20l%E4ro-medel.pdf>
- Pimm, D (1987). *Speaking mathematically: Communication in mathematics Classrooms*. London: Routledge.
- Player-Koro, C. (2011). Marginalising students' understanding of mathematics through performative priorities: a Bernsteinian perspective. *Ethnography and Education* , 6 (3), 325-340.
- Plomp, T. (2007). Educational Design research: an introduction. In T. Plomp and N. Nieveen(Eds.), *An introduction to educational design research*, Enschede: Netzdruk.
- Prediger, S., Bikner-Ahsbahs, A., & Arzarello, F. (2008). Networking strategies and methods for connecting theoretical approaches: First steps towards a conceptual framework. *ZDM*, 40(2), 165-178.
- Prediger, S. Gravemeijer, K., & Confrey, J. (2015). Design research with a focus on learning processes – an overview on achievements and challenges. *ZDM*, 7(6), 877-891.
- Pressley, M. (2000). What should comprehension instruction be the instruction of? In M.L. Kamil, P. B. Mosenthal, P.D. Pearson, & R. Barr (Eds). *Hand-book of reading research*, 545-560. Mahway, NJ: Erlbaum.
- Pressley, M. (2002). Metacognition and self-regulated comprehension. *What research has to say about reading instruction*, 3, p.291-308.
- Pressley, M., & Allington, R.L (2015). *Reading instructions that works- The Case for Balanced Teaching*. (Fourth Edition). New York: The Guilford Publications, Inc.
- Pugalee, K. (2004). A comparison of verbal and written descriptions of students' problem solving processes. *Educational Studies in Mathematics* 55, 27-47.
- Quirk, P. J. (2010). *Using reciprocal teaching and learning methods to enhance comprehension in mathematics word problems: a thesis presented in partial fulfillment of the requirements for the degree of Master of Education, Massey University, Palmerston North, New Zealand* (Doctoral dissertation, Massey University).
- Ratekin, N., Simson, M. L., Alvermann, D.E., & Dishner, E. K.(1985). Why teachers resist content reading instruction. *Journal of Reading*, 30, 432-437.

- Reehm, S. P., & Long, S. A. (1996). Reading in the Mathematics Classroom. *Middle School Journal*, 27(5), 35-41.
- Reeves, T.C. (2006). Design Research from a technology perspective, In J. Van der Akker, K. Gravemeijer, S. Mckeeney and N. Nieveen (Eds.), *Educational Design Research*. Routledge: London and New York.
- Reichenberg, M., & Lundberg, I. (2013). *Läsförståelse- genom strukturerade textsamtal*. Stockholm: Natur och Kultur.
- Reinking, D., & Bradley, B. A. (2008). *On Formative and Design Experiments*. Teacher College Press: Amsterdam and New York.
- Remillard, J. T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research*, 75(2), 211-246.
- Riddle Buly, M., & Valencia, S. W. (2002). Below the bar: Profiles of students who fail state reading assessments. *Educational Evaluation and Policy Analysis*, 24(3), 219-239.
- Riesbeck, E. (2008). *På tal om matematik. Matematiken, vardagen och den matematikdidaktiska diskursen* [Speaking of mathematics: mathematics, everyday life and school mathematics discourse]. (Doctoral dissertation, Linköping University, Sweden). Linköping: Linköping University.
- Roehler, L. R., & Cantlon, D. J. (1997). Scaffolding: A powerful tool in social constructivist classrooms. In K. Hogan & M. Pressley (Eds.), *Scaffolding student learning: Instructional approaches and issues* (pp. 6-42). Cambridge, MA: Brookline.
- Rogoff, B. (1990). *Apprenticeship in thinking: Cognitive development in social context*. New York: Oxford University Press.
- Rosenshine, B., & Meister, C. (1994). Reciprocal teaching: A review of the research. *Review of Educational Research*, 64(4), 479-530.
- Runesson, U. (2010). Learning from variation. Differences in learners' ways of experiencing differences. In B. Sriraman, C. Bergsten, S. Goodchild, C. Michelsen, G. Palsdottir, & L. Haapasalo (Eds.), *The first sourcebook on Nordic research in mathematics education*. Charlotte, N.C: Information Age Publishing.
- Ruthven, K., Laborde, C., Leach, J., & Tiberghien, A. (2009). Design tools in didactical research: Instrumenting the epistemological and cognitive aspects of the design of teaching sequences. *Educational Researcher*, 38(5), 329-342.
- Rönnerman K. (2008). Empowering Teachers. Action research in partnership between school and university. In Rönnerman, K. Moksnes Furu, E. & Salo, P (Eds.). (2008) *Nurturing Praxis: Action research in partnerships between school and university in a Nordic light*. (pp 157-174). Rotterdam: Sense Publishers.

- Saussure, F. D. (1996). *Course in general linguistics*. (1. ed.) New York: McGraw-Hill.
- Schleppegrell, M. J. (2004). *The language of schooling – A functional linguistics perspective*. London: Lawrence Erlbaum Associates.
- Schleppegrell, M. J. (2007). The Linguistic Challenges of Mathematics Teaching and Learning: A Research Review. *Reading & Writing Quarterly*, 23(2), 139-159.
- Schleppegrell, M. J. (2010). Language in mathematics teaching and learning. In *Language and mathematics education: Multiple perspectives and directions for research*, 73-112.
- Schoenfeld, A.H. 1985. *Mathematical Problem Solving*. San Diego: Academic Press.
- Segerby, C. (2014). Reading strategies in mathematics: a Swedish example. In S. Pope (Ed.) *Proceeding at BSLRM conference in Nottingham* (pp. 311-318). Nottingham: British Society for Research into Learning Mathematics. Available from <https://bsrlm.org.uk/BCME8/BCME8-Full.pdf>
- Segerby, C. (2016). Writing in mathematics lessons in Sweden. *Proceedings of the ninth Congress of the European Society for Research in Mathematics Education*, pp. 1490-1496. Prague: Czech Republic. Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education. Available from: <https://hal.archives-ouvertes.fr/hal-01287787/document>
- Segerby (submitted). *Four Grade students' comprehensions strategies when reading their mathematics textbook*.
- Segerby, C., & Chronaki, A. (submitted). *Primary students' participation in school based mathematical reasoning practices: Coordinating Reciprocal Teaching and Systemic Functional Linguistics to support reasoning in the Swedish context*.
- Segerby, C. (manuscript in process). *How does the mathematical textbook position the reader?*
- Sfard, A. (1998). On two metaphors for learning and the danger of choosing just one. *Educational Researcher*, 25(4), 4-13.
- Shanahan, T., & Shanahan, C. (2008). Teaching Disciplinary literacy to Adolescents: Rethinking Content-Area Literacy. *Harvard Educational Review* 78(1), 40-49.
- Shanahan, T., & Shanahan, C. (2012). What is disciplinary literacy and why does it matter?. *Topics in Language Disorders*, 32(1), 7-18.
- Shanahan, T. (2014). To teach comprehension strategies or not teach them. <http://www.shanahanonliteracy.com/2014/08/to-teach-comprehension-strategies-or.html>. Retrieved 2016-10-19.

- Shepherd, M. D., Selden, A., & Selden, J. (2012). University Students' reading of Their First-Year Mathematics Textbooks. *Mathematical Thinking and Learning*, 14(3), 226-256.
- Shuard, H., & Rothery, A. (Eds.) (1988). *Children reading mathematics*, London: John Murray.
- Sidenvall, J. (2015). *Att lära sig resonera: om elevers möjligheter att lära sig matematiska resonemang*. Licentiatavhandling (sammanfattning) Linköping: Linköpings universitet, 2015. Norrköping.
- Sidenvall, J., Lithner, J., & Jäder, J. (2015). Students' reasoning in mathematics textbook task-solving. *International journal of mathematical education in science and technology*, 46(4), 533-552.
- Silver, E.A., & Smith, M.S. (1996). Building discourse communities in mathematics classrooms: a worthwhile but challenging journey. In P. Elliott (Ed.). *Communicating in mathematics, K-12 and beyond* (pp.20-28). Reston, VA: National Council of Teachers of Mathematics.
- Smit, J. (2013). *Scaffolding language in multilingual mathematics classrooms*. (dissertation). Utrecht University.
- Siebert, D., & Draper, R. J. (2008): Why Content-Area Literacy Messages Do Not Speak to Mathematics Teachers: A Critical Analysis. *Literacy Research and Instruction* 47(4), 229-245.
- Sjöblom, M. (2015). *Promoting student-to-student interactions in mathematics: a study in a multilingual upper secondary classroom*. Lic.dessertation, Malmö University, Faculty of Education and Society.
- Sjöström, B., & Sjöström, J. (2016). *Prima Formula matematik.4* (second edition). Malmö: Gleerups.
- Skinner, B. F. (1976). *About Behaviourism*. New York: Vintage Books.
- Skog, K. (2014). *Power, positioning and mathematics - discursive practices in mathematics teacher education: Climbing Lion's Head*. Diss. Stockholm : Stockholms universitet, 2014.
- Skolinspektionen (the Schools Inspectorate) (2009). Rapport 2009:5: *Undervisningen i matematik – utbildningens innehåll och ändamålsenlighet* [Teaching in mathematics - educational content and appropriateness]. Stockholm: Skolverket.
- Skolverket (the Swedish National Agency for Education) (2003). *Lusten att lära – med fokus på matematik* [Desire to learn-with focus on mathematics]. *Nationell kvalitetsgranskningar 2001-2002*. Stockholm: Skolverket.
- Skolverket (2007). *Vad händer med läsningen? En kunskapsöversikt om läsundervisningen i Sverige 1995-2007*. Stockholm: Skolverket.
- Skolverket. (w. y). *Om ämnet matematik*. Retrieved from: <http://www.skolverket.se/loroplaneramen-och-kurser/gymnasieutbildning/gymnasieskola/mat?tos=gy&subjectCode=mat&lang=sv>

- Skolverket (the Swedish National Agency for Education (2011). *Läroplan för grundskolan, förskoleklassen och fritidshemmet 2011* [Curriculum for elementary school, presechool and youth leisure centre 2011]. Stockholm: Fritzes.
- Skolverket (2012a). *PIRLS 2011: läsförmågan hos svenska elever i årskurs 4 i ett internationellt perspektiv*. Stockholm: Skolverket.
- Skolverket (2012b). *TIMSS 2011. Svenska grundskoleelevers kunskaper i matematik och naturvetenskap i ett internationellt perspektiv*, No. 380. Stockholm: Skolverket.
- Skolverket (2013). *PISA 2012. 15-åringars kunskaper i matematik, läsförståelse och naturvetenskap*, No. 398. Stockholm: Skolverket.
- Skolöverstyrelsen (1969). *Läroplan för grundskolan*. Stockholm: Utbildningsförlaget.
- Skott, J. (2013). Understanding the role of the teacher in emerging classroom practices: searching for patterns of participation. *ZDM*, 45(4), 547-559.
- Smit, J., & van Eerde, A. A (2011). A teacher's learning process in dual design re-search: learning to scaffold language in a multilingual mathematics classroom. *ZDM*, 43(6-7), 889-900.
- Snow, C. E., & Juel, C. (2007). Teaching children to read: What do we know about how to do it? In M. J. Snowling & C. Hulme (Eds.), *The science of reading: A handbook* (pp. 501–520). Malden, MA: Blackwell.
- Sterner, H. (2015). *Problematisera "görandet": lärares lärande om kommunikation och resonemang i matematikundervisningen i en organiserad praktikgemenskap*. Licentiatavhandling Växjö: Linnéuniversitetet, 2015. Växjö.
- Stone, C.A. (1998). The metaphor of scaffolding: Its utility for the field of learning disabilities. *Journal of learning disabilities*, 31, 344-364.
- Stylianides, G. J. (2009). Reasoning-and-proving in school mathematics textbooks. *Mathematical Thinking and Learning*, 11(4), 258–288
- Sullivan, P., & Lilburn, P. (2002). *Good questions for math teaching: why ask them and what to ask, K-6*. Sausalito, CA: Math Solutions Publications Thomsen & Fleming.
- Sumpter, L. (2013). Themes and interplay of beliefs in mathematical reasoning. *International journal of Science and Mathematics Education*, 11(5), 1115-1135.
- Sweet, A. P., & Snow, C. (2002). Reconceptualising Reading Comprehension. In C.C. Block, L. Gambrell, & M. Pressley (Eds.), *Improving Comprehension Instruction: Rethinking Research Theory a, and Classroom Practice* (pp. 17-53). San Francisco: Jossey-Bass.
- Takala, M. (2006). The effects of reciprocal teaching on reading comprehension in mainstream and special (SLI) education. *Scandinavian Journal of Educational Research*, 50(5), 559-576.

- Teledahl, A. (2016). *Knowledge and writing in school mathematics: a communicational approach*. Diss. (sammanfattning) Örebro: Örebro universitet.
- Temple, C., & Doerr, H. M. (2012). Developing fluency in the mathematical register through conversation in a tenth-grade classroom. *Educational Studies in Mathematics*, 81(3), 287-306.
- Thomson, S., & Fleming, N. (2004). *Summing it up: Mathematics achievement in Australian schools in TIMSS 2002*.
- Thurber, R.S., Shinn, M.R., & Smolkowski, K. (2002). What is measured in mathematics tests? Construct validity of curriculum-based mathematics measures. *School Psychology Review*, 31, 498-513.
- Timperley, H. (2008) Teacher professional learning and development. Educational Practices Series 18. International Academy of Education: Brussels
- Timperley, H., & Alton-Lee, A. (2008). Reframing teacher professional learning: An alternative policy approach to strengthening valued outcomes for diverse learners. *Review of research in education*, 32(1), 328-369.
- Tjernberg, C. (2013). *Framgångsfaktorer i läs- och skrivlärande: en praxisorienterad studie med utgångspunkt i skolpraktiken*. Diss. Stockholm: Stockholms universitet, 2013. Stockholm.
- Toulmin, S. E. (2003). *The Uses of Argument*. Cambridge, U.K.: Cambridge University Press.
- Turner, R., Blum, W., & Niss, M. (2015). Using Competencies to Explain Mathematical Item Demand: A Work in Progress. In K. Stacey & R. Turner (Eds), *Assessing Mathematical Literacy* (85-115). Switzerland: Springer.
- Unsworth, L. (1997). Scaffolding Reading of Science Explanations: Accessing the Grammatical and Visual Forms of Specialized Knowledge. *Literacy*, 31(3), 30– 42.
- Utbildningsdepartementet (1994). *Läroplaner för det obligatoriska skolväsendet och de frivilliga skolformerna: Lpo 94 : Lpf 94*. Stockholm: Utbildningsdep.
- Van den Akker, J., Gravemeijer, K., McKenney, S., & Nieveen, N. (2006). Introducing educational design research. *Educational design research*, 1, 3-7.
- Van Eerde, H. A. A., & Hajer, M. (2009). The integration of mathematics and language learning in multiethnic schools. In M Cesar & K. Kompulainen (Eds). *Social Interactions in multicultural settings*, pp. 269-296.
- Vellutino, F. R. (2003). Individual differences as sources of variability in reading comprehension in elementary school children. In A. P. Sweet & C. E. Snow (Eds.), *Rethinking reading comprehension* (pp.51-81). New York: Guilford Press.
- Vetenskapsrådet (2007). *Forskningsetiska principer inom humanistisk- och samhällsvetenskaplig forskning*. Vetenskapsrådet: Elanders Gotab.

- Vilenius-Tuohimaa, P., Aunola, K., & Nurmi, J-E. (2008). The association between mathematical word problems and reading comprehension. *Educational Psychology*, 28 (4), 409-426.
- Vygotsky, L.S. (1962). *Thought and Language*. Cambridge, MA: MIT Press
- Vygotsky, I. (1978). *Mind in society: the development of higher psychological processes*. Cambridge, MA: Harvard University Press.
- Wagner, D. (2012). Opening mathematics texts: Resisting the seduction. *Educational Studies in Mathematics*, 80(1-2), 153-169.
- Watkins, J.K., Miller, E., & Brubaker, D. (2004). The role of the visual image: what are students really learning from pictorial representations? *Journal of Visual Literacy* 24(1), 23-40.
- Wedge, T. (2009). Combining and coordinating theoretical perspectives in mathematics education research. In Arzarello, F. et al. (eds.), *Proceedings of the Sixth Conference of European Research in Mathematics Education (CERME6)*, January 28 to February 1, 2009, Lyon. Lyon: Université de Lyon.
- Wedge, T. (2010). Ethnomathematics and mathematical literacy: people knowing mathematics in society. In C. Bergsten, E. Jablonka & T. Wedge (Eds.), *Mathematics and mathematics education: cultural and social dimensions. Proceedings of MADIF 7: the seventh Mathematics Education Research Seminar* (pp. 31–46). Linköping: SMDF.
- Weinberg, A., Wiesner, E., Benesh, B., & Boester, T. (2012). Undergraduate students' self-reported use of mathematics textbooks. *Primus*, 22(2), 152-175.
- Wells, G. (1994). The complementary contributions of Halliday and Vygotsky to a “language-based theory of learning”. *Linguistics and education*, 6(1), 41-90.
- Westlund, B. (2009). *Att undervisa i läsförståelse* [Teaching in reading comprehension]. Stockholm: Natur och Kultur.
- Westlund, B. (2013). *Att bedöma elevers läsförståelse: en jämförelse mellan svenska och kanadensiska bedömningsdiskurser i grundskolans mellanår*. (1. utg.) Diss. Stockholm: Stockholms universitet, 2013. Stockholm.
- Whitehead, A.N., & Russell, B. (1962). *Principia mathematica* to *56. Cambridge: Cambridge University Press.
- Wolf, M.K, Crosson, A.C., & Resnick, L.B. (2005). Classroom talk for rigorous reading comprehension instruction. *Reading Psychology*, 26, 27-53.
- Wood, D., Bruner, J., & Ross, G. (1976). The role of tutoring in problem solving. *Journal of child psychology and psychiatry*, 17, 89-100.
- Yackel, E., & Hanna, G. (2003). Reasoning and proof. In J. Kilpatrick, G. Martin & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (227-236). Reston, VA: National Council of Teachers of Mathematics.

- Yang, K-L., & Lin, F-L. (2012). Effects of reading-oriented tasks on students' reading comprehension of geometry proof. *Mathematics Educational Research Journal*, 24, 215-238.
- Åberg-Bengtsson, L. (1992). Barns förståelse av diagram – en studie av elever mellan 6 och 13 år. I G. Emanuelsson, B. Johansson, B. Rosén, & R. Ryding (Red.), *Dokumentation av 7:e Matematikbiennalen*. Göteborg: Göteborgs universitet Institutionen för ämnesdidaktik
- Österholm, M. (2006). *Kognitiva och metakognitiva perspektiv på läsförståelsen inom matematik* [Cognitive and metacognitive perspectives on reading comprehension in mathematics]. Doctoral dissertation, Linköping University, Sweden. Linköping: Uni Tryck.
- Österholm, M. (2007). A reading comprehension perspective on problem solving. In C. Bergsten & B. Grevholm (Eds.). *Developing and Re-searching Quality in Mathematics Teaching and Learning. Proceedings of MADIF 5, the 5th Swedish Mathematics Education Research Seminar, Malmö, January 24-25, 2006* (pp. 136-145). Linköping: SMDF.
- Österholm, M., & Bergqvist, E. (2013). What is so special about mathematical texts? Analyses of common claims in research literature and of properties of textbooks. *ZDM*, 45(5), 751-763.

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