ULRIKA RYAN

MATHEMATICS CLASSROOM TALK IN A MIGRATING WORLD

Synthesizing epistemological dimensions

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For Ella.

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Malmö in October 2019 / Ulrika Ryan

LIST OF ARTICLES

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- III Ryan, U. (2019). Mathematical preciseness and epistemological sanctions. *For the Learning of Mathematics*, 39(2), 25-29.
- IV Ryan, U., & Chronaki, A. (submitted). A joke on precision? Revisiting ‘precision’ in the school mathematics discourse.

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PROLOGUE

This thesis consists of four papers and a framing document of my journey as I explored three divides that manifest in endless constellations in socially and language diverse mathematics classrooms. I have not named the divides, just numbered them. In this prologue I provide some initial shattered and scattered glimpses of the divides, viewed as snapshots from a personal viewpoint. The divides are all located along a particular line of inquiry, which is the title of this thesis: “Mathematics classroom talk in a migrating world.” Divide # 1 is located in the last part of the sentence, “a migrating world.” Divide #2 is located at the centre of the sentence, “talk.” Divide #3 is located at the beginning of the sentence, “mathematics.” Only the word “classroom” is left. “Classroom” is the spatiality and temporality of the encounters of the three divides, which is where the concern of this thesis lies.

Divide #1

Many of us live our everyday lives simultaneously in several places around the globe. We Skype with relatives far away, we take part in our friends’ daily lives elsewhere on social media. For example, my neighbour is from Australia. Her sister in Australia is ill with cancer. I am concerned about her sister’s recovery.

In the autumn of 2015 a great number of people sought refuge in Sweden. Most of them arrived at the train station in Malmö, only a few metres from where I work. My daughter, whose father is Brit-

ish and lives in Oman, volunteered to support unaccompanied refugee children. I took part in some of the children's experiences that she shared with me. During the Balkan war in the 1990s, when I was studying to become a teacher, I was engaged in the reception of Balkan refugees in Malmö. It was a bewildering experience. As a child I spent several summer holidays at the house of a Croatian business associate of my father's. Many of the places the refugees came from were familiar to me. My shattered experiences of the migrating world are far from uncommon. However, they are experiences from one particular side of a great divide. They are the experiences of a privileged white Western middleclass woman in her early 50s. On the other side of the divide are experiences of the Others, those who are not necessarily privileged, nor Western, white, or middleclass. This great divide and the shattered/scattered experiences on both sides of it form Divide #1 of this thesis.

Divide #2

We talk; we use language. To make ourselves intelligible we choose the language we think our interlocutors will understand and the language we want to be understood as being speakers of. Writing this thesis, I use a particular kind of academic English, hoping to be understood and recognized as an academic. Teaching mathematics in a rural school in the south of Sweden, I used a particular kind of Swedish, hoping to be recognized as a good teacher. When talking to one of my neighbours, an elderly man who has lived in my small village all his life, I try to use yet another kind of Swedish, hoping to be recognized as a friendly neighbour. Overhearing my daughter's conversations with her friends, I realize that they speak differently from me. The words they use might also be mine, but the meaning is somehow different. I realize that I might not be able to speak like them, that I cannot be recognized as a speaker of "adolescentish."

However, in Sweden, I am at home language-wise. I speak the languages of power and hegemony. I can choose and switch between languages to make my voice heard and to make it count where I want it to count. I am on the one side of a divide. On the other side are the Others, those who speak minority, non-

hegemonic languages; their voices might not be recognized or even heard. This great divide and the shattered/scattered experiences on both sides of it form Divide #2.

Divide #3

In mathematics classrooms, there is talk of mathematics. I know mathematics. I am a mathematics teacher and a mechanical engineer. The other day I visited a friend. I spoke to her husband, who runs a small construction company. We talked about me writing a thesis on mathematics education. He said that I must really know mathematics and said that he did not. Besides, he said, to my great surprise, he never uses mathematics. As a small child, I was fascinated with logic and science. At one point I thought I understood the logic of all languages. I had learnt that the Swedish word *stjärna* is star in English. I concluded that the word for the bird species common starling, *stare* in Swedish therefore must be *stjärn* in English. I applied this logic to other words too, thinking that I had come into possession of a magnificent knowledge system. I soon learnt that my knowledge system was not an accepted one. During my first years at school, I battled with mathematics. It too did not seem to fit the logic of my knowledge systems. Today I know how to be recognized as a knower of mathematics. I am in possession of a hegemonic knowledge system. I am on the one side of a great divide. On the other side are the Others, those whose logic and knowledge systems are non-hegemonic, non-mathematical, or even a-mathematical. This great divide and shattered/scattered experiences on both sides of it is form Divide #3 of this thesis.

I have now provided some glimpses of the three divides my thesis is concerned with. Although you may recognize the divides, you might feel a bit at unease with them. So do I. “Those divides must be re-thought,” I sometimes think. But how? Along what path?

CONTRIBUTION OF THE THESIS

Below I provide a rough overview of the state of the literature in mathematics education research that is relevant to this thesis and an equally rough overview of the contributions of this thesis. A more in-depth literature review is provided in the chapter called *Language and Epistemological Diversity*. The results and answers to my research questions are further elaborated on in the chapter called *Connecting the Articles* and in the four articles. The aim of the thesis is considered in the chapter called *Discussion*.

State of the literature

- In general, studies on mathematics education in migration contexts have conceptualized the research objects as matters of dichotomization between named languages, which emphasizes the communicative and/or socio-political role of language. The role of language in these studies seems to be an adjustable variable in the classroom, while the epistemological role of mathematics often is treated as an invariable constant.
- Many studies show how the use of students' first language in mathematics learning is beneficial to their learning. However, contemporary diversified migration patterns produce complex student cohorts, where a classroom may contain multiple languages but no two students share a language other than the language of instruction.

- Recently, calls have been raised for research that considers the complex diversity of student cohorts in contemporary classrooms and embraces pluralistic views on the role of language and mathematical knowledge in mathematics classrooms.

Contribution of this thesis to the literature

- This thesis offers an updated perspective on the current research while bringing together a) linguistic inferentialism as an alternative to the representation paradigm, b) social interaction and ecological approaches on knowledge to frame the relationship between language and c) mathematics in students' talk in a classroom with a complex diversity of languages and socio-economic backgrounds.
- This thesis contributes knowledge on how students demonstrate solidarity and sometimes perform aggressive actions towards each other in their encounters with mathematical knowledge and language diversity.
- This thesis takes an ecology-based relational approach towards language and epistemology in order to provide tools for considering students' responsive translanguaging in multilingual classrooms with no shared languages (except the language of instruction).
- This thesis is the first to use inferentialism for ecology-based approaches on social epistemological issues of multilingualism in mathematics education research.

I am aware that readers, including the future I, may consider other contributions of this thesis (or none) as they will read this thesis from viewpoints that differ from my present one.

INTRODUCTION AND RATIONALE FOR THE THESIS

Usually a thesis starts with an explanation of its rationale. For instance, the doctoral student's concern might be with an issue that has been at heart since s/he encountered some particular professional or personal teaching experience. For me it was a bit different. Of course, my 20 years of experience as a mathematics teacher, mostly with students aged 10 to 13, have deeply influenced this thesis, but its rationale is not merely to be found there.

When I started to think about writing a doctoral thesis my plan was to continue to develop my interest in technology-enhanced mathematics learning and teaching and to build on my master's thesis on this subject. However, I was accepted to a PhD position on mathematics education and migration. Although I was tremendously happy to be accepted, I must admit that I also was a bit gloomy not being able, at least not the way I soon came to see it, to continue the work on Information and Communication Technology (ICT) that I had developed in my master's thesis and elsewhere together with colleagues from universities all over Sweden.

As a fresh PhD student in the autumn of 2016, I searched for literature on migration AND mathematics education in databases but got only 30 hits in ERC and 24 in ERIC. However, when I searched for mathematics education AND multilingual OR bilingual I got over 5000 hits in ERC and a similar result in ERIC. Therefore, I assumed that language diversity was a key concern in mathematics classrooms that contain students with an experience of or background informed by migration. I started to read that lit-

erature. I learnt about how bilingualism in the early 70s was considered a problem that hindered students' mathematics learning (Phakeng, 2016) and how through political struggles (Planas & Civil, 2013) multilingualism in research, and often in policymaking and practice, is now viewed as a resource (Chronaki & Planas, 2018). I learnt that language is not only a tool for thought and communication; it also privileges some while others are marginalized (Setati, 2008). I read about dilemmas that teachers might face when trying to decide what language to use in the classroom: the language of their students or the hegemonic language of academic institutionalized mathematics (Adler, 1998). I read about tensions between students' home and school language, between language policies and classroom practices, and between the formal and informal language of mathematics (Barwell, 2012).

I learnt that language proficiency of emergent speakers of the language of instruction can be divided into two categories: basic intrapersonal communication skills (BICS) and cognitive academic language proficiency (CALP) (Cummings, 2000). Emergent speakers of the language of instruction develop proficiency in BICS much faster than in CALP. Therefore, for achievement in academic school mathematics, students' first languages (L1s) ought to be used as a valuable recourse. To be able to use students' L1s as a resource, pedagogical adjustments and strategies are needed (Planas, 2012). Students and teachers switching between languages and students talking with peers in their L1s during class was advocated in the literature. When students' L1s are cultivated pedagogically, they can function as a lever in multilingual students' educational journey from informal mathematics talk in their L1s to formal mathematics talk in the language of instruction (Bose & Clarkson, 2016; Halai & Muzaffar, 2016; L. Webb & P. Webb, 2016). I learnt that the language-as-resource metaphor (Planas & Setati-Phakeng, 2014) underpins much research on mathematics education.

Moreover, the research I read about was usually conducted in settings where there was a minority language shared among the emergent speakers of the language of instruction. For instance, in the work of Planas (2011, 2012, 2014, 2018) and Moschkovich

(2002, 2008, 2015a, 2018a) the minority language was Spanish. In the work of Barwell, it was English in French immersion schools (Barwell, 2014, 2018a, 2018b).

There are six official minority languages in Sweden: Finnish, Sami, Romani, Yiddish, Meänkieli (Tornedal Finnish), and Swedish sign language. The minority languages have been legally recognized to protect the cultural and historical heritage of their respective speech communities. These communities are given certain rights on that basis, such as school education in their language. Since Sweden, like many other countries, has experienced diversified migration during different periods of time, there is a high complexity in demography (Meissner & Vertovec, 2015) reflected in a number of contemporary Swedish classrooms (Meyer, Prediger, César, & Norén, 2016). Students are first, second, or third generation immigrants from all over the world. In Malmö, which is Sweden's third largest city, people originate from no less than 182 different countries (Malmö Stad, 2019). Therefore, it is common to find school classes containing students with various degrees of mastery of Swedish, the language of instruction in most Swedish classrooms, as well as many other languages. It is likely that no student shares his or her L1 with any other student, which makes practices of, for example, code-switching insufficient. As I was reading the literature, I began to wonder how a student's L1 is determined when children grow up with several different languages. The conditions described above demand research approaches, theorizations, and classroom practices that move beyond dichotomization between one particular minority language and the language of instruction. Some research ought to embrace the complexities of global diversified migration that frame much of everyday mathematics teaching and learning in Sweden, I thought.

Although the research advocated flexible views on language use, mathematics was often approached as fixed, neutral, and universal, mirroring or re-presenting *one* 'real' world (Chronaki & Planas, 2018). This view produces the (in)formal mathematics dichotomies mentioned above, and ignores understandings that embrace flexible, pluralistic views of mathematics (Parra & Trinick, 2018). In a migrating world, children do not bring *one* mathematics to the

classroom. They bring a plurality of mathematics (Radford, 2012) whose knowledge systems are embedded in the languages they speak (Knijnik, 2012). As their languages and mathematical epistemologies encounter one particular school mathematics that originates from hegemonic Western mathematics in the language of instruction, all three divides mentioned above are present; migration, languages, and mathematics.

This thesis is an expedition into and beyond students' mathematics talk in classrooms framed by migration as a matter of dichotomization between named languages and (in)formal aspects of *one* fixed mathematics. It is an attempt to make sense of how students grapple with and move about the divides that those dichotomizations shapes. My expedition is guided by two questions:

How do students in a Grade 5 classroom framed by migration navigate language and epistemological divides when talking about mathematics?

What theoretical conceptualization of epistemological dimensions of language diversity can be used to frame the students' navigation of the divides?

'Navigation' and 'navigate' are the metaphors I use for finding one's way in spaces that do not have established paths to follow.

In this thesis epistemological divides articulate difference when individuals and/or cultures take and treat something as mathematical knowledge. They emerge when people do and talk about mathematics.

To grapple with the research questions above, I have made some incursions into the three divides with the purpose of providing a deeper understanding of how social and language diversity connects to epistemological diversity in mathematics classrooms. In this thesis I use the expression 'social and language diversity' descriptively to emphasize that epistemological dimensions of classroom talk are related socio-economic class, as well as to students' language backgrounds when students from either or both groups are emergent speakers of the academic school discourse and/or the

language of instruction (Prediger & Schüler-Meyer, 2017). My focus is not primarily on how students learn mathematics through their navigation, but rather on how the students inhabit the learning space together—how they relate to each other—as they navigate.

To learn about multiple relational aspects of how students navigate language and epistemological divides when they talk about school mathematics, I have used a flexible research design (Robson, 2002) in a multilingual, yet ‘Swedish-only’ Grade 5 (students aged 11) classroom in the south of Sweden.

The answers to the first research question are primarily found in the four articles, while the answer to the second research question is developed in the chapter called *Connecting the Articles*.

The aim of this thesis is to provide theoretical insights, illuminated by empirical ones, on some relational aspects of how students in a social and language diverse classroom grapple with language and epistemological divides when they talk about and do school mathematics, from perspectives that do not dichotomize between languages and forms of knowledges. I intend to make sense of how students grapple with the dichotomies between languages and forms of mathematical knowledge from a perspective that tries not to reproduce or reinforce the very same dichotomies.

This thesis provides a way to approaching my feeling of unease about the scattered and shattered experiences on each side of the divides. I try to re-think the divides, because avoiding dichotomization has the potential to acknowledge distinctions and highlight difference, that is to consider the particularities and differences of a whole (rather than separated shattered and scattered bits and pieces), without upholding ridged demarcations or putting them in opposing positions (Moschkovich, 2018b). I turned my focus away from first and second language separations and from separations between formal, informal, and other kinds of mathematics when I looked for answers to my research questions. Instead, I recognized distinctions in the students’ language and mathematics use to consider interrelationships among the distinctions as a way of making sense of how students navigate the divides.

LANGUAGE AND EPISTEMOLOGICAL DIVERSITY IN MATHEMATICS EDUCATION

In this chapter I review how research in mathematics education has approached issues related to language and epistemological diversity. I provide an overview of how research has grappled with and understood the divides I presented in the prologue—divides associated with language and mathematical knowledge. First, I review research that is concerned with language diversity. I then consider epistemological diversity from the perspective of ways of knowing mathematics as a body of knowledge and as a cultural activity.

My literature searches were conducted in several ways. Primarily I used the search engine Google Scholar and the databases ERC and ERIC by EBSCO. When I read key literature, I used the “snowball method” and scanned reference lists to find other relevant studies that in turn generated new literature lists. In addition, I asked researchers in the field for relevant literature and when I had the opportunity I asked to take photos of bookshelves in their offices to find relevant literature. Literature lists in some PhD courses that I took also provided essential readings. I searched recently-published books (volume collections) and I looked at the individual literature production of some researchers in relevant fields. After taking part in conferences, I scanned literature that appeared essential in relevant presentations.

The fact that I am not able to read research literature in languages other than Swedish and English has limited my literature

search. This means I have not been able to consider relevant research advances presented in other languages such as for instance Spanish.

Language diversity

The first paper in mathematics education research and language diversity was published in 1979 by Austin and Howson (Phakeng, 2016). Since then the issue of language diversity in mathematics education research has gained increasingly more attention (Barwell et al., 2016). Although some research at that point in time had already suggested that bilingualism was an asset (see, for example, Pearl & Lambert, 1962), the research field before 1979 in general concluded that children growing up with plural languages were at risk of being subjected to delayed language, cognitive, and educational development (Barwell et al., 2016; Phakeng, 2016). Hence, bilingualism was considered a deficiency. In 1979, Swain and Cummings compared studies with negative and positive findings related to learning and bilingualism. They concluded that the outcome of the studies correlated with socio-economic status and social attitudes towards bilingualism and learning in the students' learning environment (Barwell et al., 2016; Phakeng, 2016). This conclusion juxtaposed language, social, and political issues. Post-1979 literature in mathematics education tended to encourage bilingualism (Phakeng, 2016). Moreover, Adler's (see, for example, Adler, 1998, 2000) pioneering work in South Africa prompted a displacement from research on bilingualism to research on multilingualism (Barwell et al., 2016). According to Phakeng (2016), bilingualism is a matter of power struggles between the hegemonic language of instruction and students' first language in bilingual settings such as Canada (English, French) and Catalonia (Catalan, Spanish), where one language is silenced. Multilingualism embraces and recognizes all languages. Adler (1998) captured three dilemmas—one of them code-switching—that are at the heart of teaching and learning mathematics in multilingual classrooms (Barwell et al., 2016). The notion of code-switching has been promoted and explored in detail (Barwell et al., 2016; Phakeng, 2016).

Planas and Civil (2013) claimed that students' first language (L1) was a resource "for thinking and doing, and more particularly for learning and teaching mathematics" (p. 363). Indeed, earlier researchers suggested that multilingual mathematics classrooms embrace flexible views on language use and allow the possibility for learners and teachers to shift from the official language of instruction to students' L1 (to code-switch) (Adler 1998, 2001; Barwell, 2003; Garegae, 2007; Moschkovich, 1999; Planas & Setati, 2009; Setati & Adler, 2000; Setati, 2005). These practices continue to be encouraged (Bose & Clarkson, 2016; Moschkovich, 2015b; Planas & Civil, 2013; Planas & Setati-Phakeng, 2014).

Through the work in the 90s and the beginning of the 21st Century by, for instance, Adler (2006) and Setati (2005) in South Africa, Moschkovich (2002) in the southwest of the US and Gorgorió and Planas (2001) in Catalonia, language diversity has been on the agenda in mathematics education research as a resource and asset in students' mathematics learning. The significance of recognizing students' first language in mathematics education has been shown in a variety of areas. Researchers have highlighted students' first language when dealing with textbook tasks (Barwell, 2017, 2018b), in classroom conversations (Setati, 2005), in reasoning (L. Webb & P. Webb, 2016), in teacher education (Chitera, 2011; Eikset & Meany, 2018; Essien & Adler, 2016), for socio-political reasons (Chronaki & Planas, 2018; Clarkson, 2016; Svensson Källberg, 2018a, 2018b; Norén, 2015), and for justice and student agency (Chronaki, 2009; Chronaki, Mountzouri, Zaharaki, & Planas, 2016; Halai & Muzaffar, 2016; Nkambule, 2016; Norén & Andersson, 2016).

Resources in the multilingual mathematics classroom

From the perspective of students navigating language and epistemological divides, it is useful to consider what resources they might have at their disposal as they navigate and how research has addressed these resources.

Adler (2000) developed a framework of different kinds of resources available in the multilingual mathematics classroom. It comprises a) basic resources such as buildings and teachers, b) ma-

terial resources (e.g., books, calculators, or computers), c) social and cultural resources including language, and d) other resources such as teacher knowledge. In addition to the resources identified by Adler, students make use of or actualize various semiotic resources as they participate in mathematics classroom tasks. These resources include gestures, ambiguities, multiple meanings, information shown in graphs, code-switching, and metaphors (Moschkovich, 2002, 2008, 2015a, 2015b; Prediger, Clarkson, & Bose, 2016). Barwell (2018a, 2019) considered multiple interrelated sources of meaning in the multilingual mathematics classroom. Sources of meaning can be the use of named languages such as English or Swedish, genres, narrative accounts of everyday experience, mathematical vocabulary, and the voices of the students, their teacher, and/or their worksheets. Barwell's (2018a, 2019) approach focuses on the use of multiple named languages (such as Swedish or English) but also includes multiple discourses that relate to forms of socially organized languages of, for instance, particular groups of people or activities.

Research on multilingualism that highlights forms of socially organized language often draws on Halliday's (1978) work and the notion of *register* —a language variety associated with a particular situation of use (see, for example, Moschkovich & Zahner, 2018; Planas, 2018; Prediger, Clarkson, & Bose, 2016; Schütte, 2018; Wessel & Erath, 2018)—to acknowledge that multiple registers are intertwined and/or juxtaposed with multiple named languages in multilingual mathematics classrooms. The dynamic plurality of multilingual registers in multilingual classrooms calls for pedagogical approaches that embrace meaning-making as a matter of relating different registers to each other (Moschkovich & Zahner, 2018; Prediger, Clarkson, & Bose, 2016; S. Rezat & S. Rezat, 2017). Schütte (2018) pointed out that register-based approaches often (wrongly) tend to suggest that there is one correct (target) mathematical discourse that needs to be achieved.

Some authors find and address gaps (or divides) in language proficiency between low- and high-performing students as a matter of them being language learners, not merely of the language of instruction but also of the hegemonic school academic language. To

address these gaps, language-responsive teaching has been suggested (see, for example, Prediger & Zindel, 2017; Wessel & Erath, 2018). Language-responsive approaches are further discussed below.

Barwell (2012) noted that tensions arise not only in discussions on dominant versus subordinated national languages, but also with regard to registers of home versus school language. While some languages and/or registers are valued, others are not. Hence, although multiple social, cultural, semiotic, discursive, and language resources can help students navigate epistemological divides, students may avoid using some of them if they find they are not valued.

Moschkovich (2010) proposed that research ought to focus not on differences (or divides) but rather on similarities between the performances of mono- and multilingual students. Drawing on socio-cultural theories, she developed the notion of *academic literacy in mathematics (for language learners)* to integrate rather than separate cognitive aspects of mathematical proficiency (e.g., mathematical reasoning, metacognition, and conceptual development), participation in mathematical practices (problem solving, sense-making, modelling, and looking for patterns, structure, or regularity) and mathematical discourse (multiple modes of communication, sign systems, and types of inscriptions) (Moschkovich, 2015a; Moschkovich & Zahner, 2018). To move away from theorizations that thrive in the dichotomization between formal and informal mathematical discourse, she suggested *ecological approaches* to the relation between language and mathematical thinking (Moschkovich, 2017). According to Moschkovich (2017), to take on an ecological approach means to a) expand the focus on cognition to encompass situations outside school, b) to attend to resources as opposed to obstacles and c) to consider the multiple and complex social practices that students participate in inside and outside school. The overall aim of Moschkovich's (2017) ecological approaches is to escape reductionist views that attribute static qualities to languages and their speakers or to social and cultural practices and their participants.

The above have implications for this thesis. Although students may have a number of resources at their disposal as they grapple with navigating language and epistemological divides, it seems it is not a straightforward matter to apply those resources in the mathematics classroom. Since different languages and mathematical knowledges are valued unequally, tensions and dichotomizations emerge that hinder or prevent students from using their diverse resources.

Language-responsive approaches

The complexity of contemporary migration patterns has resulted in language-diversified mathematics classrooms where a number of languages may be represented but none, except for the language of instruction, may be shared. Under such circumstances language-as-resource approaches that draw on students' named languages as resources may not be applicable (Meyer, Prediger, César, & Norén, 2016). Language-responsive approaches, which have been developed recently, take this into account in their aim to close gaps in student performance based on making school academic language accessible for language learners (Prediger & Zindel, 2017). In the context of language-responsive approaches, language learners comprise emergent speakers of the national language of instruction and/or of the social language—the school academic language—that is privileged in mathematics classrooms (Prediger & Zindel, 2017; Wessel & Erath, 2018). Hence, language-responsive approaches account for social and language diversity. These approaches highlight the epistemic role of language (in contrast to the communicative role) as a learning medium, as a distributor of learning prerequisites, and as a learning goal in order to specify learning demands (Prediger & Zindler, 2017). This is operationalized by considering topic- (content) specific language demands (Prediger, Clarkson, & Bose, 2016; Prediger, Erath, & Opitz, 2019; Prediger & Krägeloh, 2016; Wessel & Erath, 2018) as there is no general *academic* language, according to Prediger and Zindel (2017).

The aim of language-responsive approaches is to provide gap-closing foundations for fostering the mathematics learning of language learners (Prediger & Schüler-Meyer, 2017). I find that these

authors provide a content-based epistemological dimension of language diversity that they approach as a medium for learning.

Language responsiveness in social and language diverse mathematics classrooms has been considered from an interactionist perspective with the aim of understanding meaning-making as a matter of interactions among individuals in social environments (Jung & Schütte, 2018; Schütte, 2014, 2018). This perspective highlights the plurality in students' diverse languages and mathematical knowledges as beneficial resources for all students when they negotiate meaning in order to reach consensus (Jung & Schütte, 2018; Schütte, 2014).

The aim of this thesis intersects with the language-responsive approaches mentioned above in that it addresses epistemological dimensions of language diversity framed in the complexity of contemporary migration (Prediger, Erath, & Opitz, 2019; Prediger & Krägeloh, 2016; Prediger & Zindel, 2017; Wessel & Erath, 2018) from an interactionist perspective (Jung & Schütte, 2018; Schütte, 2014, 2018). However, instead of highlighting gap-closing in cognitive learning outcomes, this thesis accentuates students' navigation of gaps or divides. Hence, the idea is not primarily to explore how to close gaps (or how students negotiate meaning), but to understand how students deal with the gaps themselves.

(First) Language-as-resource in the multilingual mathematics classroom

To recognize the value of students' multiple languages—to value minority languages—in multilingual mathematics classrooms, Planas and Setati-Phakeng in 2014 (Barwell, 2016a) explicitly introduced the language-as-resource metaphor into mathematics education research. The language-as-resource originates from Ruiz' (1984) language planning programme in the U.S. Its discourse is echoed in mathematics education research as well as in policy documents (e.g., OECD, 2017; Skolverket, 2016), teaching practices (Adler, 2006; Norén, 2015; Planas, 2012), and as an analytical tool in research (Planas 2014; Planas & Civil, 2013; Planas & Setati-Phakeng, 2014).

Below I consider some pedagogical aspects of the language-as-resource metaphor as they inform students' possible ways of navigating language and epistemological divides. I then zoom out to consider socio-political aspects of the metaphor that underpin theorizations and analysis of multilingual mathematics classroom talk and thus provide theoretical approaches to students' navigation of language and epistemological divides.

Pedagogical aspects of language-as-resource

In multilingual mathematics classrooms the language-as-resource metaphor is operationalized as a lever, for instance when characterising multilingual students' learning as a "journey from informal talk in their [students'] main language towards formal mathematical talk in English [the language of instruction]" (L. Webb & P. Webb, 2016, p. 198). The lever idea underpins major strands in mathematics education research on multilingualism (see, for example, Bose & Clarkson, 2016; Schüler-Meyer, Prediger, Kuzu, Wessel, & Redder, 2019; Setati, Adler, Reed, & Bapoo, 2002). Jung and Schütte (2018) claimed that a strict focus on the target resister (formal mathematics talk) risks resulting in gap gazing (Gutiérrez, 2008) and that such enterprises belongs to the deficit paradigm. Moschkovich (2018a) pointed out that it is crucial to acknowledge both informal and formal mathematics talk in students' mathematics learning since academic mathematical discourse comprises both.

Learning mathematics in a multilingual mathematics classroom is directly related to students' opportunities for communication and participation in mathematical activities (Planas & Setati-Phakeng, 2014). Thus, the flexible language use that language-as-resource calls for is intended for the extraction of learning opportunities for students whose L1 is not the language of instruction. Researchers have suggested methods such as code switching, peer talk, re-voicing, and the use of synonyms, gestures, and objects (Moschkovich, 2002, 2015a; Planas & Setati-Phakeng, 2014) to support students' participation in learning opportunities (Planas 2014; Planas & Civil, 2013; Planas & Setati-Phakeng, 2014). Learning opportunities for the whole class or for groups of students emerge when students experience "difficulties with their lan-

languages while doing mathematics” (Planas, 2014, p. 52). The linguistic discrepancies that arise in students’ language use can initiate students’ joint unpacking of conceptual meaning (Planas, 2014).

Furthermore, to “produce coordinated results” (Planas, 2012, p. 333) from participants in a multilingual classroom, Planas suggested an organizing teaching principle in which students and teachers share a joint sensitivity to (i.e., being sensitive to different ways of expressing mathematical ideas), and responsibility for (i.e., taking responsibility for each other’s learning) the production of learning and doing school mathematics.

Moschkovich (2015a) focused on conceptual understanding by providing cognitively demanding tasks to emergent speakers of the language of instruction. Such tasks offer the potential for students to make connections, understand multiple representations of mathematical concepts, communicate their thinking, and justify their reasoning. Yeong and de Araujo (2019) studied how prospective teachers set up cognitively demanding tasks. They found that the core that guided the set-ups was building a common experience. For this to happen, they needed to empower students to give their input, provide feedback, and offer strategies to support assessment of student understandings, all of which set the stage for building common experiences.

From a pedagogical perspective, language-as-resource primarily focuses on enhancing supporting learning environments for emergent speakers of the language of instruction in order for them to learn school mathematics. Language and strategies to support multiple language use are foregrounded. The agents, the speakers of languages seem to be emphasized less.

The language-as-resource metaphor plays an important role in most pedagogical and research issues on multilingualism and mathematics education (Chronaki & Planas, 2018). Pedagogical aspects of language-as-resource approaches have emphasized communicational and participatory aspects to enhance high quality learning opportunities for multilingual emergent speakers of the language of instruction.

The above-mentioned classroom approaches to multilingualism have been and still are important enactments of the language-as-

resource metaphor that move away from the idea of multilingualism as a deficit. This does not mean that language-as-resource approaches cannot or should not be problematized, which is what I do in Article I. In contrast to traditional language-as-resource approaches that draw on students sharing multiple languages, this thesis moves further to explore multilingualism in a mathematics classroom with no shared languages apart from the language of instruction.

A second wave of language-as-resource is emerging (see Article I), which signals a displacement from the conceptualization of the first wave of language-as-resource metaphor discussed above (see Barwell, 2018a, 2019; Chronaki & Planas, 2018; Planas 2018). The second wave offers theorizations specific to the mathematics classroom by for instance, addressing the notion of (re)sources, students' production of mathematics-based discursive spaces, and functional aspects of language use.

Considering how to investigate and theorize students' navigation of language and epistemological divides, there is a need to move beyond the language-as-resource metaphor for two reasons. First, the main perspective of the metaphor is languages, not agents. I am concerned with agents (students) who navigate the divides and their interactions while they conduct the navigation. Second, the metaphor presupposes dichotomizations between named languages—between students' L1s and L2s and between formal and informal school mathematics. I assume that from the perspective of students, these distinctions are blurred or perhaps even non-existent. The language-as-resource metaphor is therefore insufficient for capturing students' navigation of language and epistemological divides (see Article I).

Sociopolitical aspects of language-as-resource approaches

Norén and Andersson (2016), in discussing code switching between Arabic and Swedish, stated that “[w]hen a discourse supporting bilingualism is applied, discriminatory enactment of ‘Swedish only’ discourse [...] cannot be used as a boundary between inclusion and exclusion of students’ communication in the mathematics classroom” (p. 122). Norén (2015) found that although

classroom practices of language-support that, for instance, encourage code-switching are beneficial to the learning of both mathematics and language, such practices operate alongside processes of normalisation whereby students' L1s continue to be erased or ignored. When discussing the contributions of language-as-resource approaches in mathematics education in general and in relation to students' navigation of language and epistemological diversity in particular, it is important to pause for a moment to look for the assumptions and implications that this approach entails in informing findings like Norén's (2015).

In the politics of linguistic diversity the very concept of 'language' has been tied to the modern idea of nation-state, as Bauman and Briggs (2003) wrote. In that line of thought, to build a shared national identity and to mark territory, the idea of a national language is essential to the construction of national states (Blommaert & Rampton, 2011; Creese & Blackledge, 2015; Pujolar, 2009). To realize Ruiz's (1984) ambition of perceiving minority languages as resources, these languages must serve the needs of the national state (Ricento, 2005). The idea of language-as-resource "presupposes that language can be valued as an asset, but the values attached to the asset (languages) and the purposes for which the assets will be used are pertinent only to the needs of the state" (Ricento, 2005, p. 361).

Petrovic (2005) located language-as-recourse approaches as part of the neoliberal agenda, raising concerns about that "forces of neoliberalism are far more likely to be successful at manipulating diversity to maximize profit than cultural pluralists will be at manipulating neoliberalism to maximize diversity" (p. 404).

The use of minority languages in the classroom has been conceived of as a "temporary vehicle to promote access to full civic participation via the national language, English, and not to promote full-blown bilingualism/biliteracy" (Ricento, 2005, p. 361). This resonates with the idea that students' L1s function as a lever in the journey from informal to formal mathematics talk, as I mentioned earlier. Using this rationale, linguistic minorities are distant from the national identity that is being built or protected, since they have not mastered the national language. Their *resource* (i.e.,

language) needs to be cultivated in order for it to become useful to the national interest.

From this perspective, the idea of language-as-resource still belongs to the deficit approach, because minority languages need to be cultivated as resources to be beneficial to the state; otherwise they are burdens or even threats to the state. Hence, minority languages per se are not resources but rather encumbrances. This translates to a deficit approach to minority languages in the mathematics classroom.

Socio-political aspects of language-as-resource approaches have implications for this thesis from the perspective of how students grapple with their diverse languages in the pedagogical contexts of school mathematics. In Article I, I discuss socio-political aspects of language-as-resource as a matter of recognizing cultural and social epistemological dimensions of language diversity. In Article III, I frame language-as-resource as a matter of a multilingual student's exposure to epistemological sanctions.

A translanguaging reply to the language-as-resource metaphor

García and Wei (2014) brought forward the notion of *translanguaging* as an alternative to dichotomizing between named languages, since L1 and L2 approaches are becoming increasingly insufficient in today's mobile and blurred migrating societies. "Translanguaging is the enaction of language practices that use different histories, but that now are experienced against each other in speakers interactions as one *new* whole" (p. 21). Translanguaging is not a linguistic hybridity but a new language practice. From this point of view, there is no such thing as L1 and L2; instead there is merely *linguaging*. Translanguaging differs from the notion of code-switching in that it refers not simply to a shift or shuttle between two languages, but to the speakers' construction and use of original and complex interrelated discursive practices that cannot be easily assigned to one or another traditional definition (García & Wei, 2014). Translanguaging is always present in bi-/multilingual students' language practices, whether silenced by monolingualism or not. According to García and Wei (2014)

translanguaging serves as one of the key principles for understanding bilingualism as a dynamic process, in which the language practices of bilinguals are fluent, flexible, and diverse.

In dual lingual immersion classrooms Rubinstein-Ávila, Sox, Kaplan, and McGraw (2015) used the notion of translanguaging as a pedagogical position that does not dictate use of one particular language, but rather allows students to “utilize both languages as resources during communicative tasks” (p. 905). Farsani (2016) showed that translanguaging has the potential to make displacements in analytical focus from movements between named languages to the agent—the speaker. Removing the focus on the separation of named languages leads to an embrace of the heterogenic co-existence of languages in speakers’ performances. Farsani (2016) concluded that not only do languages flow through classroom talk, but cultural-historical (epistemological) aspects of mathematics are also present. Planas (2018) and Barwell (2018a) used the notion of translanguaging to theorize the language-as-resource metaphor. Planas (2018) noted that code switching and translanguaging voice different research discourses; code switching considers how students perform normative use of separated languages, while translanguaging focuses on what students actually do by means of all their (social) languages. Barwell (2018a) considered that multilingual students may display translanguaging behaviour as a way of interweaving language use and meaning from more than one language in educational contexts. See Article I for further details of the two theorizations. Barwell (2019) further developed the framework of sources of meaning discussed in Article 1 to include the distinction between situational and distant sources of (nuances in) meaning, which supports analyses from power-related time and space perspectives.

The discussion on language-as-resource is becoming increasingly nuanced through the notion of translanguaging. Prediger et al. (2019) found that students who were bilingual in German and Turkish used both code switching and translanguaging modes, but for different purposes. Their study considered that there are nuances in conceptual meaning across languages, which multilingual students need to keep in mind. Students used code switching to ex-

press separate meaning nuances. However, more often they used a translanguaging connecting mode to combine languages and synthesize meanings across language nuances.

This thesis adds to the discussions on the notion of translanguaging in multilingual mathematics classrooms with no shared languages except the language of instruction. In Articles I and II, I consider how students' translanguaging shapes spaces for metaunderstanding of language diversity as a way of navigating language and epistemological divides.

Epistemological diversity

In this section, I consider how cultures and individuals in and across cultures come to take something to be mathematical knowledge. This has implications for students' navigations of language and epistemological divides. I follow Planas (2018), who used the term *culture* in plural to mean norms, practices, and forms of knowledge that a) emerge in local sites, such as classrooms or among groups of family members or friends; and b) are less dynamic in broader societal contexts.

I conceive of mathematics as a plural noun, considering formal Western mathematics and school mathematics as cultures of mathematics among many other mathematics cultures (Schütte, 2018). I share Knijnik's (2012) claim that cultures and language games associate with diverse ways of living and diverse practices that still resemble one another. In this thesis, I adopt a broad understanding of epistemology as a theory of knowledge that is concerned with how individuals and cultures come to acknowledge or disregard something as knowledge and the multiple forms in which knowledge appears. That is, I am concerned with social epistemology.

Mathematics and epistemologies

Embracing a pluralistic view of mathematics is far from unproblematic, as hegemonic formal Western mathematics often is conceived of as ontologically absolute, transcendent, and universal with its epistemology shaped by Euclidian ideas about axioms used to perform deductive reasoning to prove the truth of theorems

(Skovsmose, 2005). According to Skovsmose (2005) epistemic transparency resides in the idea that knowledge can find a solid foundation in a set of axioms. To be useful for establishing truths, the axioms must state simple and obvious facts that can be grasped intuitively. Since all mathematical truths are based on the axioms, their validity can be confirmed by transparent deductive tracing of the axioms. If such tracing is successful, the idea is that truth is not merely beyond reasonable doubt; it is beyond any *possible* doubt. It is *one* precise, rational, and absolute truth (Skovsmose, 2005).

From this perspective, mathematics characterizes the very nature of rationality. Mathematics defines what precision is and offers the ability to make models and calculations, which is what makes science what science is. Thus, mathematics is the Queen of all sciences. This is what Lakoff and Núñez (2000) referred to as the Romantic Myth of Mathematics. Through mathematical proofing, the mathematician overcomes human limitations and encounters transcendent universal truths. According to Lakoff and Núñez (2000) this is a presupposition for which there is and cannot be any evidence while “[t]here is no way to tell empirically whether proofs proved by human mathematicians are objectively true, external to the existence of human beings or any other beings” (p. 342).

Since the beginning of postmodernism, the dialogical character of mathematical epistemology, based on human interaction, has increasingly been highlighted and thus Euclidian epistemology troubled. For instance, Rorty wrote that

If, however, we think of ‘rational certainty’ as a matter of victory in argument rather than of relation to an object known, we shall look toward our interlocutors rather than to our faculties for explanation of the phenomenon. If we think of our certainty about the Pythagorean Theorems as our confidence, based on experience with arguments on such matters, that nobody will find an objection to the premises from which we infer it, then we shall not seek to explain it by the relation of reason to triangularity. Our certainty will be a matter of conversations between humans, rather than an interaction with nonhuman reality. (Rorty, 1979, p. 156, cited in Ernest, 1994, p. 36).

Prediger (2002) explained that the dialogical epistemology of mathematics is concealed to the masses by the strong consensus among mathematicians established through the practice of proving. Therefore, it is not always easy to see the cultural roots of mathematics. Prediger (2002) argued that, paradoxically, it is the human desire for certainty that dehumanizes mathematics. In brief, it seems to work like this according to Prediger (2002): to be certain about mathematical propositions, mathematicians use a strong practice of coherence and consensus. That is, mathematicians agree on what are and are not appropriate mathematical propositions. By the time their agreements reach the layman, this process is hidden. The mathematicians' high coherence and wide consensus have obscured the human dimension and cultural origin of mathematical propositions, making them appear impartial.

Popkewitz (2008) argued that contemporary academic mathematics that allow epistemological uncertainty and plurality undergoes what he refers to as *alchemy* in its transformation into a school subject. The alchemy is a pedagogical translation that produces iconic images of expertise that constitute mathematical knowledge and the rules for finding it. Hence, school mathematics reinforces the Euclidean transparency epistemology. Wagner and Herbel-Eisenmann (2009) considered the mathematical discipline a powerful and sometimes repressing *myth* that positions teachers and students in particular ways that have implications for students' navigation of language and epistemological divides. They suggested that school mathematics be re-mythologized in a way that humanizes mathematics and thus avoids discourses on transcendent absolutist mathematics. Erath (2018) concluded that in whole class discussions, it is especially the teacher who takes the lead of and gives directions for students to navigate through mathematical *epistemic fields*.

Chronaki and Planas (2018) stated that in mathematics education research on language, Platonian absolutist transcendent mathematics is implicitly captured in the influential and prevailing so-called *representational paradigm*. The paradigm follows the idea that knowledge is a mirror-like representation of the physical world. The paradigm evolved around the binary notion of internal and external representations that serves to objectify the abstract

nature of mathematical ideas (Chronaki & Planas, 2018). Duval (2006) claimed that although there are a wide range of conceptualizations of *representations* ranging from individual beliefs or (mis)conceptions to semiotic signs, representations always represent some object or phenomena. Hence, representations are standing for something else—abstract transcendent mathematical concepts—that the representations are assumed to capture. Kirsh (2010) argued that mathematical thinking is an individual back and forth process between external and internal representations of abstract mathematical objects.

In the realm of the representation paradigm, social, cultural, embodied, and political dimensions of mathematical concepts are absent (Chronaki & Planas, 2018).

The above has relevance for this thesis in as much as that students' school mathematics is largely framed within the representation paradigm, which, through its reluctance to recognize social, cultural, and political aspects of mathematics, produces epistemological divides and consequently shapes the need for students to grapple with them. In Article II, I explore how students discuss their conceptions of mathematics in society—how they recognize the social and political dimensions of mathematics. In Articles III and IV, I elaborate on how attention to precision as a characteristic of mathematical epistemology influences the students' navigation of epistemological divides.

Responses to the representation paradigm

Recently, various alternative stands to the representation paradigm have been proposed. Sfard (2008) considered mathematical objects' existence as emerging among interlocutors when they discourse. The communication brings forth the mathematical object, which means that Sfard (2008) acknowledged mathematical objects as being discursive rather than representational. Based on the philosophical theory inferentialism (Brandom, 1994, 2000) some mathematics education researchers (for example, Bakker, Ben-Zvi, & Makar, 2017; Bakker & Derry, 2013; Nilsson, Schindler, & Seidouvy 2017; Schindler, Hußmann, Nilsson, & Bakker, 2017; Schindler & Seidouvy, 2019) have elaborated on the notion that concepts

caught up in statements mean something in virtue of their inferential relations to other statements (Bransen, 2002). Hence, they too distance themselves from ideas that concepts merely are mental object-like representations of an external reality. These two perspectives claim that mathematical concepts and hence knowledge are fundamentally discursive and social.

To Roth and Radford (2011) mathematical knowledge is produced through sensuous cultural-historical activity. Radford (2016) wrote, “the students’ encounters with historically constituted mathematical knowledge, materialized in the teachers’ and students’ common work, are termed processes of objectification” (p. 201). These scholars recognize a cultural-historical plurality of mathematics (Guillemette, 2017; Radford, 2008, 2012).

Apart from cultural-historical recognition of mathematical knowledge, sensuous embodied dimensions of mathematical knowledge have been proposed and explored (Chronaki, 2015; de Freitas & Sinclair, 2013, 2014, 2017; Radford, 2009).

Chronaki and Planas (2018) noted that research on language diversity in mathematics education appears not to discuss what is meant by *mathematics* and to treat mathematics as a body of knowledge that students are to ingest. They concluded that research should “look at language diversity in the mathematics classroom as the right of groups to use their languages instead of their right to produce mathematics-based discursive spaces that create opportunities for participation in learning” (p. 1104).

Considering mathematical knowledge of the Other

In mathematics education research, issues of epistemological diversity have often been addressed from a colonial perspective, investigating mathematical practices of indigenous people and/or the mathematics found in people’s non-formal daily lives (Pais, 2013). However, as part of the socio-political turn, (Gutiérrez, 2013; Valero, 2004) several recent studies on epistemological diversity attempt to move beyond such a perspective by adopting a decolonial standpoint (Gutiérrez, 2017a; Parra, 2018). Mukhopadhyay, Powell, and Frankenstein (2009) claimed that since mathematics has traditionally been taught from a narrow (Western) perspective,

contributions of most cultures and groupings of people have been ignored or trivialized. This may result in omissions from mathematics education that risk devaluing and disrespecting many students' cultural backgrounds and *funds of knowledge* generated in their homes and out-of-school activities (González, Andrade, Civil, & Moll, 2001).

Chronaki (2009, 2011) and Chronaki, Mountzouria, Zaharakia, and Planas (2016) approached epistemological diversity as a matter of students' identity work. Their research showed how pedagogical use of students' mathematical words in diverse languages grows together with learner identity-work and how culturally, socially, and politically situated mathematical knowledge is part of students' life and learning trajectories. Creating spaces for students and children—both in and out of school contexts—where they can share their languages' mathematical words and open up spaces for dialogue.

Guillemette and Nicol (2016) suggested that getting to know, hear, and respect the knowledge of the Other—learning to know *with* the Other—is an attempt to learn and teach mathematics in a socially just way. Guillemette (2017) investigated students teachers' encounters with historical texts in mathematics. He found that when the fragmented, multifaceted, and cultural character of mathematics revealed itself in the students' discussions of their readings, it helped them to consider experiences of Otherness with mathematics. Guillemette (2017) concluded that such experiences may encourage empathic movement towards the Other in the mathematics classroom, bringing open-mindedness to marginality, novelty, and difference, which in turn supports solidarity in mathematics education.

The work I have outlined above shows a potential for solidarity and recognition of Otherness in the navigation of language and epistemological divides. In Articles I and II, I investigate how students grapple with Otherness when trying grasp each other's sayings and doings.

Summing up the literature review

When students navigate language and epistemological divides, a number of aspects are at stake. Establishing norms that influence language use, providing opportunities to ask questions, finding and appreciating discrepancies as leaning opportunities, and engaging respectfully with the languages and knowledges of the Other provide spaces for students to navigate language and epistemological divides. On the other hand, prevailing epistemological discourses on school mathematics as *one* Western mathematics appear to regulate the students' navigation in ways that may jeopardize learning and identity-work. These issues have generally been investigated separately, as I have shown above. Only recently have some studies (for example, Parra & Trinick, 2018; Prediger & Krägeloh, 2016; Prediger & Zindel, 2017) made explicit connections between language diversity and epistemological discourses on (school) mathematics in multilingual contexts.

THEORETICAL TOOLBOX

In this section I describe the key theoretical ideas that have guided my journey and discuss my orientation to the three divides presented in the prologue: migration, language, and epistemological diversity. I have organized the section as follows:

First, I attend to the notion of named languages. Much research on multilingualism in mathematics education uses dichotomizing ideas that contrast named languages (Moschkovich, 2018b). Such research tracing language obstacles that may be overcome through instructional designs or research considers students' first language as one of several resources for students to use to communicate mathematics successfully (Moschkovich, 2002; Schütte, 2018). To *communicate mathematics successfully* usually means to be able to participate in the socio-cultural practice of mathematics knowers and thus qualify as one of them (for example, Moschkovich & Zahner, 2018; Sfard, 2008).

In this thesis I ask, "How do students navigate language and epistemological divides?" I ask this question not primarily from the perspective of students qualifying as participants in mathematical communities of practice, but rather from the perspective of how they bring together their languages and knowledges as they struggle to participate and qualify in these practices. This is an agent-focused, relational question that implies a need to move from a perspective that highlights *language* as the *separation* of named languages or registers to a theoretical approach that highlights *agents* and their pragmatic use of language. The notion of

translanguaging offers theoretical ideas that break free from separations of named languages to consider language diversity from the perspective of the speaker's use of all her/his languages (Creese & Blackledge, 2015).

Second, I attend to the notion of migration and epistemological diversity as a matter of ecologies of knowledges; these aspects frame students' navigation of language and epistemological divides. Reading literature on named languages and well-defined groups of students and teachers using named languages to code-switch has caused me to consider the complexities of contemporary globalized societies. I have worked to grasp theoretically such complexity in students' navigation of language and epistemological divides in a classroom framed by migration. To operationalize the two research questions in the spirit of the aim of the thesis, I needed to use theoretical thinking tools that offered the potential to approach epistemological divides in connection to migration from a position that did not dichotomize or place in opposition different ways of knowing mathematics.

Finally, since my research questions focus on students' relational actions in the pedagogical space of the mathematics classroom, I have attended to how people in face-to-face conversations deal with semantic and epistemological uncertainty. Elaborations on meaning in the mathematics classroom (and elsewhere) are far from a mere matter of spoken languages. Plural semiotic resources in addition to spoken language are at play (Barwell, 2018a; Moschkovich, 2008, 2015b). Nor is (mathematical) epistemology merely a matter of people discoursing, which I discussed in the literature review. In this thesis I have chosen to deal only with mathematics classroom *talk*, which means I am only concerning myself with the spoken in this work.

The operationalization of the theoretical constructs used in this thesis can also be found in the articles and in a section in the methodology chapter of this thesis.

Named languages

The notion of named languages has already been touched upon in the literature review, where I mentioned that the very concept of named languages is tied to the inception of the modern idea of a nation-state. For the construction of a nation-state, to build a shared national identity and to mark territory the idea of a named national language is essential (Blommaert & Rampton, 2011; Creese & Blackledge, 2015; Pujolar, 2009). Hence, named languages, sometimes referred to as systems of languages (see, for example, Planas, 2018), are social constructs designed to endorse national identity. Although the study of different named languages in contact with each other has a long history, the focus has until recently been on linguistic structural outcomes rather than socially embedded communicative processes (Collins, Slembrouk, & Baynham, 2009). A social and situated theoretical perspective on multilingualism for mathematics education research was offered by Moschovich in 2002 to help researchers grasp the full complexity of multilingual students' mathematics communication, which comprises "not only ways of talking, acting, interacting, thinking, believing, reading, and writing but also mathematical values, beliefs, and points of view of a situation" (p. 197).

According to Creese and Blackledge (2015) it is insufficient to use the notion of named languages as an analytical lens to view contemporary language practices when taking the effects of migration and globalization into account. They argue that approaches that rely on the naming and separation of languages offer a limited understanding of linguistic diversity. In line with Blommaert and Rampton (2011), Creese and Blackledge (2015) proposed an alternative stance that views languages as social resources without clear boundaries. This approach departs from the idea that people's complex language and mathematical practices do not separate into different language systems (Planas, 2018). Rather, all words and grammatical structures available to a speaker constitute the *idiolect* of that individual speaker. All speakers have their own idiolects with a particular set of words and grammatical structures. The only thing anyone actually speaks is one's own idiolect. Any two speakers using the same named language are unlikely to share the

exact same set of language features. Even among friends and family idiolects differ and do not fully overlap (Otheguy, García, & Reid, 2015). (In Article II, I use “accents” for people’s idiolects.) Idiolects differ from the notion of registers in that registers refers to language variety associated with a particular *situation* of use (Moschovich & Zahner, 2018; Planas, 2018; Prediger, Clarkson, & Bose, 2016; Schütte, 2018; Wessel & Erath, 2018), while idiolects are varieties associated with *individual* language use. The notion of idiolects therefore allows a shift in attention from the relation between the student and a (target) register toward the relations between individual students’ language use as they grapple with grasping each other’s idiolects in their encounters with the mathematical (target) register.

Idiolects are both situated and interpersonal since speakers adjust their idiolects to their interlocutors’ idiolects and to the situation. They use any linguistic and communicative resource available to them in order to enhance communication (Blommaert, 2014), which includes mathematical communication (Barwell, 2018a; Moschovich, 2002; Moschovich & Zahner, 2017). To communicate successfully in mathematics classroom discourse puts specific demands on students (Erath, 2018) elicited, for instance, by the appraisal of precision and certainty (Moschovich, 2002). Such language practices are emblematic for people who have access to only one named language (monolinguals), as well as for people who have access to more than one named language (multilinguals). However, while students who are monolingual in the language of instruction can, for example, use a large part of the set of language features constituting their idiolect, bilinguals cannot (see, for example, Norén & Andersson, 2016). Only in very particular contexts can multilinguals use their full range of language features. In most cases, it is bound by the limitations of their interlocutors’ idiolects (Otheguy, García, & Reid, 2015).

By contrast, Bagga-Gupta and Messina Dahlberg (2018) argued that a Euro-centric view is embedded in claims that connote that it is only in *particular* contexts that bilinguals can use the full features of their idiolects. In the global South, enactment of language practices containing idiolects with a variety of traces of named lan-

guages is mundane in everyday language use in institutional as well as in personal conversations. Bagga-Gupta and Messina Dahlberg (2018) stated that although the emerging interest in the global North that acknowledges, and highlights diversity is welcomed, the pressing issue for the global North is to acknowledge language diversity as being the norm and to learn from the many places in the global South where it is.

The approach to language use sketched above puts the speaker at the heart of the analysed interaction in contrast to focusing on how named languages are used separately by the same speaker(s) (Creese & Blackledge, 2015). This approach can be enmeshed by the term *translanguaging*. Otheguy, García, and Reid (2015, p. 281) described translanguaging as follows:

Under translanguaging, the mental grammars of bilinguals are structured but unitary collections of features, and the practices of bilinguals are acts of feature selection, not of grammar switch. A proper understanding of translanguaging requires a return to the well-known but often forgotten idea that named languages are social, not linguistic, objects. Whereas the idiolect of a particular individual is a linguistic object defined in terms of lexical and structural features, the named language of a nation or social group is not; its boundaries and membership cannot be established on the basis of lexical and structural features. The two named languages of the bilingual exist only in the outsider's view. From the insider's perspective of the speaker, there is only his or her full idiolect or repertoire, which belongs only to the speaker, not to any named language. Translanguaging is the deployment of a speaker's full linguistic repertoire without regard for watchful adherence to the socially and politically defined boundaries of named (and usually national and state) languages.

Because of social and political boundaries, bilinguals use their full idiolect selectively. Nevertheless, the full features of the idiolect are silently present in every instance of the communication, which means that translanguaging is always enacted, albeit not necessari-

ly explicitly and therefore not apparent to the interlocutor(s) (García & Wei, 2014).

Although Blommaert (2012), among others, has argued that the complexities embedded in the encounters of sets of language features within and between people's idiolects cannot be captured by the use of vocabulary such as *multilingual* or *translanguaging* because this vocabulary implicitly adheres to the existence of separable units, I have found both those terms useful in this thesis. The reason is that *multilingualism* is an often-used and well-established term in mathematics education research. I use *multilingual* descriptively to mean students whose idiolects comprise more than one named language. I also use *multilingualism* to recognize all languages (Phakeng, 2016). The reason for using *translanguaging* is that it is the best term I have come across to capture the complexity of communication in a migrating and globalized world, as opposed to dichotomization of language use (Creese & Blackledge, 2015). Translanguaging in educational literature has at least two dimensions (Bagga-Gupta & Messina Dahlberg, 2018; García & Wei, 2014): one that concerns the theoretical construct I draw on in this thesis and a pedagogical dimension that I do not attend to here or use in this thesis.

Migration and ecologies of knowledges

Over the past thirty-odd years migration together with neoliberal political agendas have made the world increasingly globalized, changing traditional demographic demarcations. Migration nowadays entails movement of groups of people with more varied national, ethnic, linguistic, and religious backgrounds than previously (in the global North). Sweden is no exception (SOU 2017:54). The diversification of migration channels such as work permit programmes, increased mobility due to the EU enlargement, refugee and 'mixed migration' flows, student migration, family reunions or separations and so on have resulted in increased migration.

Although migration is increasing globally, surprisingly few studies connect mathematics education and migration directly. For instance, the research I found when I searched for *mathematics education* AND *migration* in the ERIC database (34 hits) covered the

following topics, ranging from the most common to the least; achievement gaps (e.g., Neseth, Savage, & Navarro, 2009; Verhaeghe, Vanlaar, Knipprath, De Fraigne, & Van Damme, 2018), teacher mobility (e.g., de Villiers & Weda, 2017; Williams, 2008), student mobility (e.g., Robertson & Graven, 2015) web-based communication (e.g., Lloyd, 2003), epistemology (Krugly-Smolka, 2004; Prieto, Claeys, & González, 2015), and socio-politics (Gutstein, 2007). In addition, a number of articles addressed teaching mathematics by investigating zoological migration such as that of birds or whales, (e.g., Forrest, Schnabel, & Williams, 2007). In a sense, these diverse research topics echo diversified migration patterns.

Due to regulatory power exercised to uphold nation-state territories and national identities, migration has two parts—emigration and immigration—that are asymmetrically related. While it is a UN-regulated human right to leave (emigrate), it is a nation-governed matter to enter (immigrate to) a country (Miller, 2016; Oberman, 2016). This asymmetry enables nations to reject immigration for the sake of safeguarding the nation and its identity. For the same reason, nation-states may impose cultural and language integration demands on immigrants (Oberman, 2016).

Although restricted, diverging migration channels complexifies traditional migration patterns of gender, age, and human capital (such as education, work skills, and experience). This blur traditional distinctions based on ideas of geographically-located ethnic groups with particular cultures, religions, languages, and social statuses (Meissner & Vertovec, 2015). “Individuals may identify with several nationalities or racial groups and speak combinations or mixtures of several languages” (Barwell, 2016b, p. 25), which consequently dissolves any analytical focus on named languages and/or ethnic groups as discrete and distinct. This explains the need to turn to the notion of translanguaging in this thesis.

The above may be the case on ‘this side’ of what Santos (2007) called *abyssal lines* that tend to structure social realms and thus thinking. Traditionally abyssal lines were drawn between metropolitan societies and colonial territories. However, due to migration and globalization, people on *the other side of the line*, such as

the EU-migrant, the environmental refugee, or the undocumented worker, have become mobile. These lines are no longer neatly geographically ordered (Santos, 2007). They do not merely appear *between* the *West and the rest*; they are located *within* Western cities.

Abyssal lines are tied to abyssal thinking. Abyssal thinking is a system of visible and invisible distinctions, the invisible one being the foundation for the visible ones. Santos (2007) wrote

The invisible distinctions are established through radical lines that divide social reality into two realms, the realm of “this side of the line” and the realm of “the other side of the line”. The division is such that the “one side of the line” vanishes as reality, becomes non-existent, and is indeed produced as non-existent. Nonexistent means not existing in any relevant or comprehensible way of being. Whatever is produced as nonexistent is radically excluded because it lies beyond the realm of what the accepted conception of inclusion consider to be its other. What most fundamentally characterizes abyssal thinking is thus the impossibility of the co-presence of the two sides of the line. (p. 45)

To me, the quotation connects to the operationalization of the language-as-resource metaphor as a commodification of students’ first language to move them along an axis from informal towards formal mathematics talk in the language of instruction. The tension between informal and formal mathematics is a highly visible distinction; it is gazed at in mathematics education as a matter of achievement gaps (Gutiérrez, 2008). This distinction rests upon the invisible but abyssal line between mathematical knowledge that can be articulated in terms of (in)formality, and all other forms of mathematical knowledge. Although the need to value formal and informal mathematics or everyday and mathematical registers equally is highlighted (Moschkovich, 2018a, 2018b; Schindler et al., 2017), the distinction between them is usually made in students’ school language use (see, for example, Schindler et al., 2017). In these situations, students try to adjust their speech, actions, interactions, thinking, mathematical values, and beliefs

(Moschkovich, 2002) to the school academic discourse. Hence, the informal mathematics that students display is still a particular school informal mathematics *on this side* of an abyssal line. Ernest (1991) pointed out that there is a persistent conflict that cannot be avoided between the locations of mathematics in the world of students' experiences or life cultures and the need to teach and learn theoretical mathematics to provide the powerful thinking tools of abstract mathematics. This conflict applies to *the other side*, as Knijnik (2012) wrote, "forms of life outside school cannot be automatically transferred to the school form of life. To move from one form of life to another does not guarantee the permanence of the meaning" (p. 93), while meaning may reside in different epistemological rationalities (Knijnik, 2012; Wagner & Lunney Borden, 2015; Walkerdine, 1988).

Santos (2007) suggested that Western monoculture and abyssal thinking needs to be confronted with a post-abyssal thinking that moves away from dichotomization towards seizing ecologies of knowledges in which Western knowledge (such as Western mathematics) is but one way of knowing. Santos (2007) wrote, "[i]t is an ecology [of knowledges], because it is based on the recognition of the plurality of heterogeneous knowledges (one of them being modern science) and on the sustained and dynamic interconnections between them without compromising their autonomy" (p. 66). From this perspective to know something is not merely to know how to make connections among knowledge within one knowledge system—for instance, to move between mathematical representations (Duval, 2006)—but to know how that knowledge is interconnected to a plurality of knowledge systems including those of the Other(s). It is inter-knowledge among knowledge systems that makes up ecologies of knowledges.

Santos' claim has profound implications for school mathematics in a migrating world, as it offers the potential to make visible and stay with rather than trying to overcome conflicts between forms of life outside of school and school forms of life. In order to consider ecologies of mathematical knowledges, there is a need to recognize and appreciate the co-presence of mathematical knowledges at school (Moschkovich, 2018b). There is a need to consider how

learning means shaping interconnections between co-existing mathematical knowledges rather than unlearning or forgetting some kinds of knowledge (Gutiérrez, 2017a, 2017b) as the move from informal to formal mathematics connotes.

The performative notion *mathematx* (Gutiérrez, 2017a) captures a displacement from issues of what counts as grasping bodies of (in)formal mathematical knowledge toward considering how to perform ecologies of mathematical knowledges. This means to honour the mathematics of the Other, to acknowledge that all people and all epistemologies are interconnected. It means holding plural contradictory views at the same time and embracing the idea that we cannot rely solely on *one* (Western) mathematics (or any other singular knowledge) to address the problems we meet. Rather, there is a need for border crossing and a willingness and ability to bridge plural mathematical knowledges at play in the interaction among others and oneself (Gutiérrez, 2017a). From *mathematx* performances new knowledge—interknowledge—emerges. Production of interknowledge is an at once empathetic and cognitive performance that requires deep intellectual and emotional work. To hold plural mathematics in mind at the same time and to bridge them changes our relationship with the Other (Gutiérrez, 2017a). I find the performative and relational notions of *mathematx* based on the ideas of ecologies of knowledges helpful when trying to understand how students, together with their languages and knowledges, navigate epistemological divides that may propel them into *the other side* of (a)mathematical knowledge.

Dealing with semantic and epistemological uncertainty in face-to-face conversations

In social and language diverse mathematics classrooms, students talk using their own individual idiolects (Blommaert, 2014; Otheguy, García, & Reid, 2015) as they encounter the knowledges of the Other (Gutiérrez, 2017a, 2017b; Santos, 2007). Since idiolects and/or knowledges do not fully overlap, language and epistemological divides are implicit in students' conversations unless they are able to identify them and explicitly address them. The latter case may provide learning opportunities for students (for example,

Jung & Schütte, 2018; Planas, 2014). Whether the divides are explicitly addressed or implicitly embedded in talk, they mean that students reciprocally grapple with the uncertainty of understanding the Other's sayings and knowings.

Wittgenstein (2010) emphasized that the role of language is not to mirror an external true reality; rather, its role is its *use* when actions are performed or ends obtained within particular communities or forms of life. He introduced the notion of language games to capture this. He highlighted the importance of *use* in language, to accomplish communal goals and the problems associated with trying to communicate between language games. My research focus is with students grappling with divides that emerge when they take and treat something to be mathematical knowledge in the course of doing and talking about mathematics. In other words, I am interested in how language use accomplishes (or falls short in) understandings among them as they navigate difference. This focus is founded on the presupposition that the divides *can* be navigated at all, that they are not merely a two-directional matter of (in)correct representations of an external reality. Rather, my focus presupposes that grappling with divides is a matter of dealing with uncertainty.

Below I propose an inferentialist approach to discuss meaning and knowledge (Brandom, 1994, 2000), as it offers a displacement from meaning as representational towards meaning as socially inferential. A socially inferential approach to meaning allows us to consider how meaning is flexible and shaped as people use their individual idiolects to make inferences between concepts involved in different language games—how they shape interknowledge (Gutiérrez, 2017a, 2017b; Santos, 2007). I closely examine the moves made in face-to-face conversations that people perform when they encounter and use each other's claims of knowledge and form interconnections between claims.

I have chosen to use inferentialism (Brandom, 1994, 2000) because it rejects the representation paradigm. From a strongly social point of view it captures three aspects of one particular language game (*the game of giving and asking for reasons*—see below): the semantic, the epistemological, and the interpersonal positioning.

These aspects make up how students interact with each other using their languages and knowledges to navigate epistemological divides.

I have considered other theoretical approaches that highlight socially discursive aspects of language that have been more frequently applied in mathematics education research, such as Sfard's (2008) notion of commognition, systemic functional linguistics (SFL) (Halliday, 1978), positioning theory (Harré & van Langenhove, 1999), and Gee's (2005) D/discourse analysis. Below I roughly sketch how I have considered the different theoretical approaches in relation to the semantic, to the epistemological, and to interpersonal positioning.

Commognition theory (Sfard, 2008) focuses on the emergence of mathematical objects through discourse (epistemological and interpersonal—without positioning—dimensions). SFL (Halliday, 1978) highlights the social embeddedness of functional meaning (the semantic and the interpersonal). Positioning theory offers the potential to understand how discourse places people in different power-related positions and vice versa (interpersonal positioning). D/discourse analysis (Gee, 2005) unveils how social behaviour shapes and is shaped by language use (the semantic and the interpersonal positioning). To me none of these theories straightforwardly captures the interplay among the semantic, the epistemological, and the interpersonal positioning.

An inferentialist approach to meaning and knowledge

Inferentialism is a contemporary, pragmatic, and social philosophical theory. Its concern lies with social epistemological issues such as meaning, concepts, and knowledge (Bakker & Hußmann, 2017). It provides an alternative stance to a primarily referential explanation of conceptual meaning because, according to inferentialism, conceptual meaning is primarily socially inferential (Bakhurst, 2011; Bransen, 2002; Derry, 2013, 2017).

Inferentialism is concerned with the movement of thought rather than with fixed images or representations of it. "The idea of the movement of thought in the articulation of meaning is central to inferentialism since it shows that the meaning of a concept, rather

than being fixed, is fleshed out by the inferential connections that constitute it. As these connections change, so the meanings of concepts alter.” (Derry, 2017, p. 2).

Concepts are primarily understood in their role in reasoning and in inferences. To grasp a concept, one must have other concepts, “[f]or the content of each concept is articulated by its inferential relations to other concepts. Concepts then must come in packages” (Brandom, 2000, p. 15). For example, the meaning of the concept *square* can be articulated by its inferential relations to, for instance, *four sides and four corners*, *angles of 90°*, and *not a triangle* since these inferences (and others) can be derived from claiming something to be a square.

What inferences are appropriate to derive from a claim articulates the meaning of that claim and thus the meaning of the concepts used to make that particular claim (Noorloos, Taylor, Bakker, & Derry, 2017). Inferences can of course be formal, as in the mathematical example above, but they need not be. For instance, in a social practice where bringing a bag to school is the norm, no formal inferences are needed to make sense of a child asking for her school bag on a weekday morning. However, should the child pose the question on a Sunday morning she might be asked what she wanted the bag for and thus need to give reasons for her question. To infer from a child going to school that a particular bag is needed is what Brandom (2000) calls to make a materially good inference.

Material inferences need not be formally valid for us to understand each other, but they need to be good in virtue of the sense they make in the particular time and space from which they are being made (Brandom, 2000).

The notion of concepts as inferential relations—relations that are flexible and open for transformation rather than re-presenting a fixed image of the world—has been important to me because it shows that meaning is constantly in flux and is interconnected. From a language user’s point of view, there are no such things as informal and formal mathematics talk, merely sets of inferential relations that can be made explicit as we give reasons for our sayings and doings (Seidouvy, Helenius, & Schindler, 2018; Schindler

et al., 2017). Inferences that are materially or formally good or bad to derive from a claim articulate the meaning of that claim (Noorloos et al., 2017).

To be a knower is to be aware of the inferential relations that implicitly follow from the claims one makes and to be able to explicitly give normatively appropriate reasons for one's claims if asked. Relating reasons inferentially to each other is to place them in the *space of reasons* (Bakhurst, 2011). Spaces of reasons are social and normative. They are abstractions of people's concrete face-to-face conversations (Brandom, 1995). Since spaces of reasons are normative, they are shaped by the norms present in the particular conversation. For further discussions on spaces of reasons see article IV.

Social practices, norms and reasons

As we are social beings, we engage in activities together in particular ways that establish social practices. To engage in a social practice is to act in relation to others and to relevant resources in ways that may be understood as part of a pattern or system of coordination (Haslanger, 2018). Norms derive from these patterns of regularities in our social practices, while at the same time patterns of regularities (re)produce norms. We are sensitive to norms and we bind ourselves to them (Seidouvy & Schindler, 2019). Our sensitivity to norms implies that we are sensitive to what is normatively appropriate and normatively inappropriate. This does not mean that we must necessarily obey or follow norms. Nor does it mean that we are necessarily aware of (all) the norms present in the social context where we find ourselves. It merely means that we have the capacity to be sensitive to what is (in)appropriate according to norms at play in the present situation—that we have normative attitudes (Brandom, 1994).

I do not consider norms as something that people are obliged to follow; rather, they are co-constructed by interlocutors and are used by them as a structure to construct their own actions and to make sense the actions of others (Ingram, 2018). From our sensitivity to norms follows that we experience encounters between

norms that are recognised as appropriate and norms that are constructed as Other, inappropriate norms.

Noorloos et al. (2017) remarked that “every reason depends on a social context if it is to be uttered or understood. There are no reasons in the absence of others with whom to discuss, share, or establish them in the first place” (p. 449). That is, reasons emerge in the normativity of social practices and may therefore be considered a special kind of norms.

Norms, migration, time and space

In a migrating world, issues of dislocation and re-location that pertain to the *here-and-now* and *there-and-then* are present in people’s conversations (Dong & Blommaert, 2009). Therefore, migration as well as migration-related multilingualism are concerns of spatiality and temporality (Collins & Slembrouck, 2009). Time and space do not merely contextualize speech interaction. People speak in and from spaces and times that project particular values, social orders, authorities, and affective attributes. Hence, people speak in and from times and spaces that project particular norms. Time and space are not *neutral* background factors; rather, they are constitutive as they shape how people connect to each other (Barwell, 2019; Dong & Blommaert, 2009; Valero, 2007; Walkerdine, 2010) and the normative spaces of reasons that emerge among them.

The role of time and space adds to the complexity that characterizes mathematics communication in multilingual settings. The interlocutors’ speech acts do not merely use the sets of their idioms that they think will be successful in their communication, but they also speak from a plurality of times and spaces that include mathematics times and spaces (Barwell, 2019). A student utterance made *in* the space of the mathematics classroom might speak *from* a space outside the classroom or vice versa. Similarly, an utterance made in a particular lesson might speak *from* a different time. This thought draws attention to scale, as differences in position in time and space fluctuate continuously along the two axes (Barwell, 2019; Baynham, 2009).

For instance, an utterance made *in* the mathematics classroom might speak *from* a space outside it, such as a student’s home. An

utterance made *in* an interview might speak *from* a student's future. In the two examples the distance between *in* and *from* is relatively short. On the other hand, an utterance made *in* the mathematics classroom might speak *from* Plato or Euclid's mathematical time and space. In such a case, the distance is relatively large. Moreover, speech *in* the mathematics classroom *from* the mathematics of Plato or Euclid projects the values, social orders, and authority (embedded in the mathematics) from those times. In the same vein, Planas (2018) used the notion of *culture* (in plural) to capture the presence and emergence of *small cultures*—such as emerging norms, practices, and forms of knowledge of the world and people in local sites of interaction—and *large cultures*—such as more or less reified norms, practices, and forms of knowledge for broader societal representations. Thus, the norms that shape the spaces of reasons that emerge in people's face-to-face conversations are shaped by and reflect small and large cultures of different times and spaces.

Taking into account the complexity of student cohorts in mathematics classrooms, the possible time and space combinations present in classroom discourse appear to be infinite. This complexity is embedded in the normative spaces of reasons that emerge as students discourse. Hence, the normative spaces of reasons that emerge in mathematics classroom discourse comprise a far wider and more complex range of norms than the social and socio-mathematical norms (Yackel & Cobb, 1996) that often are used to characterise mathematical classroom discourse.

Knowledges and meanings co-exist in spaces of reasons. To address their co-existence, concepts needs to be placed in the space of reasons and their inferential interconnections articulated. Therefore, learning need not be matter of abandoning misconceptions or moving from informal to formal mathematics; rather, it may be about exploring and forming inferential relations between concepts, ultimately shaping interconnections between plural knowledges in epistemological ecologies (Santos, 2007).

Dealing with the uncertainty of meaning and knowledge – The language game of giving and asking for reasons

Considering translanguaging practices and that people never share the same idiolect completely, questions emerge about how we deal with the uncertainties of being understood and of understanding each other. What is in flux is not merely the semantic dimension of claims, but also the epistemological (Brandom, 2013; Derry, 2013). Before moving the discussion further, I would like to make explicit how I consider the semantic and the epistemological in this thesis.

Dealing with the uncertainties of being understood and understanding each other means paying attention to how meaning is conferred on our sayings and doings; that is, we turn our attention to semantics (Rowland, 2000). In language use, semantics is about the meaning content of the concepts we use when we express something. There are other ways of considering semantics, for example as a matter of truth conditionals, which is often the case in the philosophy of mathematics (see, for example, Buijsman, 2016; Ernest, 1991).

One way of thinking about the meaning of sayings is to approach meaning from an atomistic perspective. From this perspective the meaning of sayings is built from the bottom with the help of smaller building blocks (e.g., single terms) that make up the whole sentence. To reach the goal at the top (the full sentence), we need understand what each concept refers to or represents to put them together into a meaningful sentence (Brandom, 2000; Rowland, 2000). Each word has its own separate meaning. For example, in the claim “The book is red,” “the book” refers to an object in the world. It is from the reference that “book” gets its meaning (Rowland, 2000). This view of semantics is aligned with the notions of the representation paradigm discussed in the chapter *Language and epistemological diversity*.

Another view of semantics that I draw on in this thesis is the pragmatic top-down perspective that it is when sentences are used in social practice that meaning is conferred to the sentences and to the concepts they employ. This is a holistic, action-based perspective of semantics that connects the meaning of what is being said to the speaker’s act of saying it. Thus, it is the *speaker* who claims

“The book is red” who refers to an object, not the concept itself (Rowland, 2000).

From a holistic and pragmatic perspective of semantics it follows that knowing the meaning of concepts is to be able to apply them appropriately in the social practice where they are used (Brandom, 2000). This offers an account of *knowing* that such and such is the case, in terms of knowing how to *do* something. This leads us to the epistemological. I have already stated that in this thesis I use *epistemological* in a broad and social sense to mean how individuals and cultures come to acknowledge or disregard something as knowledge, as well as the multiple forms in which knowledge appears. *How* then do we come to take something to be *truth* and knowledge? Empiricists would say that by using our senses to experience the world and collect empirical evidence, it is possible to formulate good or valid *truths* through induction. Rationalists would say that knowledge is a matter of formulating logically valid *truths* (which connects to truth conditional semantics) by deductive reasoning (Toulmin, 2001).

The perspective I am developing here—claiming that knowing the meaning of concepts is to be able to apply concepts appropriately in the social practice where they are used—entails the act of judging the knowledge value of claims. It also involves recognizing potential reliable knowers—that is, those who apply concepts normatively appropriately. A knower (and this is not a matter of all-or-nothing, nor of always-or-never) is a claimer who gives content-related good reasons for the claim that are judged as reliable, is considered to be a reliable knower of what is being claimed, has inherited reliability from an original speaker of the claim, or is able to do something that others consider acts of knowing (Brandom, 1994; Seidouvy, Helénus, & Schindler, 2019). This reliabilist epistemology takes into account sensitivity to reasons embedded in social practices and puts normative judgement at the centre (Bransen, 2002).

Brandom (2013) explained that a traditional way of thinking about semantics and epistemology is to position them as hierarchical, considering the semantics—the meaning of the claim—as already being settled when the knowledge belief—the epistemologi-

cal—is considered (Derry, 2013). However, that is not the case according to inferentialism. Rather, in face-to-face discourse we consider the meaning of a claim at the same time as we consider its knowledge value. This means that the semantic and the epistemological are in flux simultaneously. Therefore, they ought not to be considered separately (Brandom, 2013). This is precisely what is at heart of inferentialism (Curren, 2007). Moreover, recognizing that the epistemological and the semantic are in flux simultaneously adds to the complexity of grappling with people's face-to-face use of translanguaging and individual idiolects and the reciprocal uncertainties of grasping each other's claims. Fortunately, as Curren (2007) wrote,

There are enough common *points of reference*, common *human interests*, or common *practices* (take your pick) across temporally and spatially distant human linguistic communities for us to track continuities and discontinuities in the use of concepts well enough for most purposes—sometimes only through great intellectual labour, of course. (p. 126, italics in original)

In line with the late Wittgenstein's ideas in *Philosophical Investigations* (2010) and according to inferentialism (Brandom, 1994, 2000), we play a particular social language game, the game of giving and asking for reasons (GoGAR) to keep track of continuities and discontinuities and to make them explicit. When people ask each other for reasons (explicitly or implicitly), that is when they are engaged in declarative talk; they make explicit implicit inferential relations between concepts (Brandom, 2000). When they are dealing with the meaning of a concept the semantic and epistemological are in flux. For instance, saying that some object is a rectangle because it has two pairs of parallel sides is making inferential relations—that is, the meaning of the concept—explicit. To say that something is a rectangle is to make a judgement that one can be held responsible for and asked to give reasons for in order for the interlocutor to assess the knowledge-value of the claim. Hence, there is also a social epistemological dimension to the GoGAR (Bransen, 2002; Derry, 2013).

The structure of GoGAR is built according to two normative statuses: *commitments* and *entitlements*. These normative statuses emerge in a socially articulated structure of *authority* and *responsibility*. When claiming that things are such and such one makes a commitment to a normative stand since one is putting forward a normative belief. By making a claim one also commits oneself to other normative stands or beliefs that follow implicitly from the original claim. This is the normative communicative status of responsibility. By undertaking the commitments of a claim, one also authorizes further commitments—to the inferential consequences of the propositional content of that claim (for example that it is normatively appropriate to claim that a rectangle has four sides) and also to communicational consequences with regards to how others use the original claim as premises and consequences in their reasoning. This is the normative communicative status of authorization (Brandom, 1994).

Normative assessment is the foundation of the GoGAR and from it follows a set of three moves according to which the game is played; attributions, acknowledgements, and undertakings (Brandom, 2000; Bransen, 2002). Below I give an example of how the status of a claim is changed from a commitment to an entitlement through the three moves in the GoGAR.

Let us assume that Person A make the claim “This is a rectangle” about an object. A is thereby committed to the claim. Person B attributes the commitment (including its inferential relations) to the belief about the object. Person B also attributes the commitment to Person A. Hence, Person B attributes to Person A the commitment to the belief that the claim is of the object is *true*.

Thereafter, Person B attributes to Person A an entitlement to the commitment, either by asking Person A to give good reasons for the claim, which Person B assesses as normatively appropriate, or by simply taking Person A to be a reliable knower of rectangles. Finally, Person B undertakes the commitment that s/he attributed to Person A. The claim now has the status of an entitlement, an acknowledgement that can be observed by Person B explicitly agreeing that the object is a rectangle or by Person B using Person

A's commitment (or what follows from it) in the conversation that follows (Brandom, 2000).

Having entitlements to a certain commitment means being acknowledged as able to provide normatively appropriate reasons for it, that is being acknowledged as a knower of the meaning of that particular stand. We keep track of our own and our interlocutors' commitments and entitlements in a metaphorical *scorekeeping book*, which helps us observe semantic and epistemological (dis)continuities in the idiolects we use. Hence, as we play the GoGAR we grapple with the uncertainties of understanding each other's idiolects.

The object of the GoGAR is not to win, but rather to play it well so that commitments implicit in sayings and doings can be discerned allowing players to get entitlements. "This makes playing [the GoGAR] well more than a matter of an individual player's competence: to play the game of giving and asking for reasons well, one needs a team" (Bransen, 2002, p. 387). In the social practice of the GoGAR we help each other out with discerning what follows implicitly from the use of certain concepts, unveiling their inferential relations to other concepts.

Problematizing some aspects of inferentialism

Inferentialism takes an immanent I-thou perspective, leaving out the transcendent I-we (Weiss & Wanderer, 2010). This means that the conceptualisation of norms from a sociological point of view is absent. Multilingual classrooms are often multi-ethnic and multi-racial. The stories, beliefs, attitudes, backgrounds, and foregrounds (Alrø, Skovsmose, & Valero, 2009) of students, their parents, and their teachers, together with discourses around discrimination, racializing, and Othering are intertwined in students' mathematics talk and learning (Källberg Svensson, 2018a, 2018b). Racialization—to attribute race to non-white individuals or groups—is part of constructing the Other or Othering. Othering is based on the idea that whiteness is not a race but the norm from which racialized minorities differ (Dyer, 2008). Ignoring issues about race or claiming that they are not (or no longer) relevant in societal life or in institutions such as classrooms is what Ahmed (2012) calls

Overing. Overing is to act as if issues concerning gender and race injustice are already done with.

To focus on immanent language use detached from social, gender, and race issues is a decontextualization of the classroom situation and students' interactions from which Overing might follow. Decontextualization holds that when actions and ways of being are studied and described, individuals and groups are not viewed through a socio-economic or culturally symbolic perspective (Eliassi, 2014).

Wagner and Herbel-Eisenmann (2009) discussed the radical immanent take in positioning theory. They argued that relationships, stereotypes, and discourses outside a given situation also have an immanent presence, as they are mediated through relationships with people. Taking into account that people speak in and from different times and spaces (Barwell, 2019; Dong & Blommaert, 2009; Valero, 2007; Walkerdine, 2010) has implications for the norms in play as claims are put forward in the GoGAR, but also for the normative assessment of claims. I agree with Wagner and Herbel-Eisenmann's (2009) claim that attention to immanent relationships can offer alternative understandings that a transcendent focus cannot. An immanent perspective on how students navigate language and epistemological divides offers understandings about how they grapple with uncertainty in face-to-face interaction.

Above I have argued that inferentialism offers an alternative stand to a representational paradigm. I have claimed that an inferentialist approach can embrace an epistemological (and ontological) plurality of mathematics and the co-presence of plural mathematics, taking the Platonic conception of mathematics as one mathematics among other mathematics. This is a debatable claim that ultimately, according to Radford's (2017) critique on inferentialism, resides in the distinction between realism and relativism. To address this issue, it is essential to take into account that inferentialism is sparsely used and relatively new to mathematics education research. Bakker and Derry published the first article on inferentialism in mathematics education (in statistics) in 2011. In 2017, there was a special issue on inferentialism in mathematics education research in the *Mathematics Education Research Journal*,

where some researchers accounted for and explored their use and interpretations of inferentialism, none of them concerned with multilingualism. To my knowledge, I am the first researcher to approach multilingualism using inferentialism in mathematics education research. This means that there are few discussed or pre-established understandings in mathematics education research of inferentialism and its adherence to realism or relativism. In the special issue of *Mathematics Education Research Journal*, the writers adopted a realist conception, approaching the world *as it is* (Radford, 2017). Radford's conclusion is, among other things, based on an unfortunate misquotation in one of the papers that refers to Brandom's (1994) work *Making it Explicit*. The writers in the special issue used the word *properties* to describe what it means to grasp the meaning of concepts. However, Brandom used the word "proprieties" (p. 5). There is an ontological difference between the two words. The use of the word *properties* suggests that there are in fact sets of properties that we grasp to master the real world. The use of the word *proprieties*, on the other hand, suggests normative relativism.

Radford (2017), who provided a critical reading of the articles in the special issue, concluded that although the articles reside in realism, it is hard to provide an answer as to whether inferentialism also does so. Brandom (1994, 2000) wrote about what he called a social objectivity, which has been much critiqued for being unclear (see, for example, Bowden, 2014; Hale & Wright, 2010). It is beyond the scope of this thesis to join that discussion. In this chapter I have focused on outlining my ontological and epistemological stands.

METHODOLOGY

The aim of this chapter is to provide an explicit account of the research process upon which this thesis is based. Since my experiences with the research process have influenced and shaped my choices and hence my results, it is imperative that I make my(research)self visible. By being transparent, I allow for critical examination of my knowledge claims, my intentions, and my chosen paths of engagement when meeting with people along the research process. In this I follow Valero's (2004) suggestion. One way of making myself visible is to use the pronouns *I* and *we* as I write this text. According to Knijnik (2004), such usage is a political act that underlines the assumption that knowledge production is neither value-free nor neutral; rather, it reflects deep political and moral stances.

In the prologue I presented the three divides form a personal standpoint. In doing so I made myself visible to the reader. I join researchers who follow Popkewitz's (1984) advice that "[o]ur method of research emerge from our involvement in our social conditions and provide means by which we can seek to resolve the contradictions we feel" (p. preface). Being explicit about the research process and methodological considerations involves trying to be explicit about my own involvement in various social conditions as well as contradictions that I experience as they shape the lens through which I see and think about my research and the issues I raise (Tracy, 2010). Although such an enterprise cannot be conducted in a neutral or objective way, I hope that my attempts to be transparent with my own experiences, intentions, and concerns will support critical examination of my work.

Overview of the methodology in the articles

The overview below is provided to allow critical examination of the methodological connection between aim, empirical material, and analytical tools in each article.

Article	Aim	Empirical material	Analytical tools
I. Epistemological aspects of Multilingualism in Mathematics Education: An inferentialist approach	To consider the language-as-resource metaphor and show how inferentialism can unveil an epistemological dimension of mathematics education research on multilingualism.	For illustration; field notes, interviews classroom interaction transcript.	The space of reasons and inferential relations between concepts, entitlements, and commitments (Brandom, 1994, 2000).
II. "Mathematics is bad for the society": Inferential Reasoning about mathematics as part of society in a culturally and linguistically diverse middle school classroom	To discuss how students from diverse social and linguistic backgrounds grapple with unveiling critical aspects of mathematics in society.	Intervention; Transcripts of video recordings.	The GoGAR and inferential relations between concepts (premises and consequences) (Brandom, 1994, 2000).
III. Mathematical preciseness and epistemological sanctions.	To provide findings on mathematical discursive norms that evoke epistemological sanctions in classroom talk during a group activity	Transcript from audio-recorded group activity.	Commitments, entitlements and sanctions, the GoGAR (Brandom 1994, 2000). I-language games (Beristain, 2011).
IV. A joke on precision? Revisiting 'precision' in the school mathematics discourse.	To explore the co-presence of epistemological discourses on mathematical preciseness in a joking event in the mathematics classroom.	Transcript from audio-recorded group activity.	The space of reason, theorizations of humour (e.g., Hurley, Dennett & Ad, 2011).

Table 1. Methodological overview of the articles.

To be explicit about the research process I will present a) the construction of empirical material, b) the methods of analysis, c) ethical considerations, and d) quality issues.

Empirical material

By using the expression *construction of empirical material*, I emphasize that I do not view data as a neutral object for researchers to collect and analyse. Rather, data is constructed by choices made by the researcher and by the researcher's interaction with the participants (Alvesson & Sköldberg, 2009).

It has not been a straightforward process to construct the empirical material used to answer my research questions, as it was not obvious to me from the beginning what kind of empirical material would be helpful to explore in order to theorize students' navigation of language and epistemological divides, that is, to answer the research questions. Therefore, I have used a flexible research design (Robson, 2002), which means that I have used multiple empirical materials: interviews, field notes from participant observations, audio and video recordings, and photographs. This approach is recommended for research on language and mathematics education, since mathematical activity is multi-semiotic (Moschkovich, 2018b). Multiple types of empirical material often allow different facets of problems to be explored (Tracy, 2010). This means that a flexible research design has the potential to inform plural dimensions of students' navigation of language and epistemological divides and thus allow for more complex and in-depth understandings.

Adopting a flexible research design puts demands on the researcher during the study. As the researcher uses the self as a research tool, qualities such as having an open and sensitive analytical mind and the ability to establish trustful relationships are crucial (Robson, 2002). The researcher needs to make interpretations in the field to find contradictions and to consider if more or new kinds of empirical material are needed. This means that if interactions during my research process had played out differently, I would have made different choices and a different story might have been told by the researcher(s) and/or the participants. This raises a

twofold issue about the credibility of my study. It is twofold because the issue at stake is related to what role the empirical material plays in this thesis as well as to how credibility is established (Tracy, 2010). Labaree (2012) argued that the role of the empirical material is not necessarily to be or tell the research results; its role can be to *support* the findings. Labaree explained:

The point of carrying out research is not to master every piece of trivial data on a subject and treasure every word of the related scholarly literature. Instead, the point is to find something in the data and in the literature that might be enlightening – to an academic or practitioner or policymaker or citizen. (p. 82)

This is what I have attempted in this thesis.

Giving my empirical material this particular role means that my preliminary analyses have been directed towards purposefully looking for *rich points*, which are sections of data that stand out as being unusual, different, and/or difficult to understand in the interaction (Copland & Creese, 2015). (I further account for methods of analysis and *rich points* later in this chapter.) Hence, I have concentrated less on gathering and processing large amounts of empirical material and more on establishing trusting relationships with the participants and on interpreting and reflecting on my empirical material in relation to my object of study and the aim of this thesis (Alvesson & Sköldberg, 2009).

School and classroom

Investigating students' navigation of language and epistemological divides in social and language diverse classrooms means studying student interaction in a classroom. To think about what kind of classroom to conduct my study in, I spoke to two teachers and observed two mathematics lessons they taught at two different schools in so-called socio-economically marginalized areas of a town in southern Sweden. In those classrooms, hardly any students were monolingual. Most of the students shared the same *other* language, which made code switching, translation, and peer talk possible (Bose & Clarkson, 2016; Moschkovich, 2015b; Planas &

Civil, 2013; Planas & Setati-Phakeng, 2014). Since I was interested in classrooms that were socially and language *diverse*—reflecting the complexity of contemporary migration (Meissner & Vertovec, 2015) typical in many European countries and in Sweden (Meyer et al., 2016)—I realized that I had to conduct my study at a school where students had different linguistic and social backgrounds.

To find such classrooms, I contacted a research and development coordinator on school issues in a municipality close to several larger towns in the south of Sweden. I asked if there was an interest in participating in my research project and if student cohorts at schools in the municipality matched my demands. The coordinator passed my request on to the headmasters in the municipality. One headmaster responded. We had a meeting discussing my research interest and the initial idea for the study. I met with two teachers at the school who were interested in participating—one Grade 6 and one Grade 5 teacher—to discuss the research project and the ethical guidelines that direct social science research projects in Sweden (Vetenskapsrådet, 2011). I explained that I could not in detail account for what empirical material I was interested in but that I wanted to discuss intentions and ideas with them as they emerged or changed during and between my visits. I said I hoped they would like to influence the research process (Robson, 2002). I did that to show my concern for their needs and to show that their knowledge was important to me (Tracy, 2010).

As I asked the teachers if they were interested in participating I realized that the headmaster of the school was interested in my project and wanted research to be conducted at the school. The teachers were likely aware of the headmaster's wishes. This created an unequal power relation that I found problematic because if the teachers declined to participate, this would mean the teachers would also be saying no to the headmaster's wishes. Tracy (2010) suggested that a relationally ethical researcher should "engage in reciprocity with participants and ... not co-opt others just to get a 'great story'" (p. 847). Therefore, I was careful to emphasize that I was aware of the unequal power relation as I tried to make the teachers feel comfortable with rejecting participation. Although both teachers were interested in participating, the Grade 6 teacher

and I agreed that because of the intense Swedish national testing programme for year 6, which was to occur during the time of my planned visits, the Grade 6 class would not participate.

The Grade 5 class

The Grade 5 class was located at a school in a middle-class suburban community close to two cities to which most residents commuted for work. Apart from the middle-class community where the school was located, the municipality also included a working-class area and a so-called socio-economically marginalized area. On a statistical average, the children in this municipality were disadvantaged when it came to their guardians' income, un-employment rates, health status, and level of education compared to the average child in Sweden. In 2017 the percentage of adult citizens in the municipality born abroad was approximately 30%. The reception of refugees was above the national average in this municipality (Statistiska centralbyrån, 2019). Since the two communities in the municipality were different, the distribution of the statistics presented above were skewed.

Some of the Grade 5 students travelled to school from homes located in the working class or marginalized areas. This was a choice made by their parents because "at the other schools [where I live] there are many fights and you do not learn so well there" (interview with the student Aldrin, May 2017). Thus, the class was not a homogenous middle-class group of 11-year-olds, but a socio-economically diverse group.

Most of the students had a Swedish background. Eight students had experiences of international migration in their families and said they had access to languages other than Swedish. Nine different languages were mentioned in student interviews (Albanian, Arabic, Bosnian, Hebrew, Kurdish, Norwegian, Persian, Polish, and Serbian). Having access to a language does not necessarily mean that the students were fluent or literate in the languages. It means that they at some level lived with or needed to use more than one language (Hall, 1995). No student shared a language other than Swedish with a peer. Swedish (the language of instruction) was the only language heard in the classroom. When asked in indi-

vidual interviews, none of the students claimed ever to have heard or spoken a language other than Swedish in the classroom, except from English as a school subject and as the *lingua franca* of Western culture. Only occasionally did other languages appear on the school grounds. Some students were unaware that they had classmates who spoke languages other than Swedish. Thus, the “Swedish only” discourse (Norén & Anderson, 2016) was operating strongly at the school, but not only at school. Aldrin (who spoke both Persian and Swedish at home) described how his mother brought him additional mathematics tasks in Swedish (interview in May, 2017):

U: Are the math tasks your mother brings you in Swedish or in Persian?

A: In Swedish.

U: Why do you think your mother brings you tasks in Swedish?

A: I don't know. So that I will learn more Swedish, I think.

From the interview I derived that the “Swedish only” discourse was also operating outside school, for instance in students' homes.

Although in the interviews I did not ask the students about reasons for their parents' migration, they often told me spontaneously. Reasons mentioned for migrating were multiple: political, economic, family, and so forth. The mixture of the languages the students mentioned having access to connected to countries that were typical countries of origin and time of immigration for different groups of immigrants in Sweden (see, for example, Svensson Källberg, 2018b) and hence were representative of many Swedish classrooms. Some students in the Grade 5 class were first-generation immigrants while others were second generation.

Overview of the empirical material and its place in the articles

To enhance readers' grasp and critical scrutiny of the empirical material and its role in each of the articles, I provide the following summary:

Empirical material	Quantity	Article	Role
Audio-recorded semi-structured individual interviews.	262:49 min	I-IV	Information supporting purposeful sampling and location of <i>rich points</i> . Transcripts excerpted for illustration.
Field notes and photographs from classroom observations; other field observations; (in)formal talks with students, teachers, and headmaster.	26 pics 22 pages	I-IV	Information on classroom norms and discourses. Location of <i>rich points</i> .
Audio recordings from <i>ordinary</i> classroom interaction.	102:21 min	I, III & IV	Used for discerning <i>rich points</i> and critical events. Transcripts used for analysing and illustrating.
Video recordings from small scale-project held during three sessions, in total 170 min.	335:35 min	II	Used for discerning what conceptual connections students make. Transcripts excerpted for analysing and illustrating students and teachers attempts to grasp each other's ideas.

Table 2. Overview of the empirical material and its role in the articles.

Field notes and photographs

I took field notes as I visited the classroom. I took pictures of the classroom walls, student work, and notes on the whiteboard.

Interviews

When I acted as a participant observer in the classroom, I experienced a need to learn about the students' ideas about the (lack of) multilingual language use in their classroom and at their school since language diversity did not seem to be addressed in the classroom nor was it enacted in the classroom.

I also needed to learn the students' ideas about the role of mathematics in their lives in and out of school as a way of trying to grasp epistemological discourses that were underpinning their interactions. And I wanted to get to know the students and give them the chance to get to know me to establish trusting relationships (Andersson & le Roux, 2017; Tracy, 2010). I decided to conduct individual interviews with the students because although participant observation allowed direct contact in the classroom setting, it did not allow me to have intimate uninterrupted conversations with the students (Mertens, 2005).

Focus group interviews are sometimes used to enhance a safe interviewing situation in which participants support each other in discussing difficult subjects. On the other hand, focus group interviews might silence less dominant participants (Robson, 2002). By the time I decided to conduct interviews I had visited the classroom a number of times and I was already developing a trusting relationship with the students. Therefore, I decided to conduct individual semi-structured interviews (Mertens, 2005; Robson, 2002).

I used an interview guide (see Appendix 1) that was influenced by my preliminary analyses of field notes and pictures. I discussed the interview guide with my supervisors to determine critical questions (Mertens, 2005). I asked about four major themes (Robson, 2002):

- 1) the students' ideas and experiences of school in general and of mathematics in particular, including what they thought were their parents' ideas about mathematics;
- 2) their hobbies and interests outside of school;

- 3) the students' own mastery of and ideas about languages; and
- 4) their ideas about their peers' mastery of and ideas about languages.

I conducted two pilot interviews that made me alter the initial interview guide to add questions about the students' hobbies and spare time. I did this because I realised that the students seemed to want to talk about their spare time and because it helped me to grasp and contextualize their social and language back- and foregrounds (Alrø, Skovsmose, & Valero, 2009).

When talking with the students I tried to follow Alrø, Skovsmose, and Valero (2009), who suggested using dialogue as a research approach. Through dialogue, perspectives within the students' reach and capacity may be identified, examined, and challenged (Alrø, et al., 2009). The dialogue approach offered the students opportunities to ask me questions too, which helped me in my attempts to be relationally ethical (Tracy, 2010). Although I aimed for a dialogue between the students and myself it is important to recognize that in interview situations the interviewees might respond in ways they think the interviewer wants, which affects the construction of the empirical material (Mertens, 2005).

I started every interview by informing the student that they could stop the interview at any time, that I would not tell their teacher or classmates what we talked about, and that their real names would not be revealed when I wrote about the interviews.

Classroom audio recordings

When I acted as a participant observer in the classroom and the students engaged in small group activities, I would initially move around in the classroom. As the relationship between me and the students developed I would ask a particular group if I could listen to their discussions. Since I could only listen to the conversation of one group at the time, I asked other groups if it was okay that I left a recording device at their table.

The classroom was organized in such a way that groups of four or five students sat around small tables. The discussion groups often consisted of the students sitting around the same table. I ob-

served that one of the groups appeared to be particularly interesting, as it comprised a newly-arrived student who was an emergent speaker of Swedish, one multilingual student, and two students who were monolingual (except for being learners of English as a compulsory subject at school). The four students in this group usually engaged actively in group discussions. Since the group contained the social and language diversity I was interested in, and since they were very active, I left a recording device at their table when some group discussions were conducted. Hence, I conducted purposeful sampling (Patton, 2002) based on my observations and interviews.

Purposeful sampling is used to provide insight about a phenomenon rather than an empirical generalization. To this end, empirical materials are chosen in such a way that they are particularly illuminative or information-rich in offering manifestations of the phenomenon (Mertens, 2005; Patton, 2002; Robson, 2002). I also recorded conversations between students and myself as they asked me for help with textbook tasks. I transcribed verbatim the audio recordings.

Video recordings

To make video recordings of the small-scale project in Article II, I used a Lesson-box® comprising three video cameras and three microphones. The cameras were placed along three of the classroom walls. One microphone recorded all the classroom sound and the two others were placed at student tables during the small group discussions. In total, there were five student tables. One microphone was placed on the teacher during whole class discussions. Unfortunately, one of the Lessonbox®-cameras malfunctioned. A hand-held video camera was placed at the back of the classroom as a back-up. The Lesson-box® set-up managed to cover what I perceived as all the whole-class discussions. However, as I looked through the film-clips I noticed that halfway through the third whole-class discussion the picture went black and there was no sound. Due to poor sound quality, it was only possible to analyse the group discussions at three of the five tables with students. The Lesson-box® equipment records audio and video in separate files, which allows for combining vari-

ous files according to the editor's wishes. I received some instruction and managed to do the editing myself. Critical instances (see Article II for definition) were transcribed.

Establishing relationships with the students

Adopting a flexible research design meant establishing trusting relationships with the students (Robson, 2002). The relationships between the researcher and the participants influence the opportunities that emerge for construction of empirical material (Andersson & le Roux, 2017; Mertens, 2005; Patton, 2002).

During the time I spent at the school, I had different roles. I was a participant observer, I taught during the small-scale project (see Article II), and I interviewed all the students in the class. Although the roles were different, I tried to position myself as a learning researcher who was seriously curious about and open to the students' views and experiences (Vithal, 2004). I tried to be approachable, friendly, and honest. Positioning myself like this helped me to establish trusting relationships with the students, relationships that have influenced the quality of my empirical material and thus the quality of the study (Tracy, 2010).

In Articles I and II there are examples of how I explicitly asked the students for help as I grappled with grasping what they were saying. I positioned myself as less knowledgeable, wishing to learn together with the students. I thus opened myself up to understanding the students' voices and consciousness while simultaneously letting them know mine (Radford, 2008). Although I had an epistemologically special position as the knowledgeable other with regards to school mathematics (Erath, 2018), repeatedly positioning myself as a learner of students' ways of knowing gave me the opportunity to capture *rich points* (Copland & Creese, 2015) of students navigating language and epistemological divides, overbridging and Sherpa-like actions (Article II), and moving the interlocutor into a translanguaging space (Article I).

I am aware that positioning is a reciprocal act in which participants position themselves in ways that are reflexive, relational, and contextual (Wagner & Herbel-Eisenmann, 2009) while they also position the researcher and vice versa (Andersson & le Roux,

2017). The student Arvid's comment "Mathematics is bad for society," which was the starting point for the empirical material used in Article II, I believe was uttered in response to the relationship that was emerging between Arvid and me.

During a classroom visit prior to the occasion when Arvid made his statement, he had showed some curious interest in me. For instance, once he asked me if I liked mathematics. I answered honestly saying that nowadays I like mathematics very much but that when I was his age I more or less hated mathematics. We had a small conversation about our experiences of learning mathematics and my experiences of teaching mathematics. By sharing and discussing my own experiences and thoughts with Arvid, I did not act as a researcher in the positivist subject-object-paradigm (Olsson, 2015; Vithal, 2004). Rather, a subject-subject space emerged in which we—with reciprocal respect and trust—explored our experiences with mathematics teaching and learning.

Although my intention was to position myself as friendly, approachable, and seriously curious about and open to the students' views and experiences (Tracy, 2010; Vithal, 2004), there were of course instances when I failed to do so; I sometimes found the students did not position me the way I had hoped. For instance, on one occasion I was thoughtless and insensitive as I tried to encourage a group of students (the same group of students that appear in Articles III and IV) to further develop their ideas when discussing the statement "Mathematics is bad for society." Shortly after I had spoken with the group of students, a boy came over to me and told me that I had made a girl in the group sad. I told the boy that I was grateful to him because sharing this with me made it possible for me to speak to the girl and apologize for my thoughtlessness. As they opened themselves to let me know what I had done, I was able to learn how to be together with them in their learning (Radford, 2008) as they showed me their ways of caring for each other. At the same time, they were caring for my ways of being with them. To be included in the group's care for each other was an act of trust between the students and myself. In the light of this experience, I noted several other caring actions among the students in the particular group.

The caring relationship with this group of students made the piece of interaction among the group analysed in Articles III and IV, which was filled with competitiveness and uncaring actions, stand out as contradictory and difficult to understand. My relationship with the students allowed me to identify *rich points* (Copland & Creese, 2015) that further informed my understandings of students’ navigation of language and epistemological divides (see Articles III and IV).

The above shows that although the amount of empirical material in this thesis is limited, it is based on carefully and consciously established trusting relationships that have been crucial for the construction of my empirical material and the emergence and identification of *rich points*.

Methods of analysis

Alvesson and Sköldberg (2009) suggested reflexive interpretation as a useful method of analysis, particularly when the researcher has constructed different kinds of empirical material such as interviews and observations. By reflexion Alvesson and Sköldberg (2009) meant the researcher’s ability to continuously break from a particular frame of reference, to critically ask what the frame is insufficient for. Hence, reflexive interpretation does not aim at expanding particular frames of reference or understandings. Rather, it aims at breaking consistency and looking for weak points in one’s own reasoning. The researcher moves fluidly and iteratively among four levels of interpretation (more levels could be used). I outline these in Table 3 below.

Level	Focus
Interaction with empirical material	Participant utterances, observations, etc
Interpretation	Tacit meanings
Critical interpretation	Ideology, politics, social reproduction
Self-critical and language reflection	Text production, claims, selection

Table 3. Levels of reflexive interpretation.

At the first level the researcher makes preliminary interpretations (Alvesson & Sköldberg, 2009) while conducting interviews, selecting what to record, interacting with participants, transcribing interaction, and, in my case, designing the small-scale project. Interpretation at this level is involved in the construction of the empirical material. In the previous section I accounted for the construction of my empirical material.

At the second level the researcher engages with the empirical material in order to look for tacit meanings, find patterns, look for critical instances, and develop categorizations. In Article II, I account for the analytical process by discussing how critical instances were defined and what analytical questions I used for analysis of the empirical material.

To break the consistency in the findings in Articles I and II, I used purposeful sampling (Patton, 2002) to find a disconfirming case. A disconfirming case is a case that does not fit the emerging theorizations (Mertens, 2005). I identified the empirical material used in Articles III and IV as a *rich point* (Copland & Creese, 2015). Based on my preliminary interpretations at the first level of reflexive interpretation and the analysis made in Article II, I found it difficult to understand how differently the multilingual student Samir positioned himself and was positioned with respect to mathematics and to his friend Darco compared to what I had observed previously and written about in Article II. The method of analysis for Article III at this level is described in Ryan (2018).

Article IV is based on the same piece of student interaction as Article III. The empirical material in Article IV is just a few lines comprising a joke. While I was working on analysing the material for Article III at this level of interpretation, I made an initial interpretation of the joke event. However, as I critically reflected on my interpretation (level four) I realized that it would be fruitful to explore different interpretations to provide a deeper understanding of the interaction and ultimately the tacit role of discourses on mathematical epistemology in classroom interaction, which is the objective of Article IV. At this level of interpretation field notes, individual interviews, and photographs played an important role in helping me understand norms that guided student-to-student and stu-

dent-to-teacher interactions in the classroom. For instance, the individual interviews and my field notes confirmed that an appreciation of mathematics as precise, certain, transcendent, neutral in society but valuable for success in life was manifested and occasionally troubled by the students but not by their teacher.

At the third level, the researcher makes interpretations using metatheory. Metatheory is applied to problematize the legitimacy of prevailing ideas, theorizations, and paradigms (Alvesson & Sköldberg, 2009). At this level of interpretation, I used the key notions presented in the theory section and major inferentialist ideas such as the alternative stand to the representational paradigm to problematize the notion of language-as-resource and to offer a pluralistic epistemological dimension of multilingualism. This is accounted for explicitly in Article I but it also played an important role in the other articles, as this level is reflected in the analysis of the other levels and vice versa.

At the fourth level of interpretation, the researcher reflects upon and problematizes his or her own authority and interpretations; it is a kind of self-reflexion (Farahani, 2010; Tracy, 2010). This may be done by, for instance, making oneself visible in the writing and by problematizing one's own interpretations in by opening up a text and considering a plurality of interpretations (Alvesson and Sköldberg, 2009). I do this explicitly in Article IV. In Articles I and II I analysed sections of interaction where I made visible my own monolingual lack of understanding of how to interact in silently translanguaging conversations. To perform this analysis, I had to reflect on my authority as an L1-speaker of Swedish. The interpretation at this level was reflected in Levels Two and Three. Moreover, at this level of interpretation I considered my preliminary interpretation of the joking event, which allowed me to consider the tension of being in-between in recognition and challenging of mathematical epistemological discourses present in the event.

Analyses in the four articles

To synthesize epistemological dimensions that emerged as students interacted to grapple with navigating language and epistemological

divides I have used three different analytical frameworks in the four articles (in Articles I and II the same approach is used).

I did this to illuminate some interactional aspects of students' navigation of epistemological divides from a position that recognizes language as a complex meaning-making system (Moschkovich, 2018b). I crystalized (Tracy, 2010) not merely the empirical material but also the analyses of it to "open up a more complex, in-depth, but still thoroughly partial, understanding of the issue" (Tracy, 2010, p. 844). Nevertheless, all three analytical frameworks reside in the socially normative authority/responsibility structure of the GoGAR. Therefore, in my analyses I looked for instances of students implicitly or explicitly giving and/or asking for reasons. *Giving reasons* I defined, following the general ideas of inferentialism, as declarative talk that made beliefs explicit to explain some behaviour. I have taken a broad definition of *explain* by considering turns or sequences of turns that comprise talk that clarifies previous claims, displays knowledge, or defends beliefs to be explanations, since in mathematics classroom talk it is not easy to make distinctions between explanations and arguments (Ingram, Andrews, & Pitt, 2018). Declarative talk typically includes the use of *that*- and/or *if/then*-propositions (Brandom, 2000). Below I account for the analytical frameworks that I used for the analyses in the articles and show what epistemological dimension they address.

Analytical framework in Articles I and II

The analytical framework I used in Articles I and II was designed to capture performances from the perspective of students' discursive grappling with conceptual meaning. When students discourse to grapple with meaning they articulate epistemological divides—differences that emerge when they take and treat something as mathematical knowledge (Schütte, 2018). This analytical framework was designed to find patterns of performance the students exploited to navigate the divides from a conceptual epistemological dimension. I looked for instances of student talk comprising *that*- and *if/then*-propositions that revealed patterns of discursive performance attempting to overcome language and epistemological divides when the students were grappling with meaning. Such at-

tempts I characterized as instances when students *repeatedly* asked for or gave reasons for the same doing or saying, that is, for the same behaviour.

Analytical framework in Article III

The analytical framework I used in Article III was designed to capture performances from the perspective of personal latitude that the authority/responsibility structure of the GoGAR shapes. In social epistemology, the notion of reliability is essential for the judgement of knowledge claims (Brandom, 2000). This means that making knowledge claims is a matter of the interlocutors' judgement of the claimer's reliability, which in turn shapes the personal latitude to make claims that the interlocutors recognize as knowledge claims. Hence, there is an epistemological dimension of personal latitude present when students navigate language and epistemological divides. Wagner and Herbel Eisenmann (2014) used the notion of personal latitude in a slightly different style, where they emphasized people's awareness of their own or another's personal latitude.

The analytical framework used in Article III and its operationalization I have accounted for in Ryan (2018).

Analytical framework in Article IV

The analytical framework I used in Article IV was designed to capture performance from the perspective of discursive normativity. Since I am concerned with epistemological divides—differences that emerge when individuals who are participants in cultures take and treat something as mathematical knowledge—*difference* needed to be conceptualized and addressed. I have taken difference to be the outcome of the encounter of non-coinciding norms. To understand students' navigation of language and epistemological divides it is necessary to consider the discursive normativity that underpins their interaction. There is an epistemological dimension of normativity present in students' navigation of language and epistemological divides. I looked for instances in student discourse that comprised beliefs about the character of mathematics (Moschko-

vich, 2002), as they potentially would reveal normativity that underpinned the authority/responsibility structure of the GoGAR.

Procedural ethics

When conducting research in the social sciences in Sweden, the ethical principles proclaimed by Vetenskapsrådet (2011) must be acknowledged. Vetenskapsrådet's ethical guidelines are primarily concerned with what Tracy (2010) referred to as *procedural ethics* in the production of empirical material. To meet the needs for procedural ethics, the Grade 5 teacher and I arranged for me to visit the class and inform the students about my research intentions, which I explained as an interest in how they use language as they reason about mathematics. To find out about this, I told the students that I wanted to visit their mathematics lessons, make notes, take audio and perhaps video recordings, and conduct interviews with them. I informed the students that they could say no to participation or change their mind at any time even if they might initially have agreed to participate. I explained that in writing and/or talking about them and their school I would use pseudonyms. I handed out a letter of consent for their guardians (Appendix 2) and explained what it said. Earlier, I had asked the teacher if any of the students' guardians would prefer a copy of the letter in a language other than Swedish. She suggested a version in Arabic, so I contacted a licenced translation agency, who made a copy of the letter in Arabic (Appendix 3). The guardians of all students in the Grade 5 class signed.

The quality of the thesis

The quality of methods used in quantitative studies can be described in (measurable) terms of, for instance, significance, validity, reliability, and replicability. Method-use and methodology in qualitative studies are different and diverse, yet they are still guided by qualitative signifiers. To describe the quality of methodologically diverse types of qualitative studies, Tracy (2010) suggested eight *big-tent* criteria for highly qualitative research. In this section I discuss the quality of this study by applying these eight key markers of quality: 1) worthy topic, 2) rich rigor, 3) sincerity, 4) credibility,

5) resonance, 6) significant contribution, 7) ethics, and 8) meaningful coherence. Below I consider the criteria in relation to this thesis.

According to Tracy (2010), *worthy topics* are topics that challenge prevailing norms. This study aims to challenge the prevailing language-as-resource paradigm and to offer an epistemological dimension of multilingualism that does not reside in dichotomization. This is important for two reasons.

One, the language-as-resource metaphor departs from dichotomization between named languages. Blurred migration patterns complicate distinctions between languages. The language-as-resource metaphor in its present shape has become insufficient as recent theorizations of multilingual language use dissolve such dichotomizations (Blommaert & Rampton, 2011) and offer a translinguaging (Garcia & Wei, 2014) perspective on multilingual language practices, which has implications for the metaphor. There is a need to explore multilingual language practices from a perspective that offers alternatives to making distinctions between named languages.

Two, challenging the language-as-resource paradigm may include challenging the prevailing representational paradigm that tacitly privileges hegemonic Western mathematics (Chronaki & Planas, 2018). This calls for research efforts that offer insights into and theorizations of the co-presence of a plurality of mathematical knowledges and ways of knowing mathematics in multilingual classrooms. Ultimately, such an effort is concerned with social and cognitive justice (Santos, 2007). Taking the above into account, I consider my topic a worthy one.

With *rich rigor* Tracy (2010) asked researchers to consider how the study uses sufficient, abundant, appropriate, and complex theoretical constructs, data, and time in the field, sample, context and data collection, and analysis processes. In the theory section I have presented the theoretical constructs and their interconnections used in this thesis. Previously in this chapter I addressed my analysis and construction of empirical material. I have discussed the purpose of the empirical material in this thesis as supporting the findings rather than being the findings (Labaree, 2012) and explained that

during the research process I repeatedly considered and re-considered my amount of constructed empirical material.

According to Tracy (2010), good qualitative research is not about the amount of data and the time spent in the field; rather, it is about whether the data is sufficient to answer the research questions and support the claims. Here I will focus on time spent in the field. I consider the twenty years I spent as a mathematics and science teacher in socially diverse classrooms as time spent in the field. During my 20 years as a teacher, I have had many opportunities to consider students' plural ways of knowing mathematics and I have witnessed that privileging one particular mathematics produces social and cognitive injustice.

Sincerity is about the researcher making him- or herself and her potential biases visible. I have tried to do this by using first person pronouns, sharing personal experiences, showing my own lack of meta-understanding of translanguaging practices, and making myself visible in processes of analysis.

According to Tracy (2010) the *credibility* of the results of a study ultimately lies in whether “readers feel [they are] trustworthy enough to act on and to make decisions in line with” (p. 843). Quantitative studies discuss trustworthiness and reliability, Tracy suggested the notion of crystallization for qualitative studies.

Crystallization encourages researchers to gather multiple types of data and employ various methods, multiple researchers, and numerous theoretical frameworks. However, it assumes that the goal of doing so is not to provide researchers with a more valid singular truth, but to open up a more complex, in-depth, but still thoroughly partial, understanding of the issue. (p. 844)

I have constructed a variety of different empirical materials and developed analytical tools that allow for close analysis of tacit translanguaging interactions that consider epistemological aspects. As mentioned above, Labaree (2012) argued that the role of the empirical material is not necessarily to be or tell the research results; instead, its role can be to support the findings. In this view sharp analytical tools are used and the empirical material functions

as illustrations in the researcher's attempts to substantiate claims sufficiently. Credibility lies in the thoroughly-sharpened analytical tool and in whether its application on the empirical material is conducted in a way that produces "focused and lean" (Labaree, 2012 p. 80) analyses. To trace an epistemological dimension of multilingualism to consider how students navigate epistemological divides, I have had to develop sufficient conceptualizations. In the next chapter I address those conceptualizations as answers to the second research question in this thesis.

Resonance is concerned with whether the research when presented catches the audience's attention. This is difficult for me to scrutinize. When I presented a research report (Ryan, 2018) at the 42nd Annual Meeting of the International Group for the Psychology of Mathematics (PME) the editor of the journal *For the Learning of Mathematic* was present. A couple of weeks later he contacted me, writing that he had found my presentation *mind provoking* and asking if I would be interested in writing an article for the journal (Article III in this thesis).

Ethics is discussed throughout this chapter. Finally, *meaningful coherence* is about whether or not the study achieves its aims, if methods and procedures fit its stated goals and if literature, research questions, findings, and interpretations connect meaningfully (Tracy, 2010). This has been my aim, but I think it is up to the reader to judge whether I have achieved it.

CONNECTING THE ARTICLES

In this chapter I use methodology that is inspired by cartography-making in the vein of Deleuze and Guattari (Ulmer & Koro-Ljungberg, 2015; Walkerdine, 2013) to develop replies to my second research question: “What theoretical conceptualization of epistemological dimensions of language diversity can be used to frame the students’ navigation of the divides?” The first research question is primarily elaborated in the four articles.

Ulmer and Koro-Ljungberg (2015), who wrote about cartography, noted that “[t]he data/knowledge domain [...] transfers back and forth between the map maker and map user, with the cartography itself as a conduit” (p. 139). As I wrote this chapter, I found myself moving among the four articles, alternating between being the map-maker and the map-user. In the sections named *Entanglements...* below, I develop the map-making efforts of theoretically conceptualized epistemological dimensions of language diversity, while the article summaries comprise the data/knowledge domain.

Entangled articles

This chapter moves along two major conceptualizations that inform the epistemological dimension of language in a socially diverse mathematics classroom. The conceptualizations function as conduits that connect the four papers and allow movement among them. The concepts are *mathematics based discursive spaces* (MBDS) and *meta-understanding of language diversity* (MULD). While it is these two concepts that are at heart here and used to in-

form and theorize students' navigation of language and epistemological divides in a social and language diverse yet monolingual mathematics classroom, the article summaries as well as the sections about their reciprocal entanglements will focus on these two concepts. Following the ideas of inferentialism (Brandom, 1994, 2000), my aspiration is that the two concepts will be conferred with meaning as I use them below. I therefore refrain from defining them here.

The four articles do not connect in a temporal line; they do not represent a journey that can be traced from A to B. Rather, they are entangled in a rhizomatic way that encompasses the three divides I exposed in the prologue of this thesis. The entanglements resemble a visitor's movement in a city—movements that cross themselves, re-enter points, and go back and forth in time and space (Walkerline, 2013). With these apparently haphazard movements the visitor begins to grasp the city and form a mental map of it. By moving about, I simultaneously produce and read a map that attempts to provide replies to my research questions.

Summary of Article I: A relational epistemological dimension of multilingualism

The aim of Article I was to trace an epistemological dimension of multilingualism in mathematics education and to add it to the current problematizing discussions on the prevailing language-as-resource paradigm, which conceives of students' first language as a resource in their mathematics learning (see, for example, Bose & Clarkson, 2016; Moschkovich 2008, 2015a; Planas, 2014; Planas & Civil, 2013; Setati & Adler, 2000). The rationale for this article originated from my readings of previous research on the language-as-resource (Ruiz, 1984) metaphor. In the paper, my co-author Aldo and I found a first wave of language-as-resource in mathematics education research that risked reproducing a paradigm that conceives of students' first language ultimately as a deficit, but a deficit that can be used as a resource if properly cultivated (Ricento, 2005). We concluded that the first wave of the language-as-resource metaphor used in policymaking and in research-at-large functions as a commodifier of students' first language (L1), with

the purpose of moving them from informal mathematics talk in their first language to formal mathematics talk in the language of instruction. From this perspective, language becomes a thing in itself, bounded and clearly defined as named national languages. Ultimately, students' L1s become strategic resources in nation states' pursuit of national growth and economic development (Petrovic, 2005; Valero, 2017).

This was the backdrop to our concern with a displacement from a focus on distinctions between national languages toward exploring the boundaries of the language-as-resource metaphor. We found that a translanguage perspective on language diversity dissolves dichotomizations between nation languages and allows approaching language diversity in classrooms with no shared languages apart from the language of instruction. Students who have access to plural languages always translanguage, although not always in a hearable way (García & Wei, 2014). The idea that translanguage brings about new language and new knowledge, together with my work on grasping the ideas of inferentialism (Brandom 1994, 2000), moved us towards exploring how and what mathematics is formatted through face-to-face reasoning in language diverse settings. That is, we started tracing an epistemological dimension of multilingualism.

In the article we explored our concern with the epistemological tracing, using theoretical ideas anchored in the GoGAR, spaces of reasons (Brandom, 1994, 2000), and the sensitizing of semantic and epistemological consciousness (Arazim, 2017). We concluded that analysing the plural rationalities embedded in face-to-face reasoning using inferentialism can illuminate the complexity of emerging mathematics based discursive spaces (MBDS). These issues do not belong within the scope of the language-as-resource metaphor, as they are not paths to reduce school achievement gaps or distinguish between languages. Rather, the issue at stake in Article 1 is the shaping of a meta-understanding of language diversity (MULD) that includes an epistemological dimension in which mathematics is not seized as pre-established fixed knowledge.

Summary of Article II: Solidary knowledge (re)production

The aim of Article II was to explore how and what knowledge students produced in a social and language diverse mathematics classroom. The rationale for this was to find out how to grasp connections between students' language use and practices and their knowledge production in face-to-face conversations. We explored the knowledge production the students in the Grade 5 class where I conducted my research engaged in as they reasoned about the student Arvid's statement, "Mathematics is bad for society" as part of a small-scale classroom project on critical mathematical literacy. The MBDSs that emerged in the students' reasoning concerned critical societal aspects of mathematics, intertwined with functional aspects (see, for example, Barwell, 2013; Frankenstein, 1990, 2014; Gutstein, 2006, 2016; Jablonka, 2003; Skovsmose, 2007). We approached the students' knowledge production by analysing their attempts to play the game of giving and asking for reasons well (Brandom, 1994, 2000). To play the game well means to help each other out with discerning what follows implicitly from the statements used (Bransen, 2002), which in turn was a way of unpacking implicit meaning and hence producing knowledge.

My hope was that the project would allow the students to disrupt and trouble stereotyped images about mathematics and society to the benefit of complexity and plurality. I was teaching the lessons during which the small-scale project was conducted. As it turned out, my aspiration to espouse the students' diverse *accents* or *idiolects* and personal ways of knowing (Radford, 2012) was a complex matter, particularly because of my own incapability to perform MULd. With the help of Dakro, a multilingual student, to overbridge my idiolect and Aldrin's (another multilingual student in the class), I was finally able to grasp Aldrin's claims. Hence, the *overbridging act* performed by Darko allowed us to play the GoGAR well.

In line with the ideas of Mercer, Dawes, Wegerif, and Sams (2004) about explorative talks, I invited the students during whole class discussions to question each other's claims to help unpack statements further. A newly-arrived student, Samir, had a claim questioned by another student. His peer Darko quickly gave rea-

sons for Samir's claim, protecting Samir from being exposed to the possibility of not being able to give good reasons for it in Swedish. Hence, Darko performed a *sherpa-like act* of carrying reasons for Samir's claim, which in turn helped the students to play the GoGAR well. The overbridging act and the sherpa-like act that Darko displayed are empirical examples of what I call MULd. To perform these acts seemed to take courage, confidence, and language competence.

In the article, we attended to two aspects of the GoGAR; the content based *what*-dimension and the social- and power-related *how*-dimension. We found that the semantic *whats* were interrelated to the social *hows* in the complexity of in this case, addressing and enhancing critical mathematical literacy in the language diverse Swedish-only mathematics classroom.

Entanglements of Articles I and II: MBDS and MULd

Articles I and II are entangled in the development of conceptualizing students' navigations of language and epistemological divides. As I was conducting the initial analyses of my empirical material, I struggled with developing analytical tools for tracing epistemological dimensions. I was stuck in ideas based on the separation of named languages and (in)formal school mathematics, the very same ideas I was trying to break up with. Alvesson and Sköldbërg (2009) explained that this dilemma can be tackled by creatively engaging in *shutting-out* presuppositions that characterize the research area for the purpose of creatively engaging with oppositional theoretical approaches. Critical mathematical literacy is seldom addressed in research on language diversity, hence it was unusual in that context. The small-scale project offered a disruption to the focus on functional mathematical literacy, which forced me to look for conceptualizations that did not presuppose the separation of different kinds of mathematics.

Analysing how and what knowledge the students produced about the statement "Mathematics is bad for society" suggested that MBDSs are formatted in a complex interrelated entangling of semantic *whats* and social power related *hows*. To navigate this complexity in a social and language diverse classroom, Sherpa-like

acts of articulating reasons for peers' claims or overbridging interlocutors' idiolects were helpful. Using the idea of MULD as an analytical lens helped me to locate Episode 2 in Article I in my empirical material. Using MULD and analytical tools informed by inferentialism, I uncovered the emergence of the translanguual space the student Aldrin was struggling to move me toward or invite me to in the episode. MULD became a useful concept when attempting to move away from dichotomization between languages toward approaches that embrace translanguaging and that allow for the study of translanguaging practices in classrooms with no shared languages apart from the language of instruction.

The analyses in Article II showed that the relationship between the semantic *what*-dimension and the social- and power-related *how*-dimension were interrelated, formatting MBDSs. This suggested that the relation between the meaning given and taken by interlocutors cannot be separated from what they take to be claims of knowledge. Hence, epistemological aspects of what is being said cannot be approached as though the semantic meaning of sayings are already settled. Rather, the two are simultaneously in flux as two sides of the same coin (Brandom, 2013; Derry, 2013). The entanglements of Articles I and II show that students' navigation of language and epistemological divides incorporates both MBDS and MULD, positioned as two sides of the same coin.

Summary of Article III: Epistemological sanctions

In Article III the aim was to elaborate on how the MBDS at play appeared to evoke epistemological sanctions or small aggressions directed at Samir, a newly arrived student. I have described the rationale for this article in the section immediately above this one, namely that navigation of language and epistemological divides incorporated both MULD and MBDS.

As Samir and three of his peers—Greta, Eva, and Darko—entered a group activity on angles, Samir self-confidently said, “How good I am [at mathematics].” However, at the end of the interaction his self-confidence was gone, replaced with insecurity. Submissively he begged his peer Darko to forgive him for his mathematical mistake and to trust him as a knower of mathematics

again. In the article, I used the notions of commitments and entitlements—moves in the GoGAR (Brandom, 1994, 2000)—together with I-language games or talk about oneself (Beristain, 2011) to explore instances where Samir lost his credibility to make mathematical claims of knowledge. For instance, when the girls told the boys that their claims were not mathematically appropriate, Darko repeatedly placed the loss of entitlements on Samir. This exposed Samir to what I referred to as *epistemological sanctions*.

In Article III, I claimed that since the semantic and epistemological are two sides of the same coin (as pointed out in Article I) students who use words in a style that is distant from the normatively appropriate classroom talk are the ones most likely to experience epistemological sanctions. Samir, who was newly arrived in Sweden and an emergent speaker of Swedish, held the position of being the one in least command of the normatively appropriate use of Swedish words. Samir did not reason with the other students as he tried to justify his claims; instead he told them that he was a wrong-doer and a forgetter, talking negatively about himself.

To understand the (lack of) MULD in the students' interaction that potentially could have saved Samir from talking negatively about himself, I had to consider the MBDSs, as I assumed MULD and MBDS to be two sides of the same coin—the epistemological dimension of language diversity.

I found that Samir's negative talk about himself was made possible by the kind of MBDS that the normative appreciation of precision in mathematics calls for. In such a space, claims that are imprecise or ambiguous are questioned and assessed as either right or wrong. In such a MBDS claims are precisely right or precisely wrong. If reasons that are accepted by the interlocutors as precise enough are not offered, a claim made in such MBDSs is at risk of being assessed as precisely wrong.

Being precisely wrong becomes a cognitive *crime* that may elicit an epistemological sanction, with affective consequences (negative talk about oneself) (Rowland, 2000). For emergent speakers of the language at play posing claims in a MBDS that is guided by precision might therefore be risky business.

Summary of Article IV: The place of precision in students' mathematics talk

The aim of this article was to elaborate on readings of the student Greta's ambiguous joke that she delivered during same student interaction analysed from the perspective of Samir's exposure to epistemological sanctions in Article III. The rationale for this was two-fold. On the one hand, it was to deepen the understanding of the MBDS that framed the event of the joke and the epistemological sanctions to which Samir was exposed. On the other hand, it was to grapple with how the students navigated this space.

I do not know Greta's intention with her joke. Perhaps neither does she. She delivered it on the spur of the moment during the mathematical activity on angles, which at the point when the joke was delivered had evolved into a competition in which the winner would be the student who was the most precise. See line 5 in table 4 below.

1	Greta [TO EVA]	Nej inte grader, kolla.	No not degrees, check.
2	Darko	Vad positiv du är Greta.	How positive you are Greta.
3	Greta	Den är 91 grader.	It is 91 degrees.
4	Samir	Vi är klara.	We are done.
5	Greta	Det är inte en rät vinkel... Det är en trubbig vinkel. Det är ingen rät vinkel. Den är 91 [GRADER].	It is not a right angle... It is an obtuse angle. It is not a right angle. It is 91 [DEGREES].
6	Samir	What?	
7	Greta	Jag skojade bara...Alltså om man mäter det exakt så är den...	I was just joking...That is if you measure it exactly it is...
8	Samir	Ja, ska jag lämna...	Yes, I am going to hand over...
9	Darko	[OHÖRBART] för att jag sa...	[UNHEARABLE] because I said...
10	Samir	Ja, ska jag lämna den till dom. Var så god att rätta...vänta ni har fel på [UPPGIFT] A. Ni skrev inte hur mycket grader den är.	Yes, I will hand it over to them [THE GIRLS]. Please mark this...wait you made a mistake in [TASK] A. You did not write how much degrees it is.

Table 4. Greta's joke.

The ambiguity of Greta's joke functioned for me as an invitation to further explore the character of the MBDS that framed the interaction. While humour opens up a space in which norms can be enacted, troubled, and challenged, it makes the very same norms visible (Ackermann, 2015; Chronaki & Matos, 2010; LaFollette & Shanks, 1993). Analysing events of joking can provide insight to how people cope with the norms and discourses in circulation. I assumed that Greta's joking event constituted useful methodological grounds to explore epistemological discourses underpinning school mathematics as well as how those discourses format students' actions as part of their navigation of language and epistemological divides.

The article provides two interpretations of the target of the joke and of why it fell flat. The event itself and therefore the two interpretations pivot around the idea of mathematical precision. In the first interpretation, the norm of mathematical precision was enacted by Greta to position herself as superior to Darco and Samir, who were less precise in their mathematical claims.

In the second interpretation, Greta's joke was a matter of her suddenly perceiving the norm of mathematical precision as absurd in the situation. The joke challenged the norm.

I found that regardless of which interpretation of the joke event one makes, it shows that mathematical discourses are not merely intertwined with other discourses. Rather, epistemological discourses on mathematics appear to—in a complex and ambiguous way—format students' actions. Epistemological discourses on mathematics as precise and certain open up spaces in which the competition among the four students and the joke itself is made possible at all. In a sense, Greta can be perceived as the dominant who is simultaneously dominated by the domination of hegemonic epistemological discourses on school mathematics. This complexity on the one side of the coin—the MBDS side—suggests a complexity on the MULD side too. I elaborate on this complexity below.

Entanglements of Articles III and IV (and I, II): The MBDSs and MULD

As described above, Articles III and IV are connected as they both originate from the assumption that interactions lacking MULD would further inform the concept of MULD. They are also connected because they result from the same student interaction. But how are they conceptually entangled? And how does that entanglement provide insights into how students navigate epistemological divides?

As I moved about the two articles like a city explorer, trekking back and forth between the lack of MULD, Samir's exposure to epistemological sanctions, and the two interpretations of Greta's ambiguous joke, I followed slightly different paths and suddenly found myself entering a *what if* laboratory that Haraway (2016) wrote about (cited in Chronaki, 2018). I started to imagine what-ifs. What if the interpretations of Greta's joke were not two *different* interpretations? What if the joke embodied the two interpretations at once? What if Greta was aware of that? She would then have performed a meta-understanding of the mathematics-based discursive space (MUMBDs) that shaped, for example, the epistemological sanctions imposed on Samir. Could MUMBDs have provided MULD that would avert the sanctions?

These ideas add to the complexity of students' navigation of epistemological divides because they highlight that there are not merely MBDSs, but also the possibility of a meta-understanding of those spaces, which disturbs the picture painted so far. From the perspective of Articles I and II, this implies that studying students' navigation of language and epistemological divides in social and language diverse mathematics classrooms incorporates not only MULD and MBDS but also a possible meta-understanding of MBDSs.

Synthesizing epistemological dimensions: MBDSs and MULD

In this section, I stop to look at the map. What do I see? A coin with a MBDS and a MULD side? From certain perspectives I do. On the one side I see shared translinguaging spaces, overbridging,

and Sherpa-like acts. On the other side are epistemological discourses on mathematics that echo through time and space, emerging and disappearing as the students talk, shaping their interactions, and sometimes producing epistemological sanctions.

My map provides mere fragments of a much larger map. Nevertheless, the three divides from the very beginning of this thesis are present: migration, language, and knowledge-systems. The four articles illuminate dimensions of how students in social and language diverse mathematics classrooms navigate language and epistemological divides. The map does not show how multilingual students might make a leap over language and mathematical divides. Nor is it about deconstructing the divides. Rather, it is about how students grapple with the divides, navigating them, learning to and sometimes failing to meet and learn with the translanguaging (Farsani, 2016; García & Wei, 2014) and mathematics of the Other (Guillemette, 2017; Guillemette & Nicol, 2016; Radford, 2008) in respectful ways.

In a migrating world, this is crucial learning and knowledge for *all* students (and teachers). Migration does not merely play out as a matter of language concerns for some students in the mathematics classroom. It has much more profound implications. It implies a need for cognitive justice, a re-thinking of knowledge where knowing mathematics means connecting language and epistemologically diverse mathematical knowledges in ecologies of knowledges (Gutiérrez, 2017a; Santos, 2007). It suggests the need to consider a meta-understanding of the mathematics based discursive spaces that emerge when students talk about mathematics and how those spaces are connected to meta-understanding of language diversity.

My contribution to the articles

Articles I, II, and IV are co-authored. It is important to make my contribution to those articles explicit. Although the process has been different with each article, as the first author I have been responsible for most of the research work and ideas. My responsibility comprises research design, theory development and application, construction of empirical material, transcription, analyses, and writing. During the processes, I have discussed and evaluated ideas

with my co-authors and supervisors. Ideas for Article III, of which I am the sole author, have been discussed with my supervisors. Below I account for my contribution to Articles I, II, and IV. The articles were written in different contexts of collaboration and as a result my contribution in each article is accounted for in slightly different ways.

- Article I was written by me and Aldo Parra, who at the time was a PhD student. We wrote a co-author statement that explained my contribution: “Ulrika contributed to the definition of the overall problem and provided the backgrounds for the operationalizations and theorizations of the first and second waves of the language-as-resource metaphor. She provided the theoretical approach based on inferentialism and contributed with the empirical material and analysis of episode 2 in the paper. She contributed to the closing remarks and participated in all the preliminary versions of the paper by writing, editing and reviewing it. In total, her contribution to the paper is approximately 70%.”
- Article II was written by me and two of my supervisors, Annica Andersson and Anna Chronaki. I provided research idea and questions, selected the theoretical and analytical tools, conducted the analysis, and wrote the text under the guidance of my co-authors, who also (re)wrote parts of the paper.
- Article IV was written by me and my supervisor Anna Chronaki. I provided a first approximation of the research idea and the empirical material. My co-author and I discussed and developed the theoretical, methodological, and analytical tools and approaches. I wrote the draft for the text, and my co-author contributed by restructuring the text presentation, analysing, and rewriting.

DISCUSSION

This thesis provides theoretical insights illuminated by empirical ones on some relational aspects of students grappling with language and epistemological divides as they talk about mathematics in a social and language diverse classroom from perspectives that do not dichotomize between languages and forms of knowledges. The aim is to re-think dichotomization and explore what understandings such an attempt has to offer mathematics education research. Moschovich (2018b) has called for such a recognition of the plural mathematical discourses that students participate in and bring with them to school (Schütte, 2018).

In the articles I primarily used inferentialism to unveil and explore some epistemological dimensions of multilingualism that emerged in the classroom where I conducted my study. This has implications for my results, as theories are not neutral. Rather, theories value certain constructs that are not objective but social and cultural. They are based on particular assumptions on what is of significance in the world (Popkewitz, 1984). Whether or not background theory is made explicit in research, it always guides how and what kind of objects can and will be studied (Silver & Herbst, 2007) and influences the outcome of the study (Prediger, 2019). This means, for example, that although my results are concerned with social (inter)actions as a matter of how students treat each other's claims epistemologically, I have not attended to affect

or identity work as an outcome of that treatment. Nor have I considered content-based epistemological dimensions of language diversity (Prediger & Krägeloh, 2016; Prediger, Erath, & Opitz, 2019; Prediger & Zindel, 2017; Wessel & Erath, 2018). This thesis does not make an explicit and exhaustive presentation and discussion of *all* aspects of students' navigation of language and epistemological divides. Rather, its contribution is to show that in social and language diverse classrooms, students and teachers as they interact do not merely navigate language diversity—they also navigate epistemological aspects of mathematics. These aspects may be synthesized and approached from notions of meta-understanding of language diversity (MULD) and students' production of mathematics based discursive spaces (MBDSs).

I chose to conduct my study in a multilingual mathematics classroom in Sweden where no languages were shared apart from the language of instruction, which was Swedish. My results should be interpreted in relation to this as well as to the specific sociocultural, socio-political, and historical contexts in which it was possible to conduct the present study. This means that generalisation of the results is not possible because it would not be a straightforward matter to apply them in different contexts. The value of my contribution is that it offers a *particular* re-visit to and re-thinking of epistemological aspects of social and language diverse mathematics classrooms among a plurality of such possible re-thinkings. Studies like this one are important because re-thinking prevailing *truths* offers the opportunity to unveil aspects of migration and mathematics education that otherwise might fly under the radar. This study's tracing of epistemological dimensions of multilingualism contributes knowledge about some interactional aspects of how students navigate those dimensions.

In the sections that follow, I attend to the aim and results of this study and situate them in relation to previous research. I present conclusions and discuss them in relation to mathematics classroom practice in a migrating world. Finally, I suggest implications for further research.

Students' navigation of epistemological divides in a social and language diverse classroom – A displacement from dichotomies towards ecologies

I claim that an epistemological approach to multilingualism that captures a meta-understanding of language diversity connected to the mathematics based discursive spaces that are produced as students discourse about mathematics is an approach that allows a displacement from dichotomization between languages and mathematical knowledge towards an embrace of ecologies of languages and knowledges. I discuss this below by contrasting a dichotomizing approach with an ecology-based one, to illuminate the contribution of this thesis.

To begin, I want to highlight (as mentioned in Article I) that the importance of a dichotomizing approach must not be neglected or underestimated. This approach played and still plays an important role in the struggle to acknowledge multilingual students' languages as resources rather than deficiencies (Chronaki & Planas, 2018; Norén & Andersson, 2016). It is by naming or separating minority languages that they can be recognized and discussed at all. Ahmed (2012) wrote that “to proceed as if categories *do not matter* because *they should not matter* would be to fail to show how the categories continue to ground social existence” (p. 182, italics in original), pointing out that to ignore issues about race is to act as if they were over. In the same vein, minority languages must be named in order to consider how they ground their speakers' social existence.

While named languages are tied to nation states, ethnicity, culture, and social class, naming or categorizing languages has the potential to illuminate injustice, provide resistance, and offer language-as-resource approaches as demonstrated by many researchers within the language-as-resource paradigm (e.g., Norén & Andersson, 2016; Planas & Civil, 2013; Planas & Setati-Phakeng 2014). The question at stake in this thesis is not about dichotomization as such; rather, the question is what another perspective has to offer to multilingual mathematics education and research.

A dichotomizing approach to multilingual mathematics classrooms may offer insights on how multilingual students are disad-

vantaged by the use of hegemonic language practices (Norén, 2015; Norén & Andersson, 2016; Planas & Civil, 2014) or by school language practices (Prediger & Krägeloh, 2016). It finds that students who are encouraged to use both their languages move more easily from informal to formal mathematics talk (Bose & Clarkson, 2016; Setati & Adler, 2000; L. Webb & P. Webb, 2016). It finds dilemmas and tensions that students and teachers need to deal with as they (are made to) chose one language over another (Adler, 2000, 2006; Barwell, 2012). Thus, a dichotomizing approach focuses on the language and not on the agent(s)—the speaker(s) (Creese & Blackledge, 2015; Farsani, 2016; Garcia & Wei, 2014; Moschovich, 2018b).

An ecology-based approach, on the other hand, puts its attention on the co-presence of and fluid interknowledge among languages (translanguaging) (Garcia & Wei, 2014; Otheguy, García, & Reid, 2015) and knowledges (Santos, 2007) in agents' performances. This approach allowed, for instance, Farsani (2016) to see how the co-presence of the Farsi cultural-historical idiom *Far by far, near by near* used for complex fractions and the monolingual English teacher's explanation of equations could co-exist in a multilingual student's solution of a mathematical problem. The one was not considered to be a resource for the other, but rather the student used the inter-knowledge among languages and mathematics to solve the equation easily.

It was an ecology-based approach that helped me to understand that Aldrin—in Article I—moved me into a translanguaging space in which Persian/Farsi was silently co-present in our conversation in Swedish. The overbridging of idiolects and the Sherpa-like actions discussed in Article II are agent-focused actions, which, along with Aldrin's actions, are examples of a meta understanding of language diversity (MULD), a kind of inter-knowledge.

Parra and Trinick (2018), taking a decolonial stance on ethnomathematics and language revitalization, suggested that highlighting language discrepancies (idiolects that do not quite match) has the potential to enhance inter-knowledge among languages and ways of knowing mathematics. This perspective abandons the idea that students' multiple language use functions as a resource to

achieve one unified mathematical goal—to master (Western) mathematics as a problem-solving tool. Parra and Trinick (2018) claimed that to have mastery of such tool includes understanding the tool's histories, cultures, and limitations.

In line with the findings of this thesis, Farsani (2016) and Parra and Trinick (2018) have demonstrated the inseparability of language use and epistemology. This means that considering multilingualism and mathematics education, the complexity in mathematical epistemology cannot be neglected or simplified to dichotomizations between formal and informal school mathematics (Moschkowich, 2018b).

Language responsive approaches to multilingualism (Prediger & Zindel, 2017) account for the inseparability of language use and the epistemological from the perspective of making visible content-specific school academic language demands in order to supply scaffolding for the mastery of prerequisites for discoursing about various mathematical topics in a *school fashion* (Prediger, Clarkson, & Bose, 2016; Prediger, Erath, & Opitz, 2019; Prediger & Krägeloh, 2016; Wessel & Erath, 2018). Erath (2018) studied how students and teachers navigate content based epistemic fields of explaining practices. She expected the teacher to have a particular role in the navigation as the teacher assesses student claims as matching or not matching the teacher's epistemic beliefs. This is of course the general role of a teacher and of teaching. Language responsive approaches aim at scaffolding student talk and conceptual understanding so that it will eventually match teacher (and school) epistemology.

However, in Article I, I demonstrated that abandoning that teacher role gave Aldrin the chance to move his teaching interlocutor (me) into a translanguaging space in which I could finally provide the explanation he needed for solving the task he was working on. To me this suggests two things.

First, being the knowledgeable Other (the teacher) in a multilingual classroom means being humble to the knowledge of one's students and positioning oneself as a learner of their knowledge to be able to perform meta understanding of language diversity. This finding aligns with that of Schütte (2014), who demonstrated that

teachers' competence in flexible interpretations of students claims made in and from various times and spaces is crucial for recognizing differently-framed plurality in student knowledge.

Second, ecological research approaches may capture epistemic assessment of students' (and teachers') claims from a plurality of epistemological rationalities (Knijnik, 2012; Wagner & Lunney Borden, 2015) in order to grapple with the complexity of mathematics classroom discourse in our migrating world.

Moreover, the multiplicity of students' diverse languages and knowledges are beneficial learning resources for all students as they negotiate meaning to reach consensus. Jung and Schütte (2018) and Schütte (2014) wrote about this. My findings align with theirs by showing how students navigate this diversity, but my findings differ in the sense that I do not find the aim of negotiating meaning necessarily to be to reach a consensus. The aim may also be dissensus (de Freitas & Sinclair, 2014), as dissensus has the potential to illuminate different knowledges—which is a prerequisite for fostering interknowledge (Santos, 2007)—and solidary performances that consider plural knowledges simultaneously (Gutiérrez, 2017a), such as performing meta-understandings of language diversity.

Barwell (2012) suggested that paying greater attention to the heteroglossia of human communication in the mathematics classroom will better reflect the lives and experiences of students and teachers. My findings show that the heteroglossia in communication is not merely a matter of reflecting experiences. Rather, paying attention to heteroglossia or people's idiolects is the very pediment for grasping how students grapple with uncertainty when trying to understand each other's mathematical knowledges as they engage in the production of mathematics based discursive spaces (MBDS). Such an endeavour resides in embracing a plurality of mathematics (Knijnik, 2012; Radford, 2012) and the dialogical character of mathematical epistemology (Bakker, Ben-Zvi, & Makar, 2017; Bakker & Derry, 2013; Ernest, 1994; Nilsson, Schindler, & Seidouvy, 2017; Prediger, 2002; Schindler, Hußmann, Nilsson, & Bakker, 2017; Schindler & Seidouvy, 2019; Sfard, 2008).

Barwell (2018a) and Planas (2018) recognized meaning as relational and flexible along with the co-presence of multiple meanings

when they considered multilingualism in mathematics education. They moved away from the representational paradigm (Duval, 2006; Kirsh, 2010) to explore alternative stands. In that respect the findings of this thesis are aligned with theirs. However, Barwell (2018a) and Planas (2018) appeared to adopt dichotomizing approaches, as both of them base their analyses on conceptualizations that contrast (competing) tensions between different kinds of language and knowledge modes. This means that the mathematics based discursive spaces that emerge are analysed along a particular axis of the chosen aspects of language, which of course is helpful when discussing the role of language in multilingual mathematics classrooms (Barwell, 2018a) or the value of languages other than the ones that articulate school mathematics in the language of instruction (Planas, 2018).

The ecology-based approach I embrace in this thesis means to conceive of the languages and knowledges of the Other (Guillemette, 2017; Guillemette & Nicol, 2016; Pais, 2013) not merely as roles or values, but as grounding students' production of mathematics based discursive spaces as a matter of inter-knowledge (Gutiérrez, 2017a, 2017b; Santos, 2007) among languages and plural mathematics.

In a migrating world, highlighting knowing as inter-knowledge is an increasingly pressing issue, as such approaches enable respectful and responsive meetings with the knowledges and languages of the Other. The ultimate result is cognitive justice.

Conversations with...

"So what?!?" my reader professor Núria Planas asked me at my 90% seminar held when I had almost finished this thesis. I am grateful for that question. All educational researchers must ask themselves "so what" do their findings mean to teachers and policymakers, but must they necessarily provide ready-made instructional prescriptions on "what works" to advance meaningful insights?

For quite some time I have been struggling with this question, particularly when I have been involved in professional development projects for math teachers. I keep struggling with it as I write this

thesis. On the one hand, I know that policymakers, headmasters, and teacher are looking for the “what works” solutions and that offering “what works” is helpful to teachers. On the other hand, prescribing ready-made “what works” solutions is founded in an instrumental perception of teaching that risk de-humanizing and de-professionalizing teachers (Tröhler, 2015), which is something I do not wish to contribute to. Hence, I am stuck in a dilemma.

What constitutes good education (at school and in professional development), Biesta (2009) wrote, is not merely a matter of educational efficiency and “what works” solutions. It is a matter of valuing what it works *for*, what education brings about. Biesta (2009) identified three functions that education works for. *Qualification* means providing students with knowledge, skills, and understanding that allow them to *do something*, such as serving the labour market or sustaining national cultural values. *Socialization* has to do with becoming members of particular social, cultural, and political *orders*. It is a matter of the transmission of particular norms and values in relation to the continuation of particular cultural or political traditions, such as the upholding of democracy. Finally, education is about *subjectification*, that is, the shaping of individuals, which may hint at independence from existing social orders. Biesta (2009) argued that subjectification has the emancipatory potential of resistance. In such cases the subjectification function may be in conflict with the two other functions.

This conflict explains my dilemma. My aspiration with this thesis—which is concerned with how students are together with their languages and knowledges in encounters with with school mathematics—is that it may provide some small grounds for reflection on and perhaps even raise some resistance in teachers and policymakers toward the prevailing orders that shape mathematics classroom talk and student subjectification.

With my aspiration in mind, below I will sketch what might be perceived of as a naïve or utopian picture of mathematics classroom talk that does not address “what works.” Instead the purpose is to open up “movements of resistance or counter-conduct towards new not-yet-imagined possibilities of mathematics educa-

tion in relation to policy” (Valero & Knijnik, 2016, p. 6) and thus also in relation to teaching.

My findings show that students’ navigation of language and epistemological divides are matters of dealing with co-presence and establishing interconnections by performing meta understandings of language diversity and/or meta-understandings of mathematical epistemological discourses. Learning how to grapple with such navigation—learning how to meet the knowledge and languages of the Other—is not merely a matter of pedagogical adjustments to meet the needs of multilingual students in a migrating world. It is a matter of learning for *all* students.

Planas (2012) proposed pedagogical organizing principles for multilingual mathematics classrooms in which students and teachers are sensitive and responsive to different ways of expressing mathematical ideas and take responsibility for each other’s learning. A classroom should have established norms that expect and allow students to consider the meaning of unfamiliar words and to ask for reasons and clarifications (Planas, 2014). The findings of this thesis indicate that the establishment of such norms is crucial.

However, my findings also indicate the need to move further. They indicate the need for solidary performances that embrace interknowledge among knowledges (Gutiérrez, 2017a) in multilingual mathematics classrooms. To lay the ground for such performances, classroom norms need to embrace a troubling of Euclidian epistemological transparency (Skovsmose, 2005), which, through the process of pedagogical alchemy, excludes contemporary mathematical epistemology that allows uncertainty (Popkewitz, 2008). Hence, school mathematics is conceived of as residing in Euclidian transparent epistemology. To make such troubling possible, notions of mathematics as merely universal, absolute, and precise need to be challenged in the classroom, as this is the first step toward perceiving plural mathematics. Such perception is necessary to be able to establish interknowledge, because without plural mathematics there will be nothing to interconnect, no ecologies of knowledges (Santos, 2007) to shape.

This is not an easy matter. School mathematics is powerful and repressive, positioning students and teachers in certain ways (Wag-

ner & Herbel-Eisenman, 2009) that shape classroom norms. Nevertheless, practice and policy-making need to embrace epistemological dimensions of multilingualism to enhance *mathematrix* performances (Gutiérrez, 2017a) in the vein of the meta understandings of language diversity (MULD) and meta understanding of mathematics based discursive spaces (MUMBDS) that are discussed in the four articles of this thesis and in ways that are beyond my findings.

Although it is not an easy matter, to embrace the epistemological dimensions of multilingualism is crucial in our migrating world. Some students with migrant backgrounds experience their knowledges and languages as being less or even non-valued at school (Barwell, 2012). This indicates a lack of norms that allow co-presence, which is the very pediment for inter-knowledge among languages and knowledges. As Santos (2007) wrote, migration is not merely a matter of social justice or inclusion, it is a matter of cognitive justice.

Further research

The above raises questions about how performing meta understanding of language diversity and of mathematics based discursive spaces (MULD and MUMBDS) can be fostered in multilingual mathematics classrooms and how policy making can support such fostering to achieve cognitive justice. I agree with Bagga-Gupta and Messina Dahlberg (2018), who wrote that we in the global North have much to learn from the global South in terms of acknowledging and dealing with language and epistemological diversity as mundane everyday performances. Therefore, I suggest research collaborations with the global South that specifically aim to learn how to deal with co-presence. For instance, de-colonising ethnomathematics language revitalizing programs could constitute sources of knowledge for us in the global North to learn from.

In line with Meyer et al. (2016), I also suggest further research be conducted in multilingual mathematics classrooms where non-shared languages and/or blurred translinguaging complexities exist beside the language of instruction. Due to globalisation and migration, such classroom conditions are becoming the norm in the

global North. I suggest that the notion of MULD and an increased focus on students' production of mathematics based discursive spaces, as suggested by Chronaki and Planas (2018), could be a fruitful way of conducting research in such classrooms in order to provide deeper understandings of what it means to learn and teach mathematics in social and language diverse classroom contexts.

Mathematics education research needs to engage in studies that further explore epistemological dimensions of multilingualism. In this thesis I have merely scratched the surface of this field. Rowland (2000, 2014) explored epistemological issues in the mathematics classroom from a linguistic point of view, showing that students use linguistic constructs such as *shielding hedges* to avoid exposure to epistemological sanctions when engaging with school mathematics. Language responsive approaches (Prediger & Krägeloh, 2016; Prediger & Zindler, 2017) have considered multilingualism and how the epistemic role of school language may prevent multilingual students' mathematics learning. By merging for instance the findings of Rowland (2000, 2014) and Prediger and Krägeloh (2016), the important emerging second wave of language-as-(re)source (Barwell, 2018a; Planas, 2018), the findings of this thesis and others like it, important points of departure for further research may be established in order to answer questions about multilingual students' opportunities to learn mathematics as a matter of epistemological concern.

Several possible research areas within the field of epistemological dimensions of multilingualism could emerge. These include: affect and identity work in connection to epistemological sanctions; pedagogical practices that support interknowledge among language and mathematical knowledges; development of theoretical tools to capture ecology-based aspects of epistemological dimensions of multilingualism; and monolingual students possible MULD performances.

I hope that my own and other mathematics education research in the future will engage in understanding epistemological dimensions of multilingualism, as it ultimately is a matter of justice.

SAMMANFATTNING PÅ SVENSKA

Avhandlingen tar sitt avstamp i min läsning av tidigare forskning kring matematikutbildning och migration. Under mitt läsande noterade jag att forskning kring migration och matematikundervisning till största delen begreppsliggjorts som en fråga om flerspråkighet med särskilt fokus på elevers första språk som en resurs i deras matematiklärande (Norén, 2015; Planas & Civil, 2013; Planas & Setati-Phakeng, 2014; Setati & Adler, 2000). Vidare noterade jag att de flesta studier gjorts i kontexter där ett minoritetsspråk ställts mot majoritetsspråket som matematikundervisningen bedrivits på (se till exempel Barwell, 2014; Moschkovich, 2015a; Planas 2014). Åtskilliga studier visade att då elever tillåts använda sitt förstaspråk som resurs exempelvis genom kodväxling eller samtal med en kamrat som talade samma förstaspråk kunde detta språk användas som ett hävstångslikt medel för att flytta elever från informellt matematiktalande på sitt första språk till formellt matematiktalande på undervisnings- och majoritetsspråket (Bose & Clarkson, 2016; Halai & Muzaffar, 2016; Webb, L & Webb, P, 2016). Således fungerar första-språket-som-resurs-strategier väl i matematikklassrum med elevgrupper som delar samma förstaspråk.

Den alltmer globaliserade värld vi lever i har dels inneburit en ökad rörlighet men också att dagens migrationsmönster är betydligt mer diversifierade än tidigare, vilket i sin tur medför att individer kan komma att identifiera sig exempelvis med ett flertal språkliga, kulturella, etniska och socio-ekonomiska identiteter (Meissner & Vertovec, 2015). Vad som är en individs första språk går därför

inte längre att självklart urskilja. Denna migrationskomplexitet avspeglas i svenska matematikklassrum där det mycket väl kan finnas många språk representerade men ingen att dela dem med. Det betyder att elever behöver navigera de språkliga klyftor som kan uppstå i samtal kring matematik i matematikklassrummet. Eleverna bär med sig sina språk in i matematikklassrummet, kanske inte lika uppenbara är de kunskapsbärande relationer som finns i inbäddade i deras språkanvändning som också följer med (Knijnik, 2012). Dessa kunskapsbärande rationaliteten adderar ännu en dimension till samtalet kring matematik. Således betyder det att eleverna även måste navigera de epistemologiska klyftor som kan uppstå då eleverna med olika språk samtalar kring matematik. Därför undersöker jag, utifrån ett relationellt angreppssätt, hur elever navigerar dessa klyftor samt hur deras navigation kan teoretiseras. Jag ställer följande frågor i avhandlingen;

Hur navigerar elever i årskurs fem som går i ett klassrum där nutida migrationskomplexitet finns representerad, de språkliga och epistemologiska gap som uppstår när de talar om matematik?

Vilka teoretiska begreppsliggöranden kan användas för att fånga in epistemologiska dimensioner av elevernas navigation av gapen?

Mot ovanstående bakgrund blir det tydligt att dikotomiska angreppssätt som bygger på tudelning mellan exempelvis första- och andraspråk eller formellt och informellt tal om matematik inte tillförlig kan fånga in den komplexitet som nutida migrationsmönster för med sig. Det är helt enkelt inte en fråga om svart eller vitt och nyanser av grått däremellan det vill säga ett dikotomiskt synsätt, utan det krävs betydligt fler färger för att måla bilden av hur elever navigerar språkliga och epistemologiska klyftor som uppstår i deras matematiksamtal. Därför är avhandlingens syfte att från perspektiv som inte dikotomiserar språk och matematiskt tal försöka begripa elevernas navigerande av språkliga och epistemologiska gap och därigenom försöka undvika att förstärka och återproducera dessa dikotomier.

Avhandlingens fokus är inte i första hand hur elever genom sin navigation lär matematik utan snarare *hur de relaterar till varandra* då de utför sitt navigerande.

Avhandlingen utgörs av en kappa samt fyra artiklar, vilka i det sjätte kapitlet flätas samman och besvarar avhandlingens forskningsfrågor.

För att möjliggöra teoretisk förflyttning från dikotomiska angreppssätt har jag använt en teoretisk verktygslåda som omfamnar en ekologisk syn på språk (Moschkovich, 2018b) i termer om transspråkande (García & Wei, 2014) och på (matematisk) kunskap i termer av 'kunskapsekologier' (Santos, 2007). För att kunna undersöka hur elever hanterar den semantiska och epistemologiska osäkerhet som uppstår då de navigerar språkliga och epistemologiska klyftor och hur denna påverkar deras relationella positioner har jag använt mig av den pragmatiska filosofiska teorin inferentialisms (Brandom, 1994, 2000) kärnidé, nämligen språkspelet *game of giving and asking for reasons* (GoGAR). I GoGAR förs uttalandens semantiska, epistemologiska och relationella positionerande samman, vilket gör att GoGAR lämpar sig väl för att studera hur elever är tillsammans då de navigerar språkliga och epistemologiska klyftor.

Studien är förlagd till en årskurs fem i en kommun som ligger närheten av två större städer i södra Sverige. Skolan ligger i ett medelklassområde, men flera av skolans elever bodde i socioekonomiskt utsatta områden men har på grund av det fria skolvalet valt att gå på den aktuella skolan. Det betyder att i årskursfem klassen fanns såväl elever från medelklassområdena som från socioekonomiskt utsatta områden. I klassrummet fanns nio olika språk representerade (albanska, arabiska, bosniska, hebreiska, kurdiska, norska, persiska, polska och serbiska) utöver undervisningsspråket, men ingen elev delade något språk förutom svenska med en annan elev. Läraren i klassen talade enbart svenska. Det empiriska materialet i avhandlingen utgörs av elevintervjuer, deltagande observationer, ett mini-projekt, fältanteckningar, fotografier samt ljud- och videoupptagningar. Skälet till att använda en blandning av empiriskt material, det vill säga en flexibel forskningsdesign (Robson, 2002) var att jag inte från början visste vilken typ/er av

empiriskt material som skulle vara fruktbart för att besvara mina forskningsfrågor. En flexibel forskningsdesign ställer krav på att forskaren etablerar förtroendefulla relationer med eleverna, vilket jag la stor vikt vid då jag interagerade med dem. Tack vare dessa relationer lyckades jag identifiera så kallade 'rika punkter' (Copland & Creese, 2015) (händelser som på något vis utmärker sig som ovanliga) vilka vidare analyserats i artikel III och IV. I artikel I utgörs det empiriska materialet av en ljudupptagning av ett samtal med en elev som ber om hjälp med en uppgift i matematikboken. I artikel II utgörs det av videoinspelningar som kommer från mini-projektet. Analyserna av det empiriska materialet är gjorda på flera nivåer i enlighet med reflexiv tolkning (Alvesson & Sköldberg, 2009). Detta för att åstadkomma en mångfacetterad bild av epistemologiska dimensioner i elevernas navigerande av språkliga och epistemologiska klyftor och på så sätt kunna erbjuda en mer komplex och djupgående bild som trots det inte gör anspråk på att på något vis vara komplett (Tracy, 2010).

I den första artikeln (Ryan & Parra, 2019) analyseras en första våg av språket-som-resurs-metaforen och en andra våg som är på uppgång identifieras. Vi visar att den första vågen (och möjligen även den andra) av språket-som-resurs-metaforen bygger på ett dikotomiskt angreppssätt som polariserar namngivna (så som nationella) språk och exploaterar elever från minoritetsgruppers första språk, med syfte att tillgodose det nationella kravet på goda matematikkunskaper. Vidare hävdar vi att sociala epistemologiska dimensioner går under språket-som-resurs-radarn. Inferentialism (Brandom, 1994, 2000) kan användas för att fånga sådana dimensioner då ett translanguaging perspektiv läggs på flerspråkiga elevers samtal med personer som enbart talar ett namngivet språk, i detta fallet svenska. Detta exemplifieras genom att vi visar hur en elev, Aldrin flyttar sin samtalspartner från en enspråkig- till en transspråkig samtalsrymd så att en epistemologisk klyfta kan synliggöras och därmed gemensamt behandlas. Aldrins språkhandlingar tyder på en slags meta-förståelse av flerspråkighet.

Den andra artikeln (Ryan, Andersson & Chronaki, (accepterad), som utgår från ett mini-projekt kring kritisk matematisk litteracitet med avstamp i elevcitaten "Matematik är dåligt för samhället". I

projektet diskuterade eleverna i klassen bland annat hur påståendet kan förstås utifrån vad begreppen 'matematik' och 'samhället' kan ha för innebörder i påståendet. I artikeln analyseras vilka innebörder eleverna finner samt hur de i sina resonemang brottas med de språkliga och epistemologiska klyftor som uppstår. Två ytterligare strategier, överbyggande och Sherpa-lik språkhandlingar, identifieras som exempel på meta-förståelse av flerspråkighet. En slutsats i artikeln som elaboreras ytterligare i artikel III och IV är att elevers navigerande av språkliga och epistemologiska klyftor kan förstås som två sidor av ett mynt där den ena sidan utgörs av meta-förståelse av flerspråkighet och den andra sidan utav karaktären hos de matematikbaserade diskursiva rymder som elevernas samtal äger rum i. Matematikbaserade diskursiva rymder formas av de normer kring (skol)matematik som aktualiseras i elevernas samtal. Exempelvis kan normen som säger att tala om matematik kräver att tala på ett precist sätt påverka hur den diskursiva rymd som uppstår då elever samtalar kommer att te sig.

I den tredje (Ryan, 2019) och fjärde artikeln (Ryan & Chronaki, inskickad) analyseras en ljudupptagning av en gruppaktivitet där fyra elever, varav två elever (Greta och Eva) talade enbart svenska, en elev (Darko) är född i Sverige och talade svenska och serbiska och en nyanländ elev (Samir) som var på väg att behärska svenska och som dessutom talade kurdiska, arabiska och hebreiska. Tack vare de relationer jag etablerade med eleverna hade jag innan den aktuella gruppaktiviteten vid ett flertal tillfällen noterat ett särskilt solidariskt och omhändertagande förhållningssätt eleverna emellan. Därför stod den aktuella ljudinspelningen ut som avvikande då flera aggressiva, konkurrensbetonade och nedlåtande uttalanden gjordes vilka slutligen fick Samir att be Darko om förlåtelse och om att åter lita på hans matematiska kunskaper.

Som tidigare nämnts undersöks i artikel III hur Samirs tal om sig själv (Berstein, 2011) i relation till matematikuppgiften eleverna arbetar med förändras från att han inledningsvis talade om sig själv som en matematikkompetent person. Efter att Samir har utsatts för ett antal epistemologiska sanktioner i en matematikbaserad diskursiv rymd som guidas av epistemologiska diskurser kring matematisk precishet förändras hans tal om sig själv. Detta skedde då han

tycktes inse att han i kamraternas ögon inte längre betraktades som en person med förmåga att göra matematiska kunskapsanspråk. En slutsats som dras är att elever som i lägst utsträckning behärskar innebördslig precision i matematiksamtal lider störst risk att utsättas för epistemologiska sanktioner – ett slags micro-aggressioner.

I artikel IV ligger fokus på epistemologiska diskurser kring matematisk precishet eftersom de utgör en matematikbaserad diskursiv rymd som elever förväntas navigera i då precision i matematikanvändning i exempelvis kursplaner och bedömningsmaterial ses som eftersträvansvärt och då matematik som sådan ofta ses som själva definitionen av precision. Ett potentiellt mångbottnat skämt som Greta uttalade under en gruppuppgift, blev ingången till att undersöka hur normer kring matematisk precision smyger sig in i elevers samtal och påverkar hur deras interaktion ramar in. Två olika tolkningar av Gretas skämt görs i artikeln. En där skämtet med hjälp av normer kring matematisk precision placerar Greta i en överlägsen position jämfört med Darko och Samir. I den andra läsningen förstås skämtet som att normen störs då Greta plötsligt inser det absurda i hur precision ramar in situationen hon befinner sig i. En slutsats som dras är att epistemologiska diskurser kring matematik tycks på ett flertydigt sätt forma elevernas interaktion och att det potentiellt är möjligt att Greta genom sitt skämt demonstrerar en meta-förståelse av matematikbaserade diskursiva rymder utifrån ett epistemologiskt perspektiv.

Avhandlingen bidrar teoretiskt med ett uppdaterat perspektiv på samtida forskning kring flerspråkighet i matematikklassrummet. Den för samman inferentialism som ett alternativ till det traditionella språkliga och matematiska representationsparadigmet, social interaktion och pluralistiska idéer om kunskap som ekologier för att fånga förhållandet mellan språk och matematik i elevers matematiksamtal i klassrum med en komplex diversitet av språk och socio-ekonomisk bakgrund. Avhandlingen är den första att använda inferentialism för att förstå flerspråkighet i matematikklassrum. Vidare bidrar avhandlingen med kunskap om hur elever utför språkhandlingar som kan vara såväl solidariska som aggressiva i sitt navigerande av språkliga och epistemologiska klyftor. Med utgångspunkt i elevernas språkhandlingar formas verktyg (meta-

förståelse av flerspråkighet och matematikbaserade diskursiva rymder) som kan användas för att närma sig elevers implicit transspråkiga matematiksamtal i klassrum där elever inte har någon att dela sina språk med förutom svenska.

Avslutningsvis lyfter avhandlingen fram behovet av att förstå flerspråkighet som en fråga för *alla* elever, det vill säga även de elever som enbart talar svenska samt behovet av att koppla samman handlingar grundade i (brist på) meta-förståelse av flerspråkighet som avhängigt den matematiska kunskapssyn som ramar in de matematikbaserade diskursiva rum inom vilka samtalen försiggår. Detta blir särskilt angeläget i en tid då intensifierad globalisering och migration diversifierar sammansättningen av elever i svenska matematikklassrum vilket ger elever rikare möjligheter att möta olika matematiska kunnanden i samtal om matematik. För att kunna handla solidariskt i sådana möten och att undvika att utsetta kamrater för epistemologiska sanktioner behövs meta-förståelse av flerspråkighet och potentiellt även ett ekologiskt förhållningssätt till epistemologiska matematikdiskurser.

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APPENDICES

Appendix 1

Interview guide

three major themes; the students' ideas and experiences of school in general and of mathematics in particular, including what their thought their parents ideas about mathematics was; their hobbies and interests outside school; the students' own mastery of and ideas about languages and their ideas about their peers' mastery of and ideas about languages.

Theme #1; students' ideas and experiences of school in general and of mathematics in particular

- What do you think of school? Is school important? Why? How do you know that?
- What subjects do you (dis)like? Why do you (dis)like those subjects?
- What do you think of math? Do you know why you think that? Can you give any example of when you experienced that feeling? What happened?
- Do you know what your parents think of maths? Why do you think that is their opinion on math?
- Is math important? Why/why not? Can you give me an example of a situation when math is important? Why is it important in this situation?

Theme #2; students' hobbies and outside school time

- Do you have a hobby? Would you like to tell me about it? Why do you do this hobby?
- Would you like to tell me what you do after school on an ordinary day? Where do you go? With whom do you spend time?
- What about homework? Do you have much homework? Where do you do your homework? Do you get some help with your homework?

Theme #3; the students' own mastery of and ideas about languages and their ideas about their peers' mastery of and ideas about languages

- Do you speak any other language than Swedish? What languages? When do you speak that language? With whom? How did you learn that language?
- Do you know if any of your classmates speaks another language? Do you ever hear them doing so at school or anywhere else? Do you hear any student at school, teacher or other people speak another language than Swedish at your school? Does that happen often? If no, what do you think is the reason for that?

- Ulrika shows a picture of a translingual display. Have you seen displays like this at your school? Could you picture your school with displays like this? Would it be a good idea or not? Why/why not? Why do you think they have displays with several languages at this school?
- Is there anything else you would like to tell me or perhaps ask me about?

MALMÖ HÖGSKOLA
LÄRANDE OCH SAMHÄLLE

Appendix 2

Malmö 2017-03-24

Till vårdnadshavare för elever i klass 5x och 6y

Jag heter Ulrika Ryan och har arbetat som lärare i matematik och naturvetenskap framför allt på mellanstadiet under ca 20 års tid. Sedan 2013 undervisar jag på lärarutbildningen i matematik på Malmö högskola. Jag är nu antagen som doktorand i en forskarutbildning på Malmö högskola. Det innebär att jag fått möjligheten att fördjupa kunskaper kring elevers språkanvändning i matematikundervisningen med syfte att stötta elevers lärande i matematik.

För att kunna göra det behöver jag studera elevers samtal under matematiklektioner.

I min forskning kommer jag att studera elevernas samtal i klass xx under matematiklektionerna under några terminer. Jag kommer att observera, föra anteckningar och göra ljud- och videoinspelningar och eventuellt elevintervjuer. Materialet kommer sedan att analyseras av mig tillsammans med mina handledare på Malmö högskola. I forskningsartiklar om undersökningen kommer jag att byta namn på skola, elever och lärare så det inte går att känna igen på vilken skola undersökning gjorts eller vilka elever som deltar.

Eftersom ert barn är under 15 år behöver jag ert (vårdnadshavare) tillstånd att ert barn deltar i studien.

Har du några frågor kan du vända dig till mig ulrika.ryan@mah.se, till min handledare docent Annica Andersson annica.andersson@mah.se eller till rektor vid XXskolan [REKTORS NAMN OCH EMAILADRESS]

Vänliga hälsningar

Ulrika Ryan

Forskarstuderande vid Lärande och samhälle, Malmö högskola

.....

Tillstånd från vårdnadshavare

..... får lov att delta i studien

.....

Underskrift, vårdnadshavare

Namnförtydligande

Appendix 3

MALMÖ HÖGSKOLA
LÄRANDE OCH SAMHÄLLE

مالمو 2017/3/24

إلى أولياء أمور تلاميذ صفي «5x» و«5y»

اسمي أولريكا ريان وأعمل مُدرّسةً في مادتي الرياضيات والعلوم الطبيعية، خاصةً في الصفوف 4 6 منذ 20 عاماً تقريباً. أدّرس منذ عام 2013 في مجال تأهيل المعلمين في مادة الرياضيات في جامعة مالمو. وأنا الآن مقبولة بوصفي طالبة دكتوراه في أحد البرامج البحثية في جامعة مالمو. وهذا يعني أنني حصلت على فرصة للتعمق في المعرفة المتعلقة بلغة التلاميذ المستخدمة أثناء تعليمهم الرياضيات بهدف تقوية تعلم التلاميذ للرياضيات.

أحتاج لدراسة حوارات التلاميذ أثناء حصص الرياضيات ليتسنى لي القيام بالأمر.

وسوف أدرس في أبحاثي حوارات التلاميذ في الصف xx أثناء دروس الرياضيات لبضعة فصول دراسية. وسأقوم بالملاحظة والتدوين وإجراء التسجيلات الصوتية والمرئية وربما أجريت مقابلات مع التلاميذ. سأحلل المادة بعد ذلك برفقة مُشرفي في جامعة مالمو. سأغير اسم المدرسة وأسماء التلاميذ والأساتذة في المقالات البحثية الخاصة بالدراسة بحيث لا يمكن التعرف على المدرسة التي أُجريت عليها الدراسة أو أي التلاميذ شاركوا.

أحتاج إلى موافقة (ولي الأمر) بمشاركة ابنك في الدراسة بما أن عمره أقل من 15 عاماً.

إذا كان لديك أي أسئلة يمكنك الاتصال بي ulrika.ryan@mah.se، أو بمشرفتي أنيكا أنديشون annica.andersson@mah.se (أو بالمديرة X في مدرسة X)

مع أطيب التحيات

أولريكا ريان

طالبة أبحاث في قسم التعلم والمجتمع، جامعة مالمو.

موافقة ولي الأمر

..... يحق له/لها المشاركة في الدراسة

كتابة الاسم بخط واضح

توقيع ولي الأمر

ARTICLES I-IV

I

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Epistemological aspects of multilingualism in mathematics education: an inferentialist approach

Ulrika Ryan & Aldo Parra

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Epistemological aspects of multilingualism in mathematics education: an inferentialist approach

Ulrika Ryan ^a and Aldo Parra ^b

^aDepartment of Science, Environment and Society, Malmö University, Malmö, Sweden; ^bCentro Indígena de Investigaciones Interculturales de Tierradentro, Cauca, Colombia

ABSTRACT

Recently the prevailing language-as-resource metaphor has been problematised and theorised. Using the philosophical theory of inferentialism, we trace an epistemological dimension of multilingualism in mathematics education and add it to the current language-as-resource discussions. With data from two different settings—a mathematics classroom in Sweden and a workshop in an indigenous settlement in Colombia—we show that in encounters between language practices and plural mathematics, the semantic and the epistemological are two sides of the same coin. Inferentialism captures such encounters without dichotomising either languages or mathematics. We contend that epistemological issues move beyond the scope of language-as-resource approaches, but they are not paths to improving school achievement. Neither are they matters of distinguishing between formal and informal language use. Rather, an epistemological dimension is about shaping meta-understandings of language diversity that are liberated from mathematics as fixed and pre-established knowledge.

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multilingualism;
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Introduction

Researchers in mathematics education have developed various approaches to negotiating cultural and language diversity in classrooms. While diversity has existed since the beginning of schooling, research on language diversity in mathematics classrooms began in the 1970s. Multilingualism was approached as a deficiency that hinders student achievement (Phakeng, 2016). The response was to enhance student proficiency in the language of instruction. In the 1980s, framed by globalisation, migration and political rights struggles, a new strategy to “solve the problem”, as stated by Planas and Civil (2013), was to recognise people’s language rights and allow students’ first languages (L1s) to be an additional language of learning and teaching. Discourses on students’ L1s as a resource—the language-as-resource metaphor—in the learning and teaching of mathematics emerged and still underpin much of contemporary mathematics education research on multilingualism (Barwell, 2018; Chronaki & Planas, 2018). For instance, researchers have promoted

CONTACT Ulrika Ryan ulrika.ryan@mau.se

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the pedagogical use of students' first languages in flexible and strategic ways to strengthen communication, foster participation in mathematical activities, and, ultimately, create learning opportunities (Planas & Setati-Phakeng, 2014). The language-as-resource metaphor serves to move beyond the dichotomy of "language as problem vs. language as right" (Ruiz, 1984), which is ultimately a monolingual dichotomy (the use of only L2 or only L1). Although to think of students' first language as a resource instead of a deficit still is a pressing issue in mathematics education (Norén & Andersson, 2016), to embrace language diversity as a pedagogical resource has been widely accepted (Chronaki & Planas, 2018).

A first wave of research in language-as-resource has become a leading trend during the past decade (see, e.g. Barwell, 2009; Bose & Clarkson, 2016; Moschkovich, 2008, 2015; Planas, 2014; Planas & Civil, 2013; Rubinstein-Ávila, Sox, Kaplan, & McGraw, 2015; Setati & Adler, 2000). Recently, researchers in a second wave have problematised (Parra & Trinick, 2018) and theoretically elaborated on (Barwell, 2018; Chronaki & Planas, 2018; Planas, 2018) the language-as-resource metaphor. In this paper, we join those efforts to explore the boundaries of the language-as-resource metaphor by grasping another aspect of the complexity in today's multilingual mathematics classrooms. We argue that approaching multilingualism from a translanguaging perspective raises not only language but also epistemological concerns; "[t]ranslanguaging is the enaction of language practices that use different histories [cultures and mathematics], but that now are experienced against each other in speakers interactions as one *new whole*" (García & Wei, 2014, p. 21, italics in original).

In this paper, we use illustrations from two different episodes—a workshop held during a revitalisation project in Colombian indigenous communities and a Grade 5 mathematics classroom in Sweden—to show how ideas based on the neo-pragmatic theory of inferentialism (Brandom, 1994, 2000) can illuminate epistemological dimensions in current multilingual mathematics educational contexts. We focus on the "construction and use of original and complex interrelated discursive practices" (García & Wei, 2014, p. 22) that emerge in interlocutors' face-to-face translanguaging conversations. Our intention is to trace the existence of an epistemological dimension of multilingualism—an aspect that has not been the central focus in the recent development of the language-as-resource metaphor. To focus on the epistemological dimension, in this paper we look at the ways in which interlocutors (students and teachers) in social- and language-diverse educational contexts use and treat each other's claims as they grapple with conveying meaning and shaping understandings of the world.

We have organised the paper as follows: first, we briefly address the rationale, constructs and operationalisation of the language-as-resource metaphor in what we consider its first wave. Then we review some aspects of three recent papers from the emerging second wave (Barwell, 2018; Chronaki & Planas, 2018; Planas, 2018) that problematise and theoretically elaborate on the language-as-resource metaphor in order to outline the need for an epistemological dimension. We then show how semantic and epistemological dimensions are two sides of the same coin when we abandon the idea of language mirroring the world "as it is" (i.e. a representation paradigm) (Brandom, 2008; Derry, 2013). We show that this insight is theoretically anchored in inferentialism. We then use inferentialism to elaborate on the emergence of an epistemological dimension and its social premises. We conclude the paper by proposing to include the epistemological issues that arise when rationalities of various languages meet as a central part of the

teaching and learning of mathematics. We propose a new focus for mathematics education research on multilingualism that is not achievable under the current framework of the language-as-resource metaphor. We do not adhere to the language-as-resource metaphor; rather, we offer an alternative stance that embraces mathematics classrooms as spaces where different forms of knowledge meet and, consequently, teachers and students learn to manage the plurality of languages and mathematics.

Language diversity and the language-as-resource metaphor in mathematics education research

The idea of language as a resource was introduced by Ruiz (1984), who suggested this metaphor as an alternative stance to predominant ideas about minority language use either as a right or as a problem, when developing a language-planning program in the United States. For minority languages to be useful, they had to be considered beneficial resources. Consequently, the idea of language-as-resource “presupposes that language can be valued as an asset” (Ricento, 2005, p. 361). Petrovic (2005) located language-as-resource approaches as part of the neoliberal agenda, raising the concern that “forces of neoliberalism are far more likely to be successful at manipulating diversity to maximise profit than cultural pluralists will be at manipulating neoliberalism to maximise diversity” (p. 410). Thus the original rationale of language-as-resource was part of a deficitarian approach, where the use of minority languages in the classroom was conceived of as a “temporary vehicle to promote access to full civic participation via the national language ... and not to promote full-blown bilingualism/biliteracy” (Ricento, 2005, p. 361). In the mathematics classroom, this idea could be interpreted as a commodification of multilingual students’ L1s as a means to realise their transition from informal mathematics talk (L1) to formal school mathematics talk in the language of instruction (L2). From a national perspective, students’ L1s become strategic resources in the pursuit of economic growth as they are used in the “construction of techniques to relate individual achievement in math with national and international economic development” (Valero, 2017, p. 2), an initiative manifested in testing programs such as the Programme for International Student Assessment (PISA) and the Trends in International Mathematics and Science Study (TIMSS). The instrumentalisation of language described above downplays other essential aspects such as “psychological, cultural, affiliational, aesthetic and historical [ones], among others” (Ricento, 2005, p. 362). Consequently, to emphasise language as a pedagogical resource in mathematics classroom orchestration (Planas, 2012) fails to consider language diversity in its broader cultural, social, political and historical context.

The first wave of language-as-resource

The language-as-resource metaphor, which was explicitly brought into mathematics education research by Planas and Setati-Phakeng (Barwell, 2016), is generally operationalised by characterising multilingual students’ mathematics learning as a “journey from informal talk in their main language towards formal mathematical talk in [the language of instruction]” (Webb & Webb, 2016, p. 198). This idea underpins major strands in mathematics education research on multilingualism (see, e.g. Bose & Clarkson, 2016; Setati & Adler, 2000). Planas and Civil (2013) claimed that to use students’ L1s as a resource, it is necessary to have flexible views on language use. Researchers have suggested methods such as

code switching, peer talk, re-voicing and the use of synonyms, gestures and objects (Moschkovich, 2008, 2015; Planas, 2014; Planas & Civil, 2013) to support students' participation in learning opportunities. Planas and Setati-Phakeng (2014) asserted that "[t]he extent to which language use facilitates communication and participation ... is the extent to which learning is facilitated" (p. 885). The language-as-resource metaphor is an organising principle for classroom practices with the aim of achieving learning opportunities by integrating the foci on mathematics and language; it deals with tensions between language learning and the learning of mathematics (Barwell, 2012; Planas & Setati-Phakeng, 2014; Prediger & Krägeloh, 2016). Although the language-as-resource metaphor still enmeshes pressing issues (Norén & Andersson, 2016; Planas, 2018), it appears to be operationalised primarily as a tool for providing opportunities for emergent multilingual students to learn school mathematics and/or the language of instruction. We conclude that conceptualisations of the first language-as-resource wave in mathematics education research primarily "focus on the instrumental values of heritage languages while ignoring (or downplaying) the human beings, communities, and socio-political dimensions of language acquisition, use and loss" (Ricento, 2005). Language becomes a thing in itself, bounded and clearly defined as national languages referred to as L1 or L2. Moreover, first wave language-as-resource conceptualisations appear to be based on the idea that language conveys pre-established meanings detached from social, cultural, political and epistemological aspects inherent in language.

An emerging second wave

Recently, some scholars have begun to problematise the language-as-resource metaphor and offer alternative stances. Chronaki and Planas (2018) have challenged the representational paradigm of language. This paradigm starts from the idea that language mirrors the world "as it is" and conveys pre-established meanings. Inherent in the representational paradigm in mathematics education (see, for example Duval, 2006) is a tacit ontological assumption that there is but *one* global mathematics that originates from Western cultural traditions. This assumption implicitly excludes flexible plural views of mathematics and hence prevents students from producing mathematics-based discursive spaces that incorporate plural and diverse mathematics.

In contrast to the representational paradigm, Barwell (2018) and Planas (2018) recognised translanguaging; this specifies language use as socially shaping (stratifying) meaning and recognises that semantic meaning articulates the language use of other speakers.

This discursive dimension of multiple interrelated sources of meaning that multilingual students use when engaging with mathematics tasks can include mathematics as being socially and culturally multiple (Barwell, 2018). However, it is the distinction between formal and informal talk about the *same* mathematics that comes to the forefront. In a similar way, Planas (2018) took into account dialectic tensions between, on the one hand, the *one* global school mathematics and the language of instruction and, on the other, the multi-discoursed languages and mathematics (as a plural noun) of the learners (and their teacher). In small group work, she claimed, students move back and forth in the framework of tensions as they use language as a resource to realise some of the potential mathematical meanings embedded in the immediate situation. Planas (2018) denoted potential mathematical meanings

as the substance of what is taught and ought to be learned in cultures of schooling and especially of school mathematics, recurrently shown and invoked as valued institutionalized ends in curriculum, task design and implementation, evaluation, assessment policies, and practices. (p. 220)

While the second wave of language-as-resource certainly offers a more nuanced approach to language diversity and mathematics than the first wave, we find that it still relies on an enthroned type of mathematics (the school version of the scientific discipline of mathematics) that cannot be questioned nor changed. Researchers continue to frame the school as the space in which students must achieve skills in that type of mathematics. Ideas of people having or lacking, for instance, language or mathematical proficiency are propagated. We therefore fear that the emerging second wave of the language-as-resource approach might become engulfed in the deficitarian paradigm also. In the next section, we consider the epistemological dimension as an alternative that envisions school activities as open spaces for meeting different and contrasting performances, allowing students to widen and interconnect their own mathematical knowledge and come to know those of the others.

An inferentialist view of discourse, epistemology and multilingualism

Breaking free from dichotomisations between different languages and mathematics and also ideas on students' mastery of the two calls for different ways of thinking and reasoning about language and mathematics. The way of reasoning we propose here is based on expressivism. Expressivism uses discursive reasoning practices to make explicit the socially inferential rules in our language. Making implicit rules explicit enables us to reflect on them and change them. Using language and making meaning explicit sensitises our *semantic consciousness* (Arazim, 2017). Expressivism, where inferentialism resides, is not simply about expressing something that is already there. Expressing it also gives it a shape. Although there is freedom in the shaping, there is not anarchy; the freedom is governed by our *semantic consciousness* (Arazim, 2017). The idea of semantic consciousness embraces questions about *what* and *how* mathematics is expressed and shaped in multilingual contexts, and how students and teachers become sensitised to the diversity of language and mathematics. It moves beyond questions about how language is a resource for realising (Western) mathematics.

The above discussion brings us to the connection between the semantic and the epistemological. Before addressing that connection, we want to clarify that in this paper we understand epistemology to be a theory of knowledge that studies how humans and their cultures come to take something to be knowledge and the multiple forms in which they express knowledge. Specifically related to mathematics education, authors like Ernest (1991), Hersh (1997) and Sierpiska and Lerman (1996) have recognised the epistemology of mathematics as a central concern.

The classical idea is to think of epistemological issues as if they have priority over and thus are detached from semantic ones (Derry, 2013). This hierarchy presupposes a representational view of language as having pre-established, fixed meanings (Brandom, 2008). That is, we already know the meaning of the claims we consider, and the question is merely under what circumstances they can be justified. Adopting an expressivist view means that apart from questions about what would and does justify a claim, one

cannot treat the *meaning* of the claim as having already been settled (Brandom, 2008). Therefore, the semantic and the epistemological are inseparable elements continuously in flux (Derry, 2013). This is precisely what is at heart in the semantic, neo-pragmatic theory of inferentialism (Brandom, 1994, 2000).

According to inferentialism, concepts caught up in statements mean something by virtue of their inferential relations to other statements (Bransen, 2002). They are not mental object-like representations of an external reality (Bakhurst, 2011). Inferentialism “thinks of concepts ... as inferential roles” (Brandom, 1994, p. 618) that are conferred with meaning in social practices. To grasp a concept, one must have other concepts, “[f]or the content of each concept is articulated by its inferential relations to other concepts. Concepts then must come in packages” (Brandom, 2000, p. 15). By inferences, we do not primarily mean formal logical inferences, but inferences that are normatively inherent in social languages. For instance, from the statement, “Stockholm is north of Copenhagen” we can infer that Copenhagen is to the south of Stockholm provided the social and cultural norms that regulate the concept’s use allow us to do so (Brandom, 2000).

Since the inferential relations that govern the use of language are normative, they are available for criticism and transformation (Brandom, 2000). It is precisely this transformation of the inferential relations that guides the use of concepts (for example to widen, alter, merge or revitalise conceptual meaning) which characterise translanguaging practices.

Brandom (1994, 2000) like Wittgenstein in his *Philosophical Investigations* (Wittgenstein & Anscombe, 2001) referred to the social practice of language use as language games. Knijnik (2012) used the Wittgensteinian conception of language games to discuss how mathematical language games in rural forms of life in the south of Brazil show resemblance with the globalised mathematics of the school discipline. Knijnik concluded that plural mathematics are constituted by language games whose rules are strongly involved in life culture(s) and they are marked by the rationality(ies) of that(those) form(s) of life, expressed through their own social language(s). As an example, she mentioned a case reported by Knijnik, Wanderer, and Oliveira (2005) in which a peasant explained:

when estimating the total value of what he would spend to purchase inputs for production, he rounded figures “upwards,” ignoring the cents, since he did not want “to be shamed and be short of money when time comes to pay.” However, if the situation involved the sale of the product, the strategy used was precisely the opposite: the rounding was done “downwards,” because “I did not want to fool myself and think that I would have more [money] than I really had” (Knijnik, 2012, p. 92)

From an inferentialist stance, the rationalities Knijnik mentioned are the implicit inferences inherent in social language (sayings) and in social practices (doings). To make implicit rationalities explicit, we can give and ask reasons for our sayings and doings. That is, we play the language *game of giving and asking for reasons* (GoGAR).

The GoGAR is built around two normative statuses—commitments and entitlements—that emerge in a socially articulated structure of authority and responsibility. When claiming that things are such and such one undertakes a commitment to the claim for which one can be held responsible. A commitment entails not only what is explicitly said but also what follows implicitly from it; that is, by making a claim one also commits oneself to

other normative stands (premises and consequences) that follow from the original claim. For claims to have normative status, they must be normatively appropriate in the social practice in which they are caught up. This involves assessment of claims, hence “there must be in play also a notion of entitlement to one’s commitments: the sort of entitlement that is in question when we ask whether someone has good reasons for her commitments” (Brandom, 2000, p. 43). An entitlement is a commitment that has been acknowledged and endorsed. A way of detecting whether a claim has been acknowledged and hence has the social status of an entitlement is to observe whether the interlocutor uses the claim itself and/or what follows implicitly from it in the interlocutor’s own reasoning. As we participate in social practices, we keep track of all this. We ascribe to each other commitments and entitlements to grasp what we and our interlocutors hold as meaning and knowledge. This track-keeping is about grasping the meaning of what is said (and done) in relation to the normative space in which it is uttered. This is how our semantic consciousness is sensitised.

The GoGAR has two dimensions: a semantic and epistemological *what*-dimension and a social *how*-dimension that attends to how we keep track of what other people take to be good and normatively appropriate inferences. Commitments and entitlements make up both dimensions. The *what*-dimension highlights what implicit inferential relations (premises, consequences and incompatibilities) are embedded in a claim; these guide a person’s commitments and entitlements. The *how*-dimension highlights how the status of a claim is positioned and changed (a move in the score-keeping) in the GoGAR and is a social matter of responsibility for providing reasons for claims and the social practice of (dis)acknowledging claims.

In a mathematics classroom (and elsewhere), a claim such as “This is a square” occupies positions in a *space of reasons*. From these positions, knowers of the meaning of the claim can tell what follows from it (e.g. that it is normative and appropriate to say that it has four sides) (Bakhurst, 2011). To be a knower is to navigate ways around normative spaces of reasons, to be aware of what implicitly follows from statements made (Brandom, 1995). Brandom (1995) claimed that

the “space of reasons” ... ought to be understood as an abstraction from concrete practices of giving and asking for reasons. The space of reasons is a normative space. It is articulated by proprieties that govern practices of citing one standing as committing or entitling one to another—that is, as a reason for another. What people actually do is adopt, assess, and attribute such standings. (p. 898)

It is from people’s concrete practices of playing the GoGAR that the discursive space of reasons emerges. In a mathematics classroom, this space is shaped by teachers’ and students’ language and mathematics use and by their reciprocal assessment and track-keeping of each other’s use of language and mathematics.

Epistemological dimensions of language diversity: two illustrative episodes

The episodes we use to illustrate how inferentialism can uncover some aspects of the epistemological dimension of language diversity come from two different and purposefully disparate contexts. Each episode revolved around a particular concept—the first around *unit of measure* and the second around *half as long*. The first episode took place during

workshops held by Aldo within a settlement of the Nasa indigenous people in Colombia. The second episode has two components. The first is an excerpt of a conversation between Ulrika and Aldrin—a Grade 5 student—who is multilingual. The second component is Ulrika's investigation that followed her interaction with Aldrin.

Episode 1

Episode 1 is based on field notes taken by Aldo as a collaborator in an indigenous project (Caicedo et al., 2009, 2012) conducted by the program of intercultural bilingual education of an indigenous organisation¹ in the Cauca state of Colombia. This project had the aim of researching the mathematical knowledge of the indigenous Nasa culture and supporting the revitalisation of the Nasa indigenous language (Nasayuwe). It is important to contextualise the research as an indigenous initiative, structured by Nasa culture, worldview and ways of validating and registering knowledge.² Several meetings were held with community members (elders, healers, teachers, parents and kids) not only to collect data, but mainly to analyse data and validate research results. Following the spirit of solidarity, commonality and collectiveness of the Nasa culture, the process was conducted in several places in an itinerant strategy to cover as many different settlements and territories as possible. Rules for the meeting varied in each settlement according to various types of linguistic proficiency, cultural heritage and political organisation. For instance, in the episode reported here, community members and indigenous researchers spoke mainly in Nasayuwe. Some of the participants were monolingual in Nasayuwe, while others were bilingual in Nasayuwe and Spanish. Aldo is neither indigenous nor a Nasayuwe speaker. During the meeting, indigenous researchers did not find it necessary to make audio-recordings. They only registered the final conclusions and resultant commitments. Therefore, the following history is not a transcription, but a reconstruction from the field notes Aldo took during the meeting.

Vingette based on Aldo's field notes from 2008

In the middle of a workshop organized within an Indigenous Nasa settlement, we were trying to find or create a word in Nasayuwe that can be used as translation of “unit of measure”. I mentioned properties about that concept and provided some examples. In response, a team of Indigenous researchers (all of them were bilingual) explored and discussed possible words in their own language. When they reached a consensus on one term, I asked them if the meaning of that selected word included the property of “unit of measure” of not necessarily being an object of reference, but a magnitude of reference³, and they realised that the word did not have such connotations. Therefore, another round of discussions in Nasayuwe took place and they changed the word for another called *kxteecxah*. One of the researchers explained to me that both words originally meant other things (it was the expression for “a bunch”), but they estimated that with the last word they could explain better to other Nasa the meaning of “unit of measure”. I realised then, that they had achieved not an equivalent word, but a suitable point of departure to re-create the discussion in the future with other Indigenous people, grasping the concept in their own ways.

This episode shows that to find a word in Nasa that matches the concept of interest—*unit of measure*—was far from a mere matter of translation. To find a useful Nasa word several things were done. First, the concept *unit of measure* was placed in a space of

reasons; that is, its inferential relations to other concepts were articulated. Aldo did that when he mentioned the properties of the concept and gave examples of units of time and weight. In doing so he conferred meaning to *unit of measure* by placing it in relation to the other concepts (unit is not an object, but a magnitude) he used when talking. Aldo made the inferential relations between *unit of measure* and other concepts explicit in his talk; he was engaged in playing the GoGAR. Thereafter, the indigenous researchers placed possible words in Nasa in the space of reasons to articulate their inferential relations to other concepts in Nasa. To find a suitable Nasa word entailed matching enough crucial (for the meaning of *unit of measure*) inferential relations to other Nasa concepts. When Aldo asked if another property “was included” in the Nasa word (the one that differentiates concrete objects from the quantity of magnitude that the objects have), he made explicit yet more inferential relations between the concept *unit of measure* and other concepts. The indigenous researchers concluded that a Nasa concept, *kxteeçxah*, different than the one they first had in mind, better resonated with the inferential relations of other Nasa concepts that would underpin the meaning of the concept *unit of measure* in Nasa, according to the Nasa worldview. Thus the indigenous researchers expanded the meaning of *kxteeçxah* to include meanings matching *unit of measure* to be used in contexts where a concept like *unit of measure* is needed. For the expanded meaning of *kxteeçxah* to be used as an entitled claim to knowledge by other Nasa people, new GoGARs will be played, for instance at indigenous schools. The indigenous researchers foresaw the need for new future GoGARs when they explained their refusal to use translations and borrowings in describing the meaning of *kxteeçxah*. The indigenous researchers were confident in the capacity of Nasayuwe and the Nasa worldview to express the nuances and particular characteristics of notion of a *unit of measure*. Episode 1 illuminates epistemological aspects of language use and sheds light on issues of switching between languages not merely as a matter of translation or code switching but also as a matter of discerning rationalities and worldviews inherent in language use.

Episode 2

Episode 2 starts with an audio recording of a conversation between Ulrika and a student, Aldrin, when Ulrika acted as a participant observer during a mathematics lesson in a Grade 5 (students aged 11) classroom in Sweden.⁴ The Grade 5 classroom was social and language diverse. About a third of the students claimed various degrees of speaking or having access to languages other than Swedish. Aldrin was a second-generation immigrant who spoke Persian and Swedish at home. In an interview made prior to Episode 2, Aldrin had shared that his mother was interested in mathematics and that she provided him with additional mathematics tasks to discuss and work on at home. Thus he had experience of elaborating on mathematical concepts both in Persian and in Swedish. Ulrika’s first language is Swedish. She speaks no Persian. Aldrin was working on a textbook task dealing with the length of the sides of a parallelogram. Aldrin asked Ulrika for help to clarify the task. To Ulrika, he appeared to struggle with grasping the concept *half as long*.

Part 1: the exchange between Aldrin and Ulrika

Aldrin appeared to find no way to relate inferentially the concept *half as long* to the task of finding the length of the sides of the parallelogram (Table 1). To solve the task, he had to

Table 1. Excerpt from Aldrin and Ulrika's conversation.

Turn	Speaker	Original Swedish	English translation
1	Aldrin	Två sidor är ... Två sidor ... 12.8 och sen de två övriga är hälften så långa [LÄSER HÖGT UR MATEMETIKBOKEN]	Two sides are ... two sides ... 12.8 and then the two others are half as long [READING ALOUD FROM THE TEXTBOOK]
2	Ulrika	Mmm ... är dom längre eller kortare?	Mmm ... are they longer or shorter
3	Aldrin	Längre!	Longer!
4	Ulrika	Om de är hälften så långa är de längre då?	If they are half as long are they longer then
5	Aldrin	Kortare ... nå då är de kortare	Shorter ... no then they are shorter
6	Ulrika	Ja ...	Yes ...
7	Aldrin	Men varför står det långa ... då är det väl ... är det inte 12	But why does it say long ... then it is not ... is it not 12
8	Ulrika	Vad sa du, varför står det långa är det det långa du tänker på då	What did you say, why does it say long is it that long that you are thinking about
9	Aldrin	Ja typ hälften så <i>lång</i> a	Yes kind of half as <i>long</i>

Note: In line with Andersson & le Roux (2017)'s, suggestions about ethical research writing about the Other, we account for the students' talk in Swedish with an English translation provided by Ulrika. Although Ulrika experienced doing the translations as a relatively straightforward matter, she is aware that there are nuances of the languages that may influence her own as well as the readers' focus on and interpretation of the excerpts.

infer that *half as long* is a matter of relating two lengths (the sides of the parallelograms) to each other in a 1:2 relation. In Turns 2 and 4, when trying to help Aldrin, Ulrika focused on the quantitative part (half) of the expression *half as long*, not on the mode or property to which the quantity is connected. In Turn 2, she asked if the sides that are *half as long* are (quantitatively) longer or shorter than the other two sides. In Turn 4, she again asked if the sides that are *half as long* are *longer* than the other sides—if they have a greater quantitative magnitude. She seemed to take for granted that the concept *lång*a (long) is unproblematic and that the concept use of *long* is fixed, that it mirrors one particular school mathematics *long*. She acted as though Aldrin (or anyone else) could not inferentially relate *lång*a (long) to other concepts differently than she did. Hence, she ascribed to Aldrin a commitment that excluded the possibility of other inferences than the ones she made herself. This means that in the mathematics-based discursive space that emerged through Ulrika and Aldrin's conversation, Ulrika's normative authority use of *long* did not allow the conceptual flexibility needed for her to acknowledge Aldrin's questioning of the use of *half as long* that he seemed to find normatively inappropriate. She did not entitle him to the claim that troubled the use of *long* because she did not use it in her own reasoning. Fortunately, Aldrin was persistent in his questioning of the use of *half as long*. He started to play the GoGAR. In Turn 7, he said "but why does it say long"; he was now explicitly asking Ulrika for reasons for the textbook's and Ulrika's use of *lång*a (long) by not acknowledging an entitlement to the use of *long* in her previous claims.

Simultaneously Aldrin ascribed to Ulrika a commitment that included the possibility of other inferences than the ones he made, expecting her to provide an answer. Aldrin in his track-keeping detected discrepancies between the inferences he was making from *half as long* and the ones the textbook and Ulrika made. His track-keeping of commitments and entitlements and detection of discrepancies opened up a space in which the use of *half as long* needed be troubled and critiqued in order to generate new knowledge and new languaging (García & Wei, 2014). In Turn 8, this appeared to allow Ulrika to grasp that it was the use of *long*—and not *half*—that Aldrin found troubling. She saw that it was possible to make inferential relations between *half as long* and other claims in ways that convey meanings different from her own. By clarifying this, Aldrin invited Ulrika to share the

space with him. In Turn 8, Ulrika seemed to begin to abandon her perception of *long* as having no other inferential relations than the ones she was making. She asked Aldrin to confirm this which entitled him to articulate that it was the use of *long*, not *half* that he questioned. In Turn 9, Aldrin did so. At this point, the mathematics-based discursive space changed. It now encompassed a recognition of the conceptual flexibility needed for a reciprocal acknowledgement of Aldrin's questioning of the use of *half as long* and for the assessment and unpacking of the textbook's claim: "Två sidor är 12.8 centimeter. De två övriga är hälften så långa" (Two sides are 12.8 cm. The two others are half as long). In this space, as Aldrin's questioning became entitled it was possible for him to form mathematical beliefs about the sides of the parallelogram, which is precisely what happened as the conversation between the two of them carried on. This shows that the epistemological dimension of Aldrin's assessment of the textbook claims cannot be separated from the meaning of the concepts at use (Brandom, 2008) in the conversation between Aldrin and Ulrika. Therefore, there is an epistemological dimension in translanguaging spaces. For the mathematics-based discursive space to emerge between Aldrin and Ulrika, Aldrin needed the confidence to question not only the textbook's use of *half as long* but also Ulrika's use of the term. That is, he had to talk back to the stratifying effects of language use (Barwell, 2018) and question Ulrika's authority as a user of *half as long*. As a result of him being confident enough to do so, Ulrika became sensitised to the possibility of not merely troubling the use of *half* but also of *long*, which was a premise for Aldrin's formation of mathematical beliefs about the parallelogram. Even though Ulrika is ignorant of Persian, Aldrin's troubling of the concept moved her into a translanguaging space where they could approach the dynamic and plural language rationalities embedded in their conversation. Using inferentialism, we can unveil the move into this translanguaging space.

Before we move the analysis further, we present the second part of this episode—Ulrika's investigation following her interaction with Aldrin—in order to trace some of the inferential relations embedded in the situation.

Part 2: Ulrika's investigation

When she was transcribing the audio recording, Ulrika noticed that Aldrin seemed to find it contradictory that something is *half* and *long* at the same time. Ulrika put the Swedish *hälften så lång* (half as long) into a web-based translation tool and asked for the Persian translation. She then had the Persian translation translated back to Swedish. Now the translation read *halv lång* (half long) instead of *hälften så lång* (half as long). From the Swedish concept *halvlång* (half long), it can be inferred that something is semi-long. Saying that something is semi-long is making a claim that does not put the object that is semi-long in relation to another object; it is not used to make a particular relative relation between two objects explicit. Ulrika asked a colleague who is fluent in Farsi⁵ and English for help. She explained that in Farsi the word *boland* is used for long to talk about a long book, a long time, a long road and so forth. The word *boland* has its etymological roots in old Farsi language(s). The word for measuring distance or length in Farsi is *tool*. In Swedish (as well as in English), the words *lång* (long) and *längd* (length) are phonologically related, sharing the same etymological roots, but that is not the case in Farsi/Persian, where *tool* is an Arabic loan word. Taking into account the differences and relations between *boland* and *tool* in Persian/Farsi mean mastering what can

and cannot be inferentially derived from each of the concepts. Premises and consequences that follow from using *tool* have to do with distance and length, which is not the case for *boland*. When Aldrin questioned the use of *long* in *half as long* he initiated a GoGAR that would have to make explicit the inferential relations between the concepts *length* and *half as long*. The GoGAR would have to support him in placing the two concepts inferentially related to each other differently from how *tool* and *boland* are inferentially related in order for him to grasp the text book use of *half as long*. Ulrika's investigation revealed some of the rationalities implicitly embedded in Aldrin's translanguaging. As Ulrika is ignorant of Persian, she could not play a GoGAR to make this explicit. However, she could play the GoGAR in a way that was sensitive to the language and epistemological rationalities (Knijnik, 2012) inherent in the languages and mathematics at play in the classroom, recognising that she did not yet understand these rationalities.

These two episodes illustrate in their own ways that inferentialism is an approach to multilingualism and plural mathematics that deviates from the language-as-resource metaphor. Our use of inferentialism does not reside in the existence of separate languages or between different mathematics. Rather, an inferentialist approach on translanguaging multilingualism allows elaborations on how people deal with the uncertainties of being understood and of understanding the other as a knower of plural mathematics, by keeping track of commitments and entitlements that endorse inferential relations between concepts. In the episodes, the uncertainties were about how the words *kxteeçxah* and *hälften så lång* could be used to make mathematical claims to knowledge in Nasa communities and in the Swedish mathematics classroom. In both cases, by using concepts as socially articulated inferential relations, the GoGAR and the normative statuses built around them (used particularly in Episode 2), we have shown that it is not merely a matter of translation or code switching but that there are also epistemological aspects to these uncertainties. Both episodes illustrate translanguaging in which language practices from different cultures and mathematics meet. Inferentialism can capture both the semantic/epistemological and the social dimensions of the discursive mathematics-based spaces of reasons that continuously evolve in such encounters.

Closing remarks

In this paper, we began by discussing the first wave of the language-as-resource metaphor in multilingualism in mathematics education to show the extent to which it still belongs to a deficitarian paradigm, where students' first language is commodified for the needs of the national state (Petrovic, 2005; Ricento, 2005). We then highlighted an emerging second wave of language-as-resource research by highlighting three papers that made efforts to move away from an instrumental view on language use with the purpose of questioning language as representational (Chronaki & Planas, 2018) and theorising both the production of meaning (Planas, 2018) and the sources of meaning (Barwell, 2018) in multilingual mathematics classrooms. We have proposed a shift in research focus from approaches that are concerned with how students use their L1s and informal mathematics to master formal mathematics in L2s towards approaches that embrace translanguaging (García & Wei, 2014) and can inform complexities. We suggested that inferentialism is a helpful approach to translanguaging and the continuously evolving new knowledge and languaging it connotes.

We have assumed that the GoGAR is a useful tool—as far as it provides flexible conceptions of concepts—that allows us to analyse the troubling, questioning and expansion of meaning in people's conversations. At the heart of these conversations are epistemological questions about how a plurality of heterogeneous knowledge meets and how people shape interconnections between them rather than abandoning or capitalising on some of their knowledge (Santos, 2007). The epistemological dimension allows us to study the knowledge that is generated by classroom activities, how articulation of concepts can be developed through the GoGAR, and how students place concepts in the space of reasons.

We used inferentialism in two multilingual translanguaging episodes to illuminate and answer our research question, which was to trace the existence of an epistemological dimension. The report and analysis of data uncovered the dynamic and plural rationalities embedded in face-to-face conversations and illuminated the complexity of emerging mathematics-based discursive spaces. We contend that these issues do not belong within the scope of a language-as-resource approach, in so far as they are not paths to reduce any gap in school achievement. Neither is it a matter of distinguishing between formal and informal language use. Rather, the issues are about shaping a meta-understanding of language diversity that includes epistemological dimensions in which teachers and students do not conceive of mathematics as a fixed and pre-established field of knowledge. Therefore, this emphasis on epistemology differentiates our paper from the current and recent critiques and developments of the language-as-resource approach (Barwell, 2018; Planas, 2018).

With this paper, we contribute to the research and practice of mathematics education in the contexts of language and mathematics diversity as we join efforts to problematise and elaborate on the currently trending approach—the metaphor of language-as-resource. Part of our contribution is our claim that teaching can benefit from being more sensitive to episodes when students provide reasons for concepts or perceive some sayings as contradictory. Such instances provide grounds for reflection about how students and teachers can use concepts and claims as premises and consequences in their reasoning. Within the specificity of multilingual classrooms, we have demonstrated a way for teachers and students to take advantage of episodes of mathematical and communicational uncertainty.

Notes

1. The organisation is called the Regional Indigenous Council of the Cauca (CRIC).
2. This Nasa experience resonates with the attempts of decolonising methodologies proposed by Smith (2013).
3. The same unit of measure can be present in different objects, for instance a stick or a cord that is 1 m long. You can measure with those objects, but they are not necessarily the unit of measure. The unit of measure is the quantity of magnitude present in those objects, e.g. 1 m.
4. For ethical reasons, school location is not mentioned and the student's name is a pseudonym. Swedish national guidelines for research in the social sciences have been followed (Vetenskapsrådet, 2017).
5. Aldrin named his language Persian while Ulrika's colleague said she speaks Farsi. To acknowledge their language naming both names are used.

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ORCID

Ulrika Ryan <http://orcid.org/0000-0002-9610-3682>

Aldo Parra <http://orcid.org/0000-0001-9503-0648>

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II

“Mathematics is bad for society”: *Reasoning about mathematics as part of society in a language diverse middle school classroom*

Ulrika Ryan Annica Andersson Anna Chronaki

Introduction

“Mathematics is bad for society”, said Arvid a Grade five (11 years old) student with Albanian background in a somewhat challenging tone, when we were awaiting the teacher for the math-class to start. I, Ulrika, did not have the opportunity at that moment to reply or ask him why he claimed mathematics to be bad for society. It was one of the first days that I met the class as a participant-observer. I had introduced myself as a researcher interested in reasoning in math class but not mentioned societal aspects of mathematics. Arvid’s claim got stuck in my mind. I become curious about why he made the statement. I found it surprising for such a young student to say that “mathematics is bad for society”, or even to place mathematics connected to societal matters. In my mind Arvid’s claim raised questions about middle school students’ ideas on mathematics and society.

Vignette, based on Ulrika’s field notes, 20170508

To the first author, Arvid’s statement brought forward the idea of conducting a small-scale project to explore the theme “Mathematics is bad for society” in his class. Arvid’s statement seems to exemplify concerns in critical mathematics education on how societal issues of mathematics can be addressed at school, a task that remains a challenge for mathematics educators.

Critical mathematical literacy is shaped through “...constructing knowledge of particular concepts, ideas, skills and facts [...] to help [students] recognize oppressive aspects of society” (Gutstein, 2006, p. 6). In this line of pedagogical interventions, students often engage in discussing social issues, that their teachers formulated as mathematical problems (see for example Andersson, 2011; Barwell, 2013; Frankenstein, 1990; Gutstein, 2006, 2016). Teachers assist students in translating ‘real’ world problems into mathematics, which in a sense consists a challenge as argued by Gutstein (2016). Drawing on the ideas of Paolo Freire, he argued that critical mathematical literacy contains knowledge of three types: classical academic, communal and critical. However, Gutstein (2016) identified challenges in interconnecting the different knowledge bases and asked: “[H]ow does one connect and synthesize all three knowledge bases...?” (p. 458). A pedagogical ‘dance’ that means moving between the three knowledge bases “...signifies the braided interconnections that people make between mathematics and socio-political reality” (p. 469) was suggested for formatting interconnections. We find that classical academic knowledge about mathematics, communal knowledge about society and knowledge of mathematics in society are embedded in the statement “Mathematics is bad for society”. Moreover, the word *bad* opens up a value-laden space that allows for critique.

Our endeavour in this chapter is to explore how the students in Arvid's classroom struggle to make value-laden critical interconnections between mathematics and society. The classroom can be characterised as language diverse, yet the only language heard in the classroom was Swedish. A third of the students belonged to a wide diversity of mother-tongue backgrounds. As such, the small-scale project could help to explore deeper how students from diverse language backgrounds engage in making inferences and interconnections when they reason about "Mathematics is bad for society", as those inferences allow for unpacking implicit meanings in statements (Brandom, 1994, 2000).

As we analysed the outcome of this small-scale project our interest was twofold; it was directed towards grappling with *what* knowledge the students' used, produced and inferentially interconnected in their claims, to articulate meanings of Arvid's statement and *how* that knowledge was expressed and (dis)acknowledged¹ in the language diverse, but Swedish-only (Norén & Andersson, 2016) classroom. Our interest meant that we anticipated that to achieve an understanding of knowledge, attention to its dialogical articulation is vital (Derry, 2013). Theoretically, we anchored our analysis in the language *game giving and asking for reasons* (GoGAR) (Brandom, 1994, 2000) while it comprises the two dimensions (what and how) as elements in flux in face-to-face reasoning.

The research questions explored in this chapter are:

- *What* knowledge did the students use, produce and interconnect as they reasoned about the statement "Mathematics is bad for society"?
- *How* did the students express and (dis)acknowledge knowledge when they reasoned about mathematics in society in a language diverse Swedish-only classroom?

The three knowledge bases that Gutstein (2016) referred to contains aspects of functional mathematics as well as critical knowledge about mathematics and society. Therefore, we start with some brief comments on mathematical literacy and socio-political aspects of mathematics in society. Then we give a theoretical account for the GoGAR, which we use to analyse what inferential interconnections between knowledge claims about mathematics and society the students in Arvid's classroom made and how they expressed and treated those claims as they reasoned about the statement "Mathematics is bad for society". Thereafter we describe the context and methodology of the present study, followed by the analysis of specific episodes and student claims. The chapter ends with some concluding remarks, which address the complexity of students' reasoning about mathematics in society in a language diverse classroom.

Mathematics and society

Functional mathematical literacy refers to the capacity of creating and applying mathematical knowledge when required (Jablonka, 2003). Such conceptions of mathematical literacy are closely linked to the social and economic needs of the market, as well as the individual's participation in the advanced technologized democratic society (Jablonka, 2003; Skovsmose, 2007). At the extreme end of functional mathematical literacy, mathematics is value-free and

¹ Although to some extent coinciding, in this paper we use the word 'acknowledge' less as defined in inferentialism and more as its colloquial meaning.

merely technically related to societal demands of employing mathematics (Jablonka, 2003). However, articulated in Arvid's claim "Mathematics is bad for society", mathematics is connected to society in a value-laden sense.

Critical mathematical literacy recognizes mathematics as a distributor of power in society. Mathematics acts and can be made to act (Skovsmose, 2007, Chronaki, 2010). Kolloosche (2014) claimed mathematics to be "...a body of knowledge and techniques which has served the interests of power from its very beginnings" (p. 1062) with the purpose to shape citizen behaviour. Being critically mathematical literate means for instance to hold the capacity of understanding and shaping the world using mathematics to make it socially just (Frankenstein, 1990, 2014; Gutstein, 2006, 2016), environmentally sustainable (Barwell, 2013; Hauge & Barwell, 2017), or to critically understand the political role of mathematics in the world (Andersson & Wagner, 2017; Gellert & Jablonka, 2009; Kolloosche, 2014; Skovsmose, 2007).

According to Skovsmose (2007) the distribution of power through the use of mathematics that "...refers to processes of construction, operating, consuming, marginalising which [can] be addressed functionally or critically" (p. 17). Processes of construction apply to advanced systems of knowledge and techniques used by for example engineers, economists, and computer scientists. Operating processes conducted by operators bring constructors advanced use of mathematics into operation. Operators are not necessarily aware of the specific mathematics behind their performances. Citizens consume the mathematical objects, developed by constructors, which are fed mathematical information by operators. In high technological societies, mathematics has become a necessity because "[i]f the citizens were not able to read information put into numbers, the society would not be able to operate." (Skovsmose, 2007, p. 14). However, technology black-boxes mathematics. The more technological artefacts societal life demands, the more opaque and thus powerful constructors' mathematics behind them becomes (Gellert & Jablonka, 2009). Hence, for citizens mathematics in society is double edged. On the one hand, it is a necessity-demand on the other mathematics is concealed by technology.

Taken the above into account, it is reasonable to problematize how students develop critical mathematical literacies as a matter of them interconnecting knowledge of different kinds (Gutstein, 2016) as they reason about both mathematics per se and how mathematics can be used. Although the perspective of critical mathematical education has much to offer in terms of reconsidering curriculum design for critical mathematical literacy (e.g., Frankenstein, 1983; Gutstein, 2006; Skovsmose, 1994; Skovsmose & Borba, 2004) particularly highlighting pedagogical moves for formatting interconnections between knowledge bases (Gutstein, 2016) focus here will not primarily be placed on pedagogical issues. Nor is our focus with understanding the kinds for example, academic and communal (Gutstein, 2016) or funds (González, Andrade, Civil & Moll, 2001) of knowledge students might draw on when reasoning. Rather our focus is with the students' articulation of the interconnections themselves and how those suggested interconnections are treated in their reasoning, while they allow us to think about how the students in Arvid's class place mathematics and society as critically connected. To grapple with the complexity in the students reasoning, we use the *language game of giving and asking for reasons* (GoGAR) which is at heart if the philosophical theory

inferentialism (Brandom, 1994, 2000). Drawing on Wittgensteinian ideas, inferentialism approaches meaning and knowledge from a social, pragmatic perspective emphasising that meaning must be located in real-life practices of language use, so-called *language games*. While inferentialism attends to how people make, treat and interconnect each other's claims in face-to-face conversations it allows us to think about social and epistemological complexities and dynamics in students' reasoning in a language diverse classroom. In the next section, we discuss some aspects of inferentialism in relation to the small-scale project on Arvid's statement "Mathematics is bad for society" further.

Reasoning as a language game

To grapple with how students in a language diverse yet Swedish-only classroom reasoned to interconnect claims to unpack the meaning of the statement "Mathematics is bad for society" we argue that inferentialism is fruitful while i) it sees reasoning as a language game in which implicit interconnections between knowledge claims can be articulated and made explicit; ii) it argues that such articulation is first and foremost dynamic and dialogical not formally logical and that iii) norms regulate how interconnections are made, treated and used. Conceptualizing reasoning as playing the *game of giving and asking for reasons* (GoGAR) allows for a type of analysis that is not concerned with efficiency or correctness in students' reasoning. Analysis is not geared towards eliciting individual knowledge from social reasoning or to model individual mental representations, rather it is concerned with reasoning itself. Inferentialism asks for inferences and their origins. "An inferentialist epistemology does not prioritize students' mathematical reasoning over out-of-school reasoning; nor does it view mathematics and out-of-school reasoning as separated from one another, but rather as inferentially connected to one another." (Schindler, Hußmann, Nilsson, & Bakker, 2017, p.7). By inferences, we do not mean formal logical ones, but inferences that are inherent in language itself. For instance, from the statement "Stockholm is north of Copenhagen" it can be inferred that Copenhagen is to the south of Stockholm provided the social and cultural norms that regulate the concept's use allow that. Alternatively, from a person saying, "I am hungry" it can be inferred that s/he wants something to eat. Hence, being hungry and eating as well as north and south are inferentially related to each other under particular socially and culturally normative circumstances that are inherent in the concepts use.

The scoop of the GoGAR is not to produce "...a canon or standard of right reasoning. Rather, it can help us make explicit (and hence available for criticism and transformation) the inferential commitments that govern the use of all our vocabulary, and hence articulate the contents of all our concepts." (Brandom, 2000, p. 30). That is, when we ask and give each other reasons for our claims we unpack the content of our concepts and open up a space in which concept use can be troubled, critiqued and altered. The theory of inferentialism does not argue that the content of concepts primarily represent some object or state of affair, rather concepts caught up in statements that mean something in virtue of their inferential interconnections to other statements (Bransen, 2002). Hence, Arvid's statement "Mathematics is bad for society" means something in the students' reasoning due to the inferential interconnections they make to other statements.

When Arvid and his peers asked for and gave reasons for Arvid's statement (i.e. played the GoGAR) they placed the statement in a network of inferential interconnections to other statements. Inferential interconnections are usually implicit, but as the GoGAR is played they are made explicit. Premises, consequences and incompatibilities that follow from uttering a statement or making a claim comes to the fore. For instance, a premise for claiming "Mathematics is bad for society" is that mathematics operates in society and that it is not value-free. A consequence is that mathematics may cause some kind of harm in society. Mathematics as disconnected from society is incompatible with the claim. This dimension of the GoGAR concerns the *what*-questions of our endeavour presented in the introduction.

When we play the GoGAR we help each other out with discerning what follows implicitly from statements by normatively assessing and using each other's claims, which means that "playing [the GoGAR] well [is] more than a matter of an individual player's competence: to play the game of giving and asking for reasons well, one needs a team." (Bransen, 2002, p. 387). To play the GoGAR well means that the team manage to discern (at least some of) what implicitly follows from a claim and that the team have a set of reasons that could be used to back up or question other claims with. For instance, if player A questions player B's claim, she might ask B for reasons for it to find out if B has good reasons for her claim. Giving reasons for a particular claim is to interconnect it to other claims or statements. If player A thinks that player B has good reasons for her claim player A endorse it and use it in her own reasoning. Players assess interconnections between claims according to what they find to be good, appropriate ones. What interconnections players know of and find appropriate depends on our experiences of participating in various social practices and from previously played GoGARs. This dimension of the GoGAR concerns the *how*-questions of our endeavour presented in the introduction.

Children in a language diverse classroom participate in multiple, sometimes blurred social practices, which means that "[a] critical perspective [on mathematics] cannot develop without espousing children's social and cultural backgrounds as well as their personal ways of knowing and learning as they develop and grow through experiencing the particular political contexts of living and work." (Chronaki et al., 2015, p. 150). The formation of students' critical reasoning about mathematics and society, in a language diverse classroom embracing students' personal ways of knowing is intertwined with the socially and culturally normative inferences inherent in language(s). Radford (2012) claimed that "[i]n the students of multicultural classrooms we [...] find mathematics (as a plural noun) that speak about different worlds, even if the mathematics is expressed in the same official language. We do not produce accents when talking only. We also produce accents when thinking." (p. 341). When the students reasoned about Arvid's statement, "Mathematics is bad for society", they drew on their personal ways of talking and thinking, using their own particular "accents" that are influenced by the inferences they make from concepts to other concepts. As students in language diverse classrooms, play the GoGAR and assess interconnections between claims their "accents" guide their assessments as well as the interconnections they make.

The Present Study: Classroom Context, Participants and Methods

Embedded in Arvid's claim "Mathematics is bad for society" lies the idea that mathematics acts in society and that it is not value-free as Skovsmose, (2007) pointed out. Therefore, this very

statement was taken as a starting point for group and whole class explorative talks (Mercer, Dawes, Wegerif, & Sams, 2004). In groups of 4 or 5, students were asked to talk and reason about the statement. The outcomes of each group discussion were elaborated on in whole class talks orchestrated by Ulrika. She drew a concept map on the board to summarize the students' findings. The orchestration of the 40-minute plus 20-minute lessons² organised by Ulrika is shown in the flow chart below.

Picture 1. Flow chart of lesson orchestration.

The whole class talks were inspired by interactive, talk-based teaching (Hufferd-Ackles, Fuson & Sherin, 2004) in which practices of explorative talks are characterised in a way that:

- all relevant information is shared;
- all members of the group are invited to contribute to the discussion;
- opinions and ideas are respected and considered;
- everyone is asked to make their reasons clear;
- challenges and alternatives are made explicit and are negotiated;
- the group seeks to reach agreement before taking a decision or acting.

(Mercer, et al., 2004, p. 362)

was used to provide good chances for students to make, question, use and interconnect each other's claims synthesizing knowledge bases to unpack the meaning of "Mathematics is bad for society".

Arvid's classroom was situated in the south of Sweden in a suburban community close to two cities, which most residents commute to for work. The school authorities of the municipality incorporated both middle class and socio-economically deprived areas. In this specific grade 5 school³ class some students travelled to school from homes located in the deprived areas, a choice made by their parents to avoid the rowdy local schools. Most students had Swedish as their mother tongue, but over a third of them (8) claimed to have various degrees of access to languages other than Swedish, in total eight different ones (Albanian, Arabic, Bosnian, Hebrew, Kurdish, Norwegian, Persian, Polish and Serbian). The formal language of teaching and learning is Swedish. Despite the fact that about 20% of the students at the school had one or both parents born abroad the Swedish-only-discourse (Norén, 2010; Norén & Andersson, 2016;

² The lessons were video recorded using a Lessonbox[®], i.e. a set of three cameras and three microphones.

³ For ethical reasons school location and students' names in the paper are pseudonyms.

Skog & Andersson, 2014) was strongly operating. For instance, there were no signs or books in other languages than Swedish, and other languages than Swedish were seldom heard in the corridors, classrooms or at the school yard.

The short descriptions of **Samir**'s and **Elsa**'s backgrounds below exemplify how several of the students in Arvid's class had cultural, social and linguistic backgrounds that were complex and blurred. **Samir** spoke Arabic, Hebrew and sometimes Kurdish at home. His parents were well-educated refugees who arrived from the Palestine about 2.5 years ago. **Elsa**, a girl with divorced parents altered on weekly basis between living with her well-off CEO father who spoke Serbian and came to Sweden during the Balkan war, and her Swedish working-class mother who lived in a socio-economically deprived area. The students were used to group and whole class talks during math class since that was part of their teacher's pedagogy. From individual interviews Ulrika learnt that they were not used to talking about mathematics critically.

To grapple with the students' reasoning about critical interconnections between mathematics and society we attend to *what* they inferred from "mathematics" and "society" in the claim "Mathematics is bad for society". We also analysed inferential interconnections, which were used to articulate critical aspects of mathematics in society. Critical instances chosen for analysis were when students made explicit aspects of mathematics in society following from "Mathematics is bad for society". Analysis of the critical instances pivoted around the questions below.

- What inferential interconnections were discerned when the students played the GoGAR about the statement, "Mathematics is bad for society"?
 - How were those inferential interconnections used as premises and consequences in claims about mathematics in society?
 - How did those interconnections articulate critical aspects about mathematics in society?

Moreover, we analysed some transcripts that illustrated some episodes on *how* the students express, treat and (dis)acknowledge claims as they reasoned.

- How were attempts to play the GoGAR well made, e.g., (how) did the students help each other out with discerning what follows implicitly from statements by normatively assessing and using each other's claims?
 - What were the outcomes of those attempts?

In the following section, we account for the results of our analysis.

Reasoning about 'mathematics in society'

When Ulrika wrote "Mathematics is bad for society" on the board there was a little surprised fuss since the students were not used to talking about mathematics as something bad. However, quickly the students engaged in the group and whole class talks with curious interest.

Students' inferential interconnections between mathematics and society

In this section, we elaborate on how the students reasoned about society and mathematics and how they located mathematics within their reasoning about society and vice versa. Below is a picture of the concept map drawn on the board to summarize the students' reasoning (picture 2) and an English translation of it (picture 3), which provided the empirical material for the analysis.

Picture 2. Concept map on the white board showing the inferential interconnections that the students made.

Picture 3. An English translation of the concept map on the white board showing the inferential interconnections that the students made.

To make explicit what follows implicitly from using the concept “society” in the statement, the students discerned inferential connections focusing the environment; “school, house, trees, where we live, pollution”. They placed “society” as inferentially related to humans as social beings; “us”, “humans shape it”, “where we live” and to humans’ power distributing actions; “we change it together”, “what we do”, “we affect society with our pollution”.

When they reasoned about “mathematics” as used in “Mathematics is bad for society”, the students discerned connections to do with mathematics as actions; “problem-solving”, “measure”, “learning”, “comparing”, “coding”, “building” and conceptual constructs; “time”, “money”, “weight”, “volume”, “numbers”. They also located mathematics to physical objects; “work-places”, “shops”, “machines”, “recipes”.

The students’ derived inferential interconnections put on the board revealed a stereotypical middleclass jargon about society as something an undefined “we” (the students?) participate in, have the agency to impact and are obliged to care for environmentally. When the students’ knowledge about mathematics interconnected with their knowledge about society, they made inferential interconnections showing that mathematics operates as means for the working, consuming and problem-solving citizen in such a society. Hence, their reasoning reflects and reproduces functional mathematical literacy but lack critical aspects of mathematics in society. This is precisely why it was important for the students to reason about Arvid’s claim “Mathematics is bad for society” - it troubles mathematics as merely functional.

Further, the articulated prevailing ideas about society and mathematics did not appear to accommodate the students’ linguistic, cultural and social diversity. The case was rather that diversity seemed silenced. One of Ulrika’s hopes when designing the mini-project was that talks about “mathematics as bad for society” might disrupt and trouble stereotyped images and allow for complexity and plurality to emerge in the students’ talks as suggested by Chronaki et al. (2015). Despite that, the mind map on the board did not show conflicting ideas or culturally diverse images speaking about different worlds using various “accents” (Radford, 2012).

There could be several reasons for this being the case. Small group talks preceded the whole class one. From those talks, a group spokesperson summarized each group contribution to the mind map. In the small group talks ways of applying concepts, in this case society and mathematics, were normatively assessed by the students it is likely that the Swedish-only-discourse excluded linguistic and cultural troubling, while the students assessed them as normatively inappropriate in the group talks. Hence, the group spokesperson did not report them in the whole class talk. Language, social and cultural diversity did not appear to be norms that regulated the talks. The fact that Ulrika is a middle class native Swedish speaker and a former mathematics teacher with many years of teaching experience shapes the ‘accent’ she used to understand and account for the students’ summarizations, which adds to the complexity of the mind map making. The episode below illustrates some complexity that emerged in the classroom as Ulrika grappled with making the concept map.

Attempts to play the GoGAR well – episode 1; Overbridging different ‘accents’

The excerpt⁴ below shows how Ulrika struggled to grasp what Aldrin, a speaker of Persian and Swedish, wanted her to add to the map about the concept “mathematics”. Darko, who spoke Serbian and Swedish, assisted their conversation by using Ulrika’s Swedish formal mathematical ‘accent’ (turn 3; “equations”, turn 10; “assignments”, turn 13; “problem-solving”) to interconnect or overbridge Aldrin’s claims and Ulrika’s grasping of them. By the

⁴ In line with Andersson & le Roux (2017) suggestions about ethical research writing about the Other, we account for the students talk in Swedish with an English translation provided by the authors.

help of Darko, the three interlocutors formed a team that managed to play the GoGAR so that they at least seemingly could discern enough of what follows from Aldrin's claim in turn 1 for Ulrika to grasp it, namely that we have numbers at all, is due to mathematics.

Excerpt 1			
1	Aldrin	Ja ett tal med ett tal det är lika med någonting. Det är matematik.	Yes one number together with another number equals something. That is mathematics.
2	Ulrika	Räknesätt menar du?	An operation of arithmetic you mean?
3	Darko [interrupts]	Ja det är ekvationer.	Yes it is equations.
4	Aldrin	Räknesätt är ju plus minus och sådant.	Operations of arithmetic are plus and minus and such stuff.
5	Ulrika	Ja, det var det du menade?	Yes is that what you meant?
6	Aldrin	Att man tar ett tal och ett annat tal...ja typ som ett räknesätt	That you take one number and another number...yes well kind of like operations.
7	Ulrika	Eller är det inte det du menar?	Or is that not what you meant?
8	Aldrin [hesitant]	Ja...	Yes...
9	Ulrika	Nu förstår jag inte riktigt vad du menar.	Now I do not quite understand what you mean.
10	Darko	Uppgifter.	Assignments.
11	Ulrika [to Aldrin]	Du löser uppgifter...?	You do assignments...?
12	Aldrin	Det är som det...	It is that which it...
13	Darko	Problemlösning	Problem-solving.
14	Aldrin	Det som... [OHÖRBART]	That which... [UNHEARABLE]
15	Ulrika	Var det det du menade?	Is that what you meant?
16	Aldrin	Typ	Kind of.
17	Ulrika	Typ fast inte riktigt. Det är jag som inte fattar vad du menar. Säg en gång till så ska jag försöka se om jag förstår hur du menar.	Kind of but not quite. It is I who do not get what you mean. Say it once more and I will try to see if I can get what you mean.

18	Aldrin	Typ att man tar ett tal med ett annat tal så lägger man ihop det och så..	Kind of that you take one number and add it to another number and then...
19	Ulrika	Alltså <i>att</i> det finns tal som man kan använda och lägga ihop och så?	That there <i>are</i> numbers which we can use and add and so on?
20	Aldrin	Ja!	Yes!

Episode 1. Overbridging different ‘accents’.

While the above interaction was ongoing, Ulrika found herself in a dilemma. On the one hand, to grasp Aldrin’s claim, which she thought might be unclear not only to herself but to other students in the classroom as well, she needed to find reasons for it. Specifically, in turns 2, 9, 11, 17 she implicitly tried to elicit what reasons Aldrin had for his initial claim (turn 1). On the other hand, while doing so she realized that she assessed the goodness of inferential interconnections Aldrin brought forward using her own middleclass mathematics teacher ‘accent’, for example when she claimed not to understand what Aldrin said in turns 9 and 17.

When Ulrika asked the students for reasons for their statements in order to grasp and comprise them to put them on the board her assessment of the claims shaped what were put into the concept map. This resulted in Ulrika acting quite the opposite of her aspiration of allowing the students to disrupt and trouble stereotyped images about mathematics and society and to let complexity and plurality emerge. Hence at a close look, teachers’ good intentions of espousing students’ diverse ‘accents’ or personal ways of knowing and talking about mathematics (Radford, 2012) and society appears to be a complex matter. However, students’ interrupting solidarity actions in the vein of Drako’s ‘accent’ overbridging attempts can underpin such espousing.

Students deriving inferences from “Mathematics is bad for society”

In this section we analyse four student claims to highlight how the students discerned what could follow implicitly from the statement “Mathematics is bad for society” to articulate critical aspects of mathematics as power distributor in society.

Felix’s claim

Felix said, “Man kan ha programmerat fel på trafikljusen så att det blir [OHÖRBART] krsch”. [“Someone could have made a coding mistake with the traffic lights, so it gets [UNHEARABLE] krsch.”]. Felix used inferential interconnections, conceptualized as humans in action, driving cars and mathematics as actions materialized as coding located in physical objects i.e. traffic lights. Making those interconnections unveiled Felix’s awareness of mathematical algorithms operating behind the switching traffic lights. Algorithms designed by constructors who intentionally command the traffic and in the end hold power over people’s lives. Thus, he made inferential interconnections between the statement “Mathematics is bad for society” and critical aspects of black boxed mathematics (Gellert & Jablonka, 2009).

Edin’s claim

Edin said: “Man kan räkna ut något fel och då kan det bli dåligt för samhället.” [“You can make a calculus mistake and then it can be bad for society.”]. As premises for his claim, Edin used inferences interconnected to humans as social beings who’s actions affect each other.

Mathematics in his claim, is related to systems of thought, calculus. Edin's claim made explicit constructors distributing power using mathematics. He does not place mathematics as neutral actions (Skovsmose, 2007) nor as neutral systems of thought (Kollosche, 2014).

Elsa's claim

Elsa said, "...på sjukhusen finns maskiner och om de blir felprogrammerade då så kanske de gör något som är farligt för människorna eller typ skadar dem." ["... at the hospitals there are machines and if they get wrongly programmed they might do something which is hazardous to people or kind of hurt them."]. We interpret Elsa's claim as if it is erroneous in-data ("wrongly programmed") put into the hospital machines by operators that causes the harm and not the algorithms themselves (as in the case with the traffic light). Elsa recognized that operators put information into machines and from that information, the machine "makes decisions". Thus, there is human action on two levels (constructors and operators) involved in the decisions of the machines. Elsa used inferences interconnected to society as physical places (hospitals) and human actions (giving care). She relates mathematics as inferentially located to physical objects (machines) and conceptual constructs (data to be put into the machines).

Aldrin's claim

Aldrin said, "Jag tänker så här att om man kör bil och sen så 20 [OHÖRBART] 20 på den där vägen och så får man köra 70 då måste man kunna matte..." [I think like this that if you drive a car and then 20 [UNHEARABLE] 20 on that road the speed limit is 20 and you may drive 70 you need to know maths..."]. Aldrin's claim places mathematics inferentially interconnected to regulatory power that shape citizen behaviour. He recognized that constructors, who controls traffic behaviour uses mathematics to place individuals as regulated subjects (Kollosche, 2014). Precisely how his claim was given as a reason for "mathematics being bad for society" is unclear. Perhaps he would prefer no speed limits. If so, his claim could be interpreted as an objection to the regulatory powers of mathematics. Aldrin's claim was inferentially related to societal distribution of power. He inferentially interconnected mathematics to conceptual constructs to do with time and distance, in other words speed.

The analysis reveals how the students provided reasons for why "mathematics is bad for society" as a matter of articulating inferential interconnections that showed how mathematics acts and is made to act in society. For instance, mathematics operates as a regulating power in people's lives in Felix's and Aldrin's claims. In Edin's and Elsa's claims, mathematics is made to act when hospital employees use machines or when calculus is used in general. Edin's and Elsa's claims show that mathematics in society distributes responsibility. In Elsa's claim lies implicit that hospital machines make decisions rather than humans, hence the machines will be held responsible in case of an accident. Edin's claim shows that humans rely on mathematics to tell the 'truth', that is unless a calculus mistake has been made. In Edin's case, mathematics is a trusted 'truth'-teller (who is held responsible) when things go right. Should things go wrong, i.e. should mathematics not tell the 'truth', then it is a human calculator who is responsible for the 'mistake'.

Above, the students used their own world observations to reason about mathematics in society in contrast to teachers providing students with *their* world observations for the students to

reason about. We argue that critical mathematics education does not only mean that students engage in talks on social issues formulated as mathematical problems or approached mathematically by the help of teachers (see for example, Andersson, 2011; Barwell, 2013; Frankenstein, 1990; Gutstein, 2006, 2016). In addition, we think that critical mathematics education means providing classroom activities where students put to the fore their own world observations and reason about them as a matter of interconnecting them to knowledge on mathematics in action in society. Moreover, as the students' claims above show, such activities provide opportunities for students to disrupt discourses in and about mathematics as merely a value-free functional tool, be it used to engage in issues for social justice. At the same time, such activities may highlight students' personal 'accents', shaping and sharing ways of talking and thinking about critical aspects of mathematics in society.

Attempts to play the GoGAR well – episode 2; Peers struggling to make grasp each other's claims

As the students started the group talks focusing on “society” in the statement “mathematics is bad for society” Samir, an emergent Swedish-speaker, claimed “I know what society is. It is my brother's school.”. None of the three other students in the group responded to Samir's claim or used their own claims. For a while Samir was silent. Then he re-entered the group talk telling the other students that he did not understand what “society” is. Excerpt 2 below shows how the group talk proceeded thereafter.

Excerpt 2			
9	Samir	Jag fattar inte vad samhälle är.	I don't get what society is
10	Darko	Samhälle det är Fältvidda...det är ett samhälle...samhälle är som en ort.	Society it is Fältvidda [the community where the school is located]...that is a samhälle [in Swedish the same word, samhälle, is used both for society and community) ...samhälle is a place.
11	Greta	Jag tycker att det är människorna runt omkring en som innebär ett samhälle.	I think that it is the people around you that means a society.
12	Alla/All	Ja, ja.	Yes, yes.
13	Samir	Kan ni förklara för mig?	Can you explain to me?
14	Greta	Samhälle det är alltså typ så här jag tänker att det är människorna runtomkring en som bildar ett samhälle som är en del av samhället.	Society it is kind of I picture it as the people around you together make a society which is part of the society.
15	Samir	Jag tycker samhälle är en skola som min bror går.	I find that society is a school that my brother attends.
16	Greta	Ok [SLÅR UT MED HÄNDERNA]	Ok [PUTS HER HANDS OUT]
17	Darko	Tror du att det där är samhälle? [SKAKAR PÅ HUVUDET]	Do you think that is society? [SHAKES HIS HEAD]
18	Samir	Det är gymnasiet.	It is upper secondary school.
19	Darko	Solajmon [SLANG?]	Solajmon [SLANG?]

		TYSTNAD	SILENCE
20	Oscar	Ja men ja det Greta sa.	Yes, but yes that what Greta said.
21	Samir	Vad sa du? [TITTAR PÅ GRETA]	What did you say? [LOOKING AT GRETA]
22	Greta	Att människorna är... [BLIR DISTRAHERAD AV DARKO]	That the people are... [GET'S DISTRACTED BY DARKO]
23	Samir	Människorna är...?	The people are...?
24	Greta	Det är människorna runt omkring en som bildar ett samhälle.	It is the people around you who form a society.
25	Samir	Jag fattar inte.	I don't get it.
26	Darko	Samir Samir du ser de husen där borta?	Samir Samir do you see the houses over there?
27	Samir	Jepp jepp.	Yeah, yeah.
28	Darko	Det är samhället...Fältvidda är ett samhälle.	That is society/community...Fältvidda is a society/community
29	Greta	För det är människor i Fältvidda så bildar de ett samhälle.	Because there are people in Fältvidda they form a society.
30	Samir	Fältvidda är ett samhälle. [HOPPAR SITTANDE UPP OCH NER PÅ SIN STOL]	Fältvidda is a society. [JUMPS UP AND DOWN SITTING ON HIS CHAIR]

Episode 2: Samir and his peers struggling to grasp each other's claims.

We find this an attempt to play the GoGAR well so that Samir and his peers could discern what follows implicitly from each other's claims in order to grasp them.

In line 9, 13, 21, 23 and 25 Samir urged his classmates to give reasons for their claims and in line 10, 11, 14, 24, 26 and 28 they tried to do so. The problem was that they did not manage to use Samir's claim about his brother's school as premises or conclusions in their own claims and vice versa. Samir's claim is inferentially related to the upper secondary program that Samir's brother attended, which is called "Samhällsprogrammet" [Social science program]. Literally this translates "Society program", which explains Samir's attempts to inferentially interconnect his brothers school program with his peers' claims about 'society'. However, the rest of the group members did not find it appropriate to make inferential interconnections between somebody's school program and what can be inferred from 'society', according to their previously played GoGARs. Samir, on the other hand, did not appear to assess the rest of the group members claims as appropriate, though it might seem so. In line 30 Samir says "Fältvidda is a society" which could indicate that he had assessed Darko's claims in line 10 and 28 as appropriate. However, Samir jumps up and down sitting on his chair uttering the words in a chanting style indicating that he was repeating Darko's words rather than undertaking the conceptual meaning of his claim.

The missing inferential interconnections, the inferences from claims in line 10, 11, 14, 24, 26 and 28 to Samir's claim about his brother's school were not made explicit. This made the group unable to provide Samir with inferential interconnections which would be useful for him in

order to place his own and his peers' claims interconnected to each other and vice versa. For the students to play the GoGAR well, a player who's 'accent' comprises the missing inference to overbridge the claims needs to enter the game to help them discern what follows implicitly from their different claims so they can be inferentially related to each other. In the previous episode Darko's 'accent' comprised the inferences needed to overbridge Aldrin's and Ulrika's claims. In this episode, such an 'accent' was missing, which could be a reason for the unsuccessful outcome of the students' attempts to play the GoGAR well.

Attempts to play the GoGAR well – episode 3; A peer gives reasons for an emergent Swedish-speaker's claim

In line with explorative talk (Mercer et al., 2004) Ulrika invited the students to challenge each other's claims. Samir disagreed with a peer who claimed that cashiers who operate tills use mathematics. Darko, sitting next to Samir in the classroom, entered the whole class talk right after Samir's objection. In the video recording, it was possible to hear the two of them engaging quietly in an intense conversation right before Samir made his objection. What they said was unfortunately not hearable in the recording. It is likely that it was reasons for Samir's objection they discussed.

Excerpt 3			
1	Ulrika:	Jag frågar om det är någon som vill protestera jag frågar vad ni tycker.	I ask you if somebody would like to object I ask what you think.
2	Samir:	Det här som vi sa jag tycker inte att alls det är så.	This what we said. I do not think that is how it is.
3	Darko:	Affär för där har man ju en dator som räknar ut åt en man räknar ju ut på en dator man scannar ju så får man talet där så plussar man så så har man ju en dator som räknar ut det istället	Shops, because there you have a computer that counts for you. You count on the computer. You scan and you get the number and you add so so you have a computer to do that for you instead.
4	Ulrika:	Och då är det inte matematik menar du eller?	And then that is not mathematics you mean?
5	Darko:	Ja	Yes

Episode 3: Samir makes a claim and gets entitlements to it through support from a peer.

Darko, who was born in Sweden, immediately gave reasons for Samir's (who was an emergent speaker of Swedish) claim using previously articulated inferential interconnections between mathematics and shops and computers. While doing so Darko acted as a carrier of reasons for Samir's claim and as such he made it possible for Samir to provide good reasons for his initial claim. Should Darko refrained from giving reasons for Samir's claim, it could have been questioned and Samir would risk not being able to give good reasons for it (in Swedish). Furthermore, the GoGAR would not have been played so well as Samir's contribution to the unveiling of inferential interconnections would have been left unarticulated. As it turned out, Samir's claim, for which reasons were articulated by Darko, was questioned by other students. Hence, yet more reasons for it were given and thus inferential interconnections between Samir's and other students' claims were discerned.

In this episode, students discerned conflicting reasons for locating mathematics either in people's head or in machines. Finally, they were able to discern good reasons for claiming both locations. The analysis shows attempts of playing the GoGAR well when a peer acted as a carrier of reasons, which to some extent overbridged the skew distribution of language access caused by the Swedish only discourse (Norén, 2010; Norén & Andersson, 2016; Skog & Andersson, 2014).

Closing remarks

Of course, the small-scale project analysed in this chapter do not equip the students with means to format the interconnections between knowledge bases that Gutstein (2016) discusses. However, we find that it allowed the students to locate mathematics and society as interconnected in order to grasp the political role of mathematics in society, i.e. to be critically mathematical literate. If students cannot place mathematics and society as interconnected, teachers' efforts to engage students with critical mathematics activity might risk becoming nothing but another mathematics assignment to students. For the students in Arvid's classroom to make inferences between mathematics and society meant that they not only had to unpack implicit meanings of mathematics in society they had to make displacements from prevailing ideas/discourses about mathematics as being neutral or doing good in society towards disturbing the very same ideas. It requires courage and the confidence and language competence to articulate and unpack such ideas – a complex challenge particularly in a language diverse classroom. In our study Darko appears to be aware of that challenge since he displayed a meta-understanding of language diversity and acted accordingly by for instance overbridging Ulrika's and Aldrin's 'accents' and by giving reasons for Samir's claim.

As students and teachers grapple with unpacking critical aspects of mathematics in society in a language diverse Swedish only classroom, their reciprocal assessment of claims based on their 'accents' of knowing and talking about mathematics and society, shape both their actions and the outcome of their efforts. Hence, the critical aspects of mathematics in society that come to the fore in a language diverse mathematics classroom are intertwined with students' and teachers' (lack of) meta-understanding of language diversity and attempts to make their ideas explicit in order to put forward and grasp each other's claims. As normative assessment guides the outcome of their attempts, prevailing power is present in the reasoning. In other words, the semantic *whats* are dialectically entangled with the social and power related *hows* in the complexity of addressing and enhancing critical mathematical literacy in a language diverse classroom. We find that further research that address this complexity is needed to deepen and widen the understanding of how critical mathematical literacy becomes formatted in classrooms like Arvid's.

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III

Mathematical preciseness and epistemological sanctions

Ulrika Ryan

Precision is important in mathematics. What happens when a mathematics student is not as precise as expected? I am especially interested in how students in social and language diverse contexts relate to mathematical preciseness.

In this article, I examine how a focus on preciseness in the mathematics classroom could affect group activity. The preciseness I am interested in relates to the ways mathematical concepts, in this case angles, are described in discourse between students. In this context, I consider how ‘micro-invalidations’, seemingly minor but potentially damaging criticisms, can limit students’ opportunities to learn, particularly in a social and language diverse classroom. This article is about Samir, an eleven-year-old, multilingual emergent speaker of Swedish, which is the language of learning and teaching in his classroom. Samir and three of his peers were working on a group activity on angles. Samir entered the group activity confidently saying, “How good I am” when solving a task with his peer Darko. However, at the end of the activity Samir’s talk about himself was completely changed from self-confidence to insecurity. Submissively he begged his peer Darko to rely on his mathematical knowledge saying “Forgive me...please trust me.” During the group activity, Samir was exposed to micro-invalidations. At the end of the activity, Samir’s peer Darko do not trust Samir’s ability to make reliable mathematical claims any longer. Samir has lost the authority to make mathematical claims of knowledge. His mathematical claims have become invalid in the eyes of his peers. I explicate the idea that Samir’s loss is linked to discourses on (Western) mathematics that embrace mathematical rigidity and preciseness.

Mathematical concepts used in formal mathematics are rigid, based on absolutist, axiomatic conceptions. Their use elicits discourses that comprise preciseness and certainty, acknowledged aspects of the Romance of (Western) mathematics (Lakoff & Núñez, 2000). Such discourses influence school mathematics. Preciseness impede flexible conceptions of mathematics and evokes ideas about mathematical rigidity. This resembles an absolutist view

of mathematics that reject mathematics as a cultural product. Albeit the absolutist approach has been challenged, for instance by Lakatos, it is not always easy to see the cultural roots of mathematics. Prediger (2002) argues that it is (paradoxically) the human desire for certainty that dehumanizes mathematics. In brief, it works like this according to her; to be certain about mathematical propositions mathematicians use a strong practise of coherence and consensus. That is, mathematicians agree on what is appropriate and what is not appropriate mathematical propositions. By the time that their agreements reach the layman, this process is hidden. The mathematicians' high coherence and the wide consensus has obscured the human dimension and cultural origin of mathematical propositions, hence making them appear impartial.

In the realm of the mathematics classroom aspects of preciseness, rigidity, coherence and consensus play out as a matter of being right or wrong. Although, as pointed out by Wagner (2009), it is when the act of forbidding the “wrongs” is being challenged that significant mathematics might emerge, classroom mathematics is more often about fostering consensus on what is easily assumed as “rights” (deFreitas & Sinclair, 2014). Therefore, putting forward mathematical claims of knowledge can be a risky business because the chance of being “precisely wrong” is at stake. Being wrong may expose the claimer to micro-invalidations. Students whose claims repeatedly are invalidated are at the risk of ceasing to perceive themselves as potential participants in school mathematics practices (Andersson, Valero & Meany, 2015). Judging claims based on whether or not they are to be acknowledged as claims of knowledge is an epistemological matter. Therefore, the micro-invalidations that Samir are exposed to I define as small epistemological sanctions.

Multilingual students' participation in verbal reasoning group activities may provide both mathematics and language learning opportunities (e.g., Moschkovich, 2015; Planas, 2011, 2014). Albeit such participation is crucial for emergent speakers of the language of learning and teaching, it is far from unproblematic. In Samir's case, it means potential exposure to epistemological sanctions, which affects the ways he speaks and feels about himself when doing and thinking about mathematics and perhaps in the long run his mathematics learning. One way of grappling with this issue is to provide language- and content-integrated learning opportunities that take into account the epistemic role of school language discourse (Prediger & Krägeloh, 2016). Another way is learning to meet the Other in the classroom. That is, learning to meet marginal and minority ways of engaging with mathematics in open and respectful ways (Guillemette & Nicol, 2016) or creating dialogical spaces (Chronaki, 2011).

Yet another way is re-thinking what kinds of discursive spaces absolutist mathematics shapes and how students (and teachers) navigate those spaces (McGarvey & Sternberg, 2009). By no means, none of these ways excludes the other.

In this paper, which draws on a previous research rapport (Ryan, 2018), my concern is to understand how discursive spaces based on an absolutist mathematics shape Samir and his peers' interactions. I argue that discourses of preciseness in mathematics may elicit micro-invalidations directed towards students who do not justify their claims in a (school) mathematically precise way. Possibly because they are not participating in the epistemic school language discourse.

I draw on the later Wittgenstein's ideas on language games (Wittgenstein & Anscombe, 2001) to grapple with Samir and his peers' reasoning about angles and Samir's talk about himself during the activity. I use the notion of I-language games and the game of giving and asking for reasons, which I account for below.

I-language games and the game of giving and asking for reasons

I-language games does not raise questions about what "I" am nor what it is to be "me", rather they depart from the question "how do I talk about me?" presupposing that I do not merely talk in one way about myself. I change my I-language games constantly depending on where I am, whom I talk to, how I feel, what I am doing, etc.

All of the language games in which I use the word 'I' or when I talk about myself are latticed. Together they form a whole and ever-changing structure. By the use of the word "I" we draw attention to ourselves. We need not explicitly be talking about ourselves; who we are or how we feel. We draw attention to ourselves also when we do and talk about other things than ourselves. For instance; while we do mathematics, we use the word I explicitly and/or implicitly. By doing so we share our "mental/psychological states, experiences, feelings, thoughts" (Beristain, 2011, p. 108) while we are busy doing mathematics. Hence, to study students' I-language games allows for approaching their ways of sharing experiences and feelings about themselves when they, for example as in the present paper, participate in a group activity reasoning about angles. This means that studying Samir's I-language games has the potential to illuminate his ways of sharing feelings and thoughts about himself in response to epistemological sanctions such as micro-invalidations. Feelings and thoughts, which might lead to stigmatization and eventually perceived marginalized membership in mathematical communities (Guitérrez, 2017).

When students engage in reasoning in mathematics class their I-language games are latticed with other language games as the *game of giving and asking for reasons* (GoGAR). The GoGAR is at the heart of inferentialism (Brandom, 1994, 2000) – a neo-pragmatic philosophical theory of language use and meaning. The GoGAR concerns people’s practical face-to-face reasoning, that is the kind of reasoning that may occur when students deal with group activities in mathematics. The GoGAR is not about how to build good arguments based on data, warrants and conclusions. Rather, it attends to how we deal with the uncertainty and anxiety of making ourselves understood and of understanding the O/other when we reason. To grapple with this uncertainty, we keep track or score of our interlocutors’ doings and sayings. This score-keeping is built around two normative statuses that we assign claims; commitments and entitlements. The idea is that when we claim that things are such and such we undertake a commitment to the claim. This does not mean that we cannot change our mind and commit us to another kind of claim the next instance. It merely means that while I am committed to a particular claim I can be held responsible for it in the sense that I can be asked if I have good reasons for it.

To keep track of if our interlocutors’ have good reasons for their claims involves assessment. Hence “there must be in play also a notion of entitlement to one’s commitments: the sort of entitlement that is in question when we ask whether someone has good reasons for her commitments” (Brandom, 2000, p. 43). To think that an interlocutor has provided good reasons for a claim means that we find the claim normatively appropriate in the realm of the social practice that the claim is caught up in. If we are satisfied with the reasons given and hence find our interlocutor reliable, we acknowledge the claim and undertake it ourselves. An entitlement is a social status that a commitment has received within a community, e.g. a mathematical community and/or a classroom community or a practice.

There are epistemological aspects to the GoGAR, while it is concerned with how interlocutors keep score of commitments and entitlements and judge what they and others take to be claims of knowledge. If we acknowledge claims, we take them to be knowledge-claims that we are prepared to use in our own reasoning. Thus, in practical face-to-face reasoning, to take something to be a claim of knowledge we assess the reasons that articulate a claim or we simply rely on the claimer to have good reasons for it, before we acknowledge it and use it as a piece of knowledge ourselves.

The assessment of claims is normative. It is shaped by the norms present in the situation. This means that there are sayings and doings that are normatively appropriate as well as

normatively inappropriate from the perspective of the meaning of the claim. For instance, claiming that an angle is 100° and acute is usually considered normatively inappropriate in a mathematics classroom. Claiming that the same angle is obtuse is normatively appropriate. If I commit myself to that claim, my claim has the social status of entitlement due to the norms of the mathematical community.

From an absolutist perspective of mathematics, norms comprising preciseness shape the assessment of mathematical claims. Imprecise use of mathematical concepts violates the norms of mathematics as precise and certain. Such violation (i.e. lack of entitlement to a claim) calls for some kind of sanction which can be internal, external or both (Brandom, 1994). This resonates with findings by for example Rowland (2000). External sanctions like exclusion, nullification or disregarding of a person's beliefs or statements are micro-invalidations which, when constituting statuses in the GoGAR, shape epistemological sanctions. Epistemological sanctions might lead to disqualification from counting as eligible to undertake commitments, a kind of authority loss. Hence, a student who fails to give reasons for a specific claim involving, for example, a mathematical concept which her/his interlocutors assess as inappropriate risks being exposed to external epistemological sanctions in the form of micro-invalidation due to the failure. Being repeatedly exposed to external epistemological sanctions could affect a student's I-language games. Changed feelings about oneself could like in Samir's case, evoke I-language games to change from positive to negative ones. In other words, "the 'crime' is cognitive but the penalty affective" (Rowland, 2000, p. 123). To avoid exposure to epistemological sanctions interlocutors can attach vagueness and uncertainty to their claims. That is, they can use shielding hedges. To use a shielding hedge is for instance to say, "*I think* that the angle is 90° " instead of saying, "The angle *is* 90° ". Using "I think..." means a weakened commitment to the claim and thus a protection from being held responsible for it. This reduces the risky business of making mathematical claims. Or, by using "I think" the claimer shows uncertainty and thereby risks undermining his/her authority as a producer of entitled knowledge claims for interlocutors to use in their own reasoning. Moreover, "I think"-talk is part of I-language games and hence affects the way a person draws attention to her- or himself.

From "How good I am" to "Forgive me...please trust me" – context and three snapshots from a group activity on angles

"How good I am [at solving mathematical problems]" was one of the first turns recorded by the device I had left at Samir's (S) and his peers Eva (E), Greta (G) and Drako's (D) desk

when I was a participant observer during a regular math class in their social and language diverse grade 5 classroom (students aged 11). In the classroom, the language of teaching and learning is Swedish. Although multiple languages are represented in the classroom, none but Swedish is heard. According to Barwell (2005) this is a common thing when the teacher, as in this case, do not share the languages of the students. In addition, in this particular class no student shares the same “other” language with a peer except for Swedish. Among the four students, Darko born in Sweden by immigrant parents speaks Serbian and Swedish at home. Samir, who arrived from the Palestine 2.5 years ago, is an emergent Swedish speaker who speaks Arabic and Hebrew at home. Greta and Eva speak only Swedish at home. During the lesson, the four students were working on two tasks that entailed drawing angles which the other pair of students were to measure and denote (task 1) and judge which is right, acute and obtuse and give reasons for their judgement (task 2). Samir and Darko formed one pair and so did Greta and Eva. Below are three snapshots¹ from the interaction that illustrate how Samir’s I-language games changes from positive to negative ones as he is exposed to epistemological sanctions when the students engage in the GoGAR.

Prior to the lesson, I interviewed Samir. He told me that he likes mathematics and that he is good at it. This attitude was occasionally shared in the classroom. In other words, Samir’s I-language games when latticed with mathematics, were usually positive.

Snapshot 1

Samir and Darko measured the angles drawn by Greta and Eva. They wrote the magnitude of the angles on a piece of paper, which they handed over to the girls. They denoted one of the angles as 80/100°, the two numbers placed above each other on a protractor scale. Greta asked them to give reasons for claiming that the angle is 80/100°.

G:	Yes, but no. It cannot be both [80° AND 100°]
S:	Yes, because they are above each other
G:	Yes, but they...it does not mean that it is the same...it [THE ANGLE] is not 100 slash 80. It must be one of them.
S:	Yes, yes...I get it, I made a mistake...where is the protractor...

¹ Each snapshot comprises a transcript that I translated from Swedish to English. Although I experienced doing the translations as a relatively forward matter, there is a translation challenge in Snapshot 3 discussed below. Moreover, I am aware that there are nuances of the languages that may influence my own as well as the readers’ focus and interpretation of the excerpts.

Samir then used the protractor to re-measure the angle. He suggested that it is 100° and Darko that it is 80° . Following Darko's suggestion, Samir wrote 80° on the piece of paper. The boys' discussion caused Greta to thoroughly give reasons, hence engage in the GoGAR, for claiming that angles are denoted using only one number and explaining how the scales of the protractor works. Though both Samir and Drako initially were committed to claiming the double denunciation, when failing to give normatively appropriate reasons for the claim and thus realizing that they were not entitled to such a claim, Darko told Samir "Why didn't you say so.", holding him responsible for the loss of entitlements. Hence, Samir was held responsible for the loss of the social status as a reliable claimer of mathematical knowledge.

Snapshot 2

The four students were occupied with task 2; drawing one right, one obtuse and one acute angle that the other pair of students were to judge which was which. Samir decided that he should draw the right angle. Right after he had finished drawing, Greta urged Eva to use the protractor to check that their right angle was *exactly* 90° . Such preciseness was not necessary while task 2 was about looking at the angles and judging which was which. Probably inspired by Greta's urge for their right angle to be precisely 90° the following turns were uttered among the boys.

- D: Well done...What degrees is it [THE RIGHT ANGLE THAT SAMIR DREW]
- S: It is...eh eh I am good at forgetting.
- D: Mmm...yes you are good at forgetting.
- S: Yes, that is why my name is Forgetty.

Darko wanted Samir to measure their right angle, i.e. he wanted Samir to commit himself to a claim of the angle being exactly 90° . Samir was just about to do so when he started to say "It is..." but appeared to change his mind and claimed instead to be "good at forgetting". Samir seems to avoid being responsible for claiming the angle to be precisely right which might (like in snapshot 1) cause him entitlement loss and exposure to epistemological sanctions.

Snapshot 3

Although task 2 was not about measuring angles that is what the students did. Greta questioned the boys' claim about the magnitude of an angle they had measured. The boys

reasoned with Greta about whether to denote it 145° or 155° . Samir claimed it being 155° , a claim that Darko initially supported.

- D: Wait [TO GRETA]...look it *is* 155.
- G: Not [1]55. [1]55 is there [SHOWING ON THE PROTRACTOR]...this is [1]45.
- S: I thought² [*trodde* in Swedish] it was...
- D: I am not going to trust you anymore.
- S: I thought [*trodde* in Swedish] it was so...
- D: You cannot just think [*tro* in Swedish] so.
- S: May I...
- D: I asked you specifically and you just said yes.
- S: Forgive me...please trust me.

Greta justified her claim by showing Samir and Darko where on the scale of the protractor 145° and 155° respectively the two angles are located. Samir apparently undertook Greta's claim and simultaneously stated that his initial claim was based on that he "thought it was" an appropriate one. To avoid being exposed to sanctions due to a possible lack of entitlement for claiming the angle to be 155° Darko dismissed Samir's claim and seemed to argue that "think so" is not enough to justify a claim, hence challenging Samir's reliability. In the last turn Samir appears to think that his reliability and thus authority to make claims that will be acknowledged and thus given the social status of an entitlement is lost and he begs Darko to forgive him and to reassign him reliability.

Throughout the three snapshots the normative appreciation of preciseness in mathematics opens up a space which makes it possible for Greta to claim that an angle cannot be both 80° and 100° . A straight angle must be measured and is precisely 90° and an angle not 155° but 145° instead of merely obtuse. This very space, inherent in Western mathematical ideas of angles and the artefacts used to measure and denote them, licence the entitlement losses Samir

² In this snapshot Samir and Darko use the Swedish word *tro* (think) and its past tense *trodde* (thought). *Tro* is one of the three possible Swedish translations of the English word think. The other Swedish words are *tänka* and *tycka*. *Tycka* refers to having a personal opinion. *Tänka* refers to mental activity. *Tro* refers to holding a belief that one is not sure about.

is held responsible for as the students play the GoGAR. From the entitlement losses follows epistemological sanctions, which makes Samir change his I-language games from positive to negative talk of being a forgetter, a wrong-doer and a person without reliability. It causes him to play negative I-language games to avoid further exposure to potential epistemological sanctions. Samir do not use shielding hedges (Rowland, 2000; 2014) in any of the snap shots. Instead, in snap shot 2 he claims to be “forgetty”. In snap shot 3 he adds uncertainty to his previous claims by saying that “he thought it was so”. This does not work out though while Darko do not seem to accept shielding hedges being attached to claims afterwards. Attaching shielding hedges is a kind of epistemic school discourse (Prediger & Krägeloh, 2016) that students use as ways of dealing with the risky business of making mathematical claims when preciseness is dictating the epistemic discourse. Either Samir do not want to undermine his authority as a producer of knowledge claims or he is not aware of the epistemic school discourse that use of shielding hedges are part of.

Moreover, the normative appreciation of preciseness appears to prevent the four students from reasoning about alternative ways of denoting angles or from exploring each other’s conceptions of angles. The concealment of mathematicians’ consensus (Prediger, 2002) is efficient in that it appears to have excluded any appreciation of dissensus (deFreitas & Sinclair, 2014). The concealment of consensus, the dehumanization of mathematics, moves the students into a space in which their pursuit of preciseness and universal certainty is possible at all.

For an emergent speaker of the language of learning and teaching to engage in mathematical reasoning activities, that is to engage in the GoGAR, in a Swedish-only classroom where preciseness guides the interlocutors’ reasoning, appears to be particularly risky business while her/his resources underpin giving reasons for claims that are diminished by mono-lingual, mono-cultural and mono-mathematical normativity. When inviting students to group activities involving reasoning, educators need to be sensitive to the exposed situation of emergent speakers and provide student awareness on potential epistemological sanctions evoked by enthroned preciseness. Although it is Samir, who is exposed to epistemological sanctions, it could in fact happen to any student, whose ways of expressing mathematical knowledge differs from the “pursuit-of-preciseness-and-certainty” normativity. I claim that emergent speakers and students whose social languages differ from the school language are at greater risk of exposure to epistemological sanctions than other students are. I elaborate on the theoretical reason for this below.

It is common to think of language and epistemological issues as separate, hierarchically ordered, hence presupposing that when the epistemological is considered the semantic meaning is already settled. That is to think that we already know the meaning of the claims we consider and that the question is merely under what circumstances they could be justified. However, according to inferentialism one cannot treat the *meaning* of a claim as already settled before the knowledge content is judged (Brandom, 2008). We consider the meaning of a claim at the same time as we consider its knowledge value. Hence, the semantic and the epistemological are inseparable elements that constantly are in flux (Derry, 2013). Consequently, the person who is in least command of the language at play in face-to-face reasoning, is potentially the one who's claims are most likely to be asked reasons for while that person's claims are at the greatest risk of being the least accurate or precise according to the norms that guide the situation. At the same time, the emergent speaker of a language (be it a social or national language) is the one who might be the least equipped language wise to give reasons for her/his claim in order for it to potentially be acknowledged by the interlocutors and thus get the status of entitlement. Hence, the claims of an emergent speaker of the language of learning and teaching are at a greater risk of being nullified or disregarded than claims of students in command of the language at play. Therefore, learning to meet the Other, to respect his/her knowledge and ways of expressing it (Guillemette & Nicol, 2016) is crucial in social and language diverse contexts. It is particularly important when preciseness guides the norms that shape the face-to-face reasoning. Learning to meet the Other and her/his knowledge when engaging in the GoGAR has the potential of saving Samir and others at risk from playing negative I-language games about themselves when reasoning about mathematics.

To close, as stated by Gutiérrez and Dixon-Roman (2011, p. 32),

“[w]e need to be constantly considering the forms of mathematics and what they seek to deal with. As society presents new demands, new technologies, new possibilities, we must ask ourselves whether our current version of mathematics is adequate for dealing with the ignorance that we have”

and what it shapes in social and language diverse contemporary mathematics classrooms. This is imperial at present as global migration is reaching new stages, diversifying local practises such as mathematics classrooms. This paper suggests that in social and language diverse classrooms, students and teachers do not merely navigate language diversity as an outcome of migration; they also navigate epistemological aspects of mathematics. Learning to meet the

knowledge of the Other, learning to avoid epistemological sanctions calls for meta understanding of language diversity which embrace epistemological aspects of mathematics.

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This thesis is an expedition into and beyond students' mathematics talk in classrooms framed by migration as a matter of dichotomization between named languages and (in)formal aspects of one fixed mathematics. It is an attempt to make sense of how students grapple with and move about the divides that those dichotomizations shapes. I ask;

- How do students in a Grade 5 classroom framed by migration navigate language and epistemological divides when talking about mathematics?
- What theoretical conceptualization of epistemological dimensions of language diversity can be used to frame the students' navigation of the divides?

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205 06 MALMÖ, SWEDEN
WWW.MAU.SE